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Approximating the Mass Moment of Inertia of Human Body

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Abstract- Mass moment of inertia (MMI) is critical in the study of motion of rigid bodies in dynamics. However, its precise determination can be challenging when dealing with complex shapes such as vehicles, airplanes, trains, and human bodies. To determine the moment of inertia, several analytical studies have been conducted and many experimental methods applied. Those experimental methods, however, require a substantial amount of work, materials, and time, resulting in high cost and danger. Therefore, the need for simple and accurate mathematical models, confirming the results of experimental methods, is warranted. The main objective of this paper is to formulate a general equation of MMI to be validated and corrected using the experimental data from Matsuo [7] and Tozeren [9]. The outcome of the application of the obtained general equation to an African university student population and to a larger set of US college students' sample is also investigated.

Keywords- Mass Moment of Inertia, Dynamics, Experiments, Models, Human body

I. INTRODUCTION

Mass moment of inertia (MMI) plays a vital role in the study of motion of rigid bodies in dynamics. However, its precise determination can be challenging, especially in cases of complex shapes, such as vehicles, airplanes, trains, and human bodies [1].

In the area of human sports, the moment of inertia is an important parameter in the kinetic analysis of human body motion. MMI allows for the prediction of the performance of athletes. Almost all human motion in sports require rotation and achieving the prescribed amount of rotation is critical to the successful completion of many desired sport skills. In sports such as ice skating, diving and gymnastics, athletes are constantly changing their body configuration, subsequently changing the entire body moment of inertia [2]. In a study conducted by Richards on young female gymnasts, it was found that higher velocities were achieved by gymnasts with lower moment of inertia values [3]. It was also observed, in a case study of finding the moment of inertia for a backward and a forward pike dive, that by increasing the radius from the axis of rotation, the moment of inertia increases, thus slowing down the speed of rotation. If athletes wish to enhance rotational speed, they can reduce the radius by moving the body's segments closer to the axis of rotation, therefore lowering the radius and mass moment of inertia [4].

To determine the moment of inertia, several analytical studies have been conducted, and many experimental methods are carried out. Three principal methods have been used to determine the human body's moment of inertia. The first method observed is the compound pendulum method. This technique helps find the mass moment of inertia of an object by oscillating the object about a fixed suspension point and defining the moment of inertia from the measurement of the period of oscillation. It has been used in a study to determine the moment of inertia of a sample of 66 living male subjects in eight different body positions [5]. The second method used by Griffiths [6] was the turntable method which consisted of rotating table where the human body was placed in different positions using a turntable and video motion-analysis equipment. And Matsuo used a third method called method of the oscillating table illustrated in Fig. 1, that was carried out on teenagers. This method established a correlation between the moment of inertia, the height, and the mass [7].

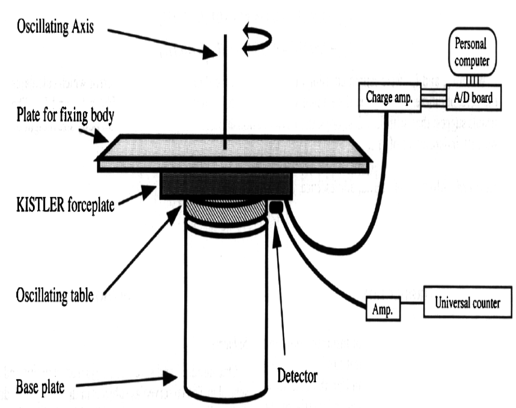

Fig. 1 : Illustration of the measurement device for moment of inertia of human body. The measurement system for moment of inertia used in this study was composed of a horizontal oscillating table, a Kistler force plate, and a platform to fix the body of the subject [7]

Experimental methods require a substantial amount of work, materials, and time, resulting in high cost and danger for the researchers and the subjects being tested [8]. Consequently, mathematical methods were used to model the results obtained from experimental work. The goal of this approach was to simplify the procedure of determining the human body moment of inertia and gain a better understanding of the values obtained. The main parameters of the mathematical model were the mass (W) and the height (H) of the subject. It is important to point out that the values of the coefficients in Matsuo's mathematical model were obtained from a Japanese population sample of adolescents. Tozeren extended Matsuo's work to a small size of US airmen population to verify Matsuo's findings and derived another mathematical model [9].

It is worth noting that none of these methods, to the best knowledge of the authors, was extended to an African sample and no general formulation was attempted.

The main objective of this paper is to use Matsuo's work in the formulation of a general and theoretical expression, apply the model to a set of African students and American students' data, and compare the results between the two sets of populations.

