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Application of Geometric Method for the territorial planning of sports spaces. Case Study: Delta Campus of the University of Guayaquil

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Abstract– ***The educational community at a higher level has direct influence with the use of public space and its use characteristics translated into recreational, contemplative or sports activities; The latter generates a high influence in determining the quality of life and health status of the student at the University of Guayaquil, which is currently a paused process; since the provision of public spaces exists, but the renewal of sports furniture, equipment and infrastructure is fragmented for two main reasons such as the lack of maintenance and comprehensive provision that promotes a perception of the sports space as sectoral, by only counting according to the perception of the student community as a space within the Faculty of Physical Culture. The lack of maintenance of sports infrastructure and strategic planning for the creation of new spaces and interconnection of current sports spaces, which prevents sports activities from being carried out normally. The objective of this scientific article is to design a planned territorial system of sports spaces applying characterizations of the geometric method based on the evaluation of the central place theory to develop a comprehensive planning scheme with buildable space and green areas. As a conclusion, it is obtained that the interconnection of sports spaces with green areas and public spaces improves the quality of the urban-architectural design through a strategic planned approach.***

Keywords-- geometric method, public sports space, social interaction, strategic planning.

I. Introduction

Territorial planning focused on the development of sports spaces is seen as an approach of social interest and quality of health because sport generates an active physical dynamic that is attributed to the pertinent improvement of the conditions of psychic-sensory perception, which is why it is recommended that passive educational spaces be integrated with active spaces so that the conventional process of the education model generates quality of student life and contributes to the cognitive development and integration of the student community [1].

Sport contributes to well-being regardless of age, gender or ethnicity. From this precept, sport is established as an integrated means of societies in which everyone has an enjoyment of it and its contribution is unmatched in the architectural spatial perception since it generates opening of spaces [2].

In the Ecuadorian national environment, sport is established as a right and a healthy practice endorsed by both Organizations such as the WHO, the UN and finally aligned with the Sustainable Development Goals related especially to health (Goal 3: Guarantee a healthy life and promote the well-being of all at all ages).

Physical activities and sport, combined with the curriculum, are necessary for a complete education (Goal 4: Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all). Considering the benefits that sport offers for personal and social development, increasing access and participation is a primary development objective. Sport at its most basic form encourages balanced participation and could promote gender equality (Goal 5: Achieve gender equality and empower all women and girls). Through sport and physical activity, women and girls can be empowered, who can also benefit from its positive effect on health and psychosocial status [3].

Traditional territorial planning is changing, new components of the territory are being evaluated that contribute to its integral and cohesive development, geometric models are established as a method for the design of the territory to influence the maximum physical-spatial contribution of the qualities of land use. One of the geometric analysis models that has framed a substantial development today in urban processes is proposed to the model of Christaller and Losch with their theory of central place [4] and hierarchies that establish a systematic order of development based on a typical centrality that establishes multicentralities and through each centrality a containment is established that provides a diagrammatic structure of the territory [5].

The strategic planning of sports spaces should currently be a priority at both the national and provincial levels. Currently, the city of Guayaquil has less than 3.24% of sports spaces per kilometer, which entails a deficit of public sports spaces at the service of the community, causing sedentary lifestyle, drug consumption, health problems and low rate of social interaction [6]. Therefore, strategic actions must contemplate rescuing sports infrastructures and promoting dynamic actions to be able to recompose the dynamics of the sports system articulated by paths, corridors, tracks and in general green spaces.

In the higher education community as such, the influence of sport specifically at the University of Guayaquil is a paused process; since the provision of public spaces exists, but the renewal of sports furniture, equipment and infrastructure is fragmented for two main reasons, such as the lack of maintenance and comprehensive provision that fosters a perception of the sports space as sectoral, by only counting according to the perception of the student community as a space within the Faculty of Physical Culture.

The adoption of planning for the design of sports spaces as previously proposed must operate in sports infrastructures as an articulating element of public space to be able to deploy its functions distributed around specific sports developments. Frequently, stadiums, coliseums and other spaces dedicated to sports become attractions within an educational community, thanks to their architectural design and their capacity to host various events. For this reason, governments and public institutions allocate a significant part of their resources to the creation of sports spaces that promote citizen integration and participation in sports activities.

Sports spaces are much more than simple infrastructures; They are the setting where physical activity is developed, health is promoted, communities are built, and values are cultivated. These places, designed and conditioned for the practice of sports and exercises, play a fundamental role in the lives of people and societies.

II. Materials and Methods

A. *Christaller and Losch model*

The mentalizer of this model was the German geographer Walter Christaller, who proposed a hierarchical organization of cities, which operate as central places that offer goods and services in an isotropic, that is, homogeneous space.

The hierarchy of cities is determined by the threshold of the central place. The threshold is understood to be the radius around the central location that covers the minimum population that a company requires to reach its equilibrium point in sales. Its model is inductive since it started from the observation of reality