II. DATA COLLECTION

Two characteristics of the human body, namely the mass (W) and the height (H), were considered in the mathematical model that would help determine the MMI of the human body. In the anonymous collection of data in Africa (DR Congo), the mass of the body was obtained by using a 300 kg weight scale and the height was obtained by using a tape measure. Data was randomly collected from 44 volunteer students, whose age ranged between 19 and 21 years old. Gender was not considered in the study. The tallest subject was 1.93 m, and the heaviest subject weighed 142.1 kg. The average mass of the sample is 65,67 Kg and the average height 1.74 m.

A second anonymous set of data larger than Tozeren's three airmen was collected in US from 44 volunteer students. The tallest subject was 1.93 m and the heaviest subject 113.40 kg. The average mass and height of the sample were 78.54 Kg and 1.77 m, respectively.

III. MATHEMATICAL MODELING OF MMI

Studies have been conducted to find the mass moment of inertia more precisely using mathematic modeling. Matsuo's work [7] proposed the following equations for determining the mass moments of inertia of Japanese adolescent boys between the ages of 13 and 18:

$$I_{11} = 3.52H^2 + 0.125W - 7.78 \quad (1)$$

$$I_{33} = 3.44H^2 + 0.144W - 8.04 \quad (2)$$

Where I_{33} and I_{11} are the mass moments of inertia with respect to the center of mass around the posterior-to-anterior and right-to-left axis (kg.m²), H is the height of each boy in meters (m) as shown in Fig. 2, and W is the mass of each boy in kilograms (kg).
The axes of rotation in equations (1) through (4) are also shown in Fig. 3 below.

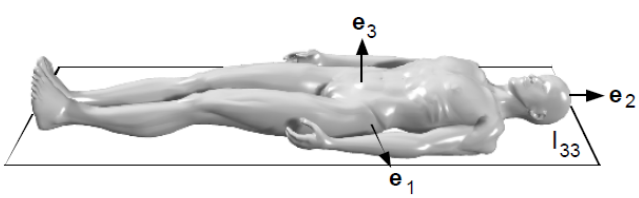

Fig. 2 : Illustration of posterior-to-anterior and right-to-left axes on human body [9]

IV. CYLINDER APPROXIMATION OF MODELING

In this paper, the human body is assumed to be a cylinder, with the axes of rotation passing through its center as illustrated in Fig. 3 below.

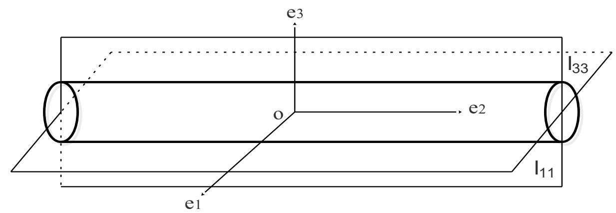

Fig. 3: Model of a human body as a cylinder

Therefore, the equation for the MMI for a cylinder is given in dynamics textbooks by:

$$I_{C11} = I_{C33} = \frac{W}{4}\left(R^2 + \frac{H^2}{3}\right) \quad (3)$$

Where I_{C11} *and* I_{C33} which are respectively the mass moments of inertia along the axes 1-1 and 3-3 for each student (kg.m²). W is the mass of each student in kilograms (kg), H is the height of each student in meters (m), R is the equivalent radius for each student (m)

It will be assumed that the equivalent radius or the radius of gyration of the body can be written as:

$$R = \sqrt{\frac{W}{\pi.\rho.H}} \quad (4)$$

Where: ρ *is* the density of the human body 1062 kg/m³ [10], R: equivalent radius (m).

Note that this model assumes Ic_{11}=Ic_{33}, because of the cylinder approximation. In reality, the human body is not cylindrical and the values of I_{11} are different from I_{33}. The model is used primarily for its simplicity.

To gain a perspective of how different they were, a percentage errors was calculated for each student using the following equations:

$$Error(\%)_{33} = \left|\frac{I_{33} - I_{C33}}{I_{33}}\right| \times 100 \quad (5)$$

$$Error(\%)_{11} = \left|\frac{I_{11} - I_{C11}}{I_{11}}\right| \times 100 \quad (6)$$

Then the average of the results was taken to obtain the percent errors of 36.6 % on the 3-3 axis and 45.2 % on 1-1 axis. To reduce this margin of error, an adjustment of the cylinder formula was investigated using the following ratio:

$$\frac{I_{33}}{I_{C33}} = \alpha \quad (7)$$

$$\frac{I_{11}}{I_{C11}} = \beta \quad (8)$$

The values of the computed ratios were $\alpha = 0.749$ and $\beta = 0.66$. It was noted that those were close respectively to the values of $\frac{3}{4}$ and $\frac{2}{3}$. Therefore, the revised equation for the mass moment of inertia of a cylinder is:

$$I_{cc11} = \frac{2}{3} \times \frac{M}{4}\left(R^2 + \frac{H^2}{3}\right) \quad (9)$$

$$I_{cc33} = \frac{3}{4} \times \frac{M}{4}\left(R^2 + \frac{H^2}{3}\right) \quad (10)$$

At this stage, after correction of the cylinder model, the errors are of the order of 3.2% along the 1-1 axis and 2.5% along the 3-3 axis. These acceptable error margins allowed the authors to validate the corrected cylinder model.

However, because equations (11) and (12) contain an additional term R, a direct comparison cannot be made with equations (1) through (4). Assuming the same second-degree trend in H for the mass moment of inertia of the African population, the expected MMI equations will be of the form:

$$I_p = aH^2 + bW + c \quad (11)$$

With constants a, b, c to be determined using any three points in the African data set and solving the ensuing system of three equations and three unknowns. Therefore, the equations for the African set of data are the following:

$$I_{p11} = 3.7H^2 + 0.17W - 11.74 \quad (12)$$
$$I_{p33} = 4.13H^2 + 0.19W - 12.74 \quad (13)$$

Repeating the same process with the US larger set of data, the following expressions are obtained:

$$I_{pa11} = 4.33H^2 + 0.18W - 14.24 \quad (14)$$
$$I_{pa33} = 4.87H^2 + 0.21W - 16.02 \quad (15)$$

V. Results

Figs. 4 and 5 represent the moments of inertia between the Matsuo model and the uncorrected cylinder model. A strong deviation between these two models is observed, which is due to the approximation of the human body as a perfect cylinder.

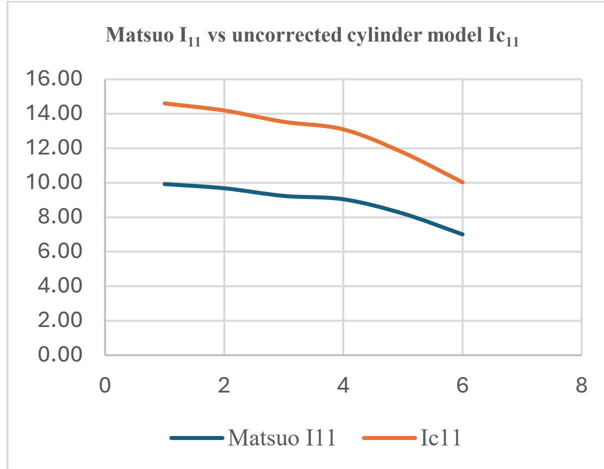

Fig. 4 : Comparison of Matsuo and uncorrected cylinder model's Moment of inertia along axis 1-1

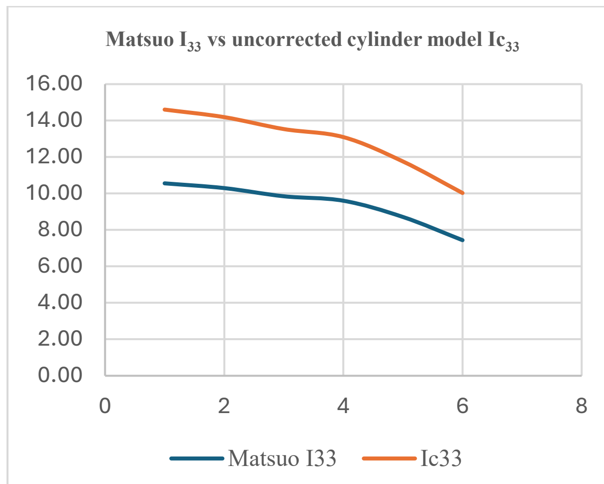

Fig. 5 : Comparison of Matsuo and uncorrected cylinder model's Moment of inertia along axis 3-3

Figs. 6 and 7 represent the moments of inertia along the 1-1 and 3-3 axes between the Matsuo model and the corrected cylinder model.

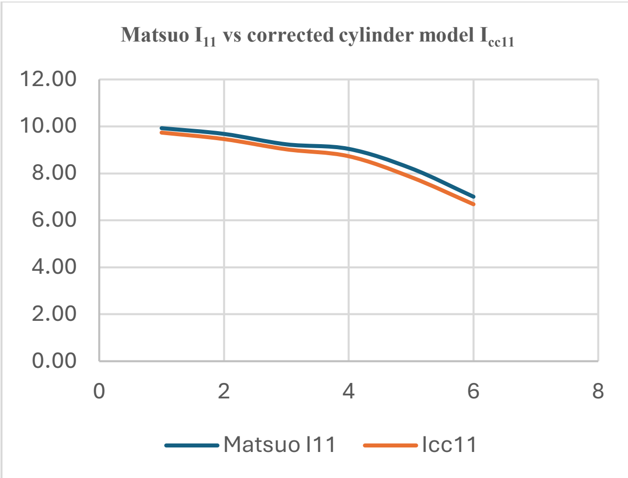

Fig. 6 : Comparison of Matsuo and the corrected cylinder model's Moment of inertia along axis 1-1

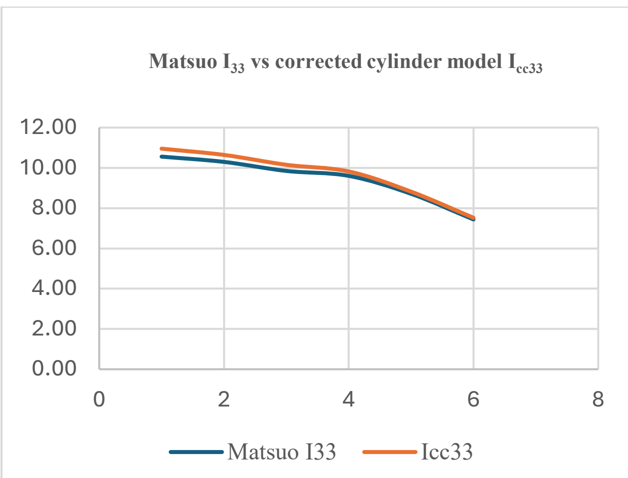

Fig. 7 : Comparison of Matsuo and the corrected cylinder models

The corrected model was transformed to propose models that determine the moment of inertia of students at New Horizons University (UNH) in Africa and students at Metropolitan State University of Denver (MSU) in the United States. The moment of inertia of these populations was compared. Figs. 8 and 9 show the trend of the moments of inertia along the 1-1 and 3-3 axes of both UNH and MSU students. It is noticed that the moments of inertia of MSU students are higher than those of UNH students as could be anticipated by comparing the average values of the mass and the height of the samples.

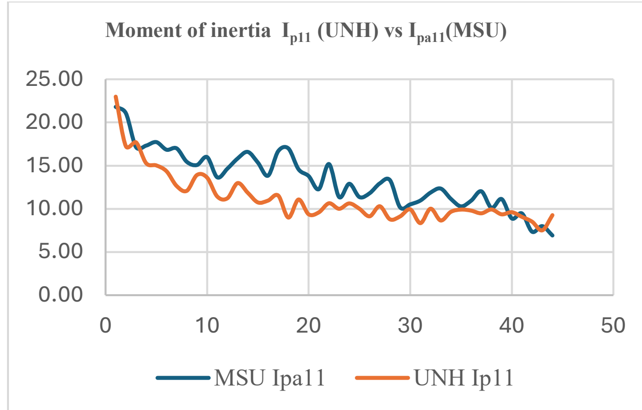

Fig. 8 : Comparison of UNH and MSU students' moment of inertia along the 1-1 axis

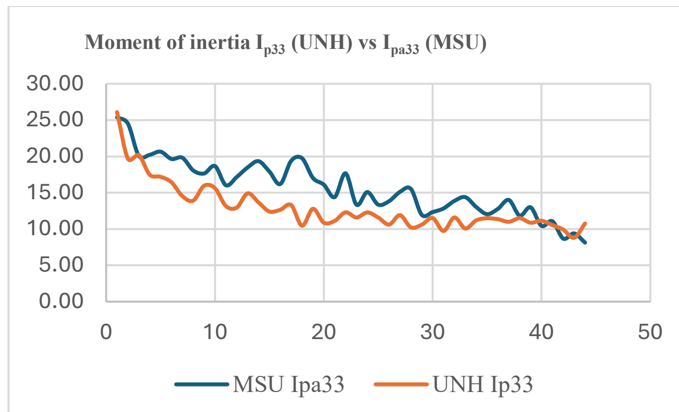

Fig. 9 : Comparison of UNH and MSU students' moment of inertia along the 3-3 axis

Table 1 and Table 2 show samples of the data collected (weight and height) respectively on UNH and MSU students, the uncorrected cylinder moment of inertia Ic, the corrected cylinder moment of inertia Icc, and the proposed equations of moment of inertia Ip (for African students) and Ipa (for USA students).

TABLE I
ANALYSIS OF UNH'S STUDENTS' SAMPLE WITH MMIS.

Weight and height of UNH students			uncorrected cylinder model $I_{c3} = I_{c1}$	corrected cylinder model I_{cc3}	corrected cylinder model I_{cc1}	I_{p33} aH^2+bW+c UNH	I_{p11} aH^2+bW+c UNH
№	Weight *(kg)*	height *(m)*	***kgm²***	***kgm²***	***kgm²***	***kgm²***	***kgm²***
1	142,1	1,69	26,261	26,04	23,14	26,09	23,25
2	93,6	1,88	21,026	20,94	18,61	19,68	17,52
3	92,1	1,93	21,771	21,69	19,28	20,18	17,97
4	89,2	1,79	18,196	18,11	16,10	17,48	15,55
5	80,5	1,88	18,041	17,98	15,98	17,19	15,29

TABLE 2
ANALYSIS OF MSU'S STUDENTS' SAMPLE WITH MMIS

Weight and height of MSU students			uncorrected cylinder model $I_{c3} = I_{c1}$	corrected cylinder model I_{cc3}	corrected cylinder model I_{cc1}	I_{pa33} aH^2+bW+c MSU	I_{pa11} aH^2+bW+c MSU
№	Weight *(kg)*	height *(m)*	***kgm²***	***kgm²***	***kgm²***	***kgm²***	***kgm²***
1	113,40	1,90	34,62	25,97	23,08	25,37	21,80
2	108,86	1,91	33,39	25,04	22,26	24,51	21,07
3	104,00	1,71	25,82	19,36	17,21	20,06	17,14
4	97,40	1,80	26,69	20,02	17,80	20,21	17,32
5	95,25	1,85	27,53	20,65	18,36	20,65	17,72

VI. DISCUSSIONS

When looking at the four different representations of MMI on Figs. 8 and 9, Tables 1 and 2, it appears that they all follow the same general decreasing trend of MMI as the weight decreases because the data is sorted by weight from biggest to smallest. Although there are some outliers, the trend is still confirmed. This trend is expected and makes sense based on the definition of MMI.

As stated previously, the MMI measures a body's resistance to change in a rotational direction and is dependent on the distribution of mass in a body with respect to an axis of rotation. As mass moves further away from the axis of rotation, it becomes more difficult to change the angular velocity of the body. Since the chosen axis of rotation for this experiment are located from the posterior-to-anterior and the right-to-left of the body, the taller the body, the more amount of mass is farther from that axis and a higher MMI is expected.

The most significant difference between the different representations is that the MMI of the cylinder is significantly higher than the MMI based on Matsuo's model, hence the need for the corrected representation that matches Matsuo's model more closely.

The corrected MMI of the cylinder can accurately predict the MMI of a human body within reasonable limits, the percentage of error is less than 5% between the corrected cylinder and the experimental values obtained by Matsuo. These errors could be attributed to the extra limbs, the uneven weight distribution, and the variance of body types.

The large spikes on the graphs of Fig. 8 and Fig. 9 - for instance samples 9, 13, and 19 for UNH I_{p11} and samples 10, 14 and 22 for MSU I_{p11} - have a much higher MMI than the surrounding sample points because they have a considerably higher mass than the surrounding points. This also makes sense considering that more MMI means that there is more mass concentrated at a further distance from the center of gravity, compared to a body with the same height. There is also a greater discrepancy between the different models for these points. This is likely because there is more mass for these samples to vary, since it was not examined where the mass is unevenly distributed.

CONCLUSIONS

A study on MMI has been done on Congolese and USA students aged between 19 and 21 years old. To determine the MMI, the human body was considered as a cylinder. A mathematical model was proposed and compared to a previous mathematical model by Matsuo. Based on the data collected, an error of less than 5% was recorded.

It was also found that the MMI of the human body obtained by the mathematical model where the human body is assumed to be a cylinder was significantly higher than the previous mathematical models, hence the need to correct the cylindrical mathematical model to match Matsuo's model more closely and formulate mathematical models with expression that only consider the weight and the height of subject. These new models (one for UNH students and the other for MSU Students) were the proposed mathematical models by the authors.

In general, when looking at the four different representations of MMI on a graph it is obvious that they all follow the same general trend of a decrease in MMI as weight decreases. Although there are some outliers, the plots have similar trends. This trend is expected and makes sense based on the definition of MMI.

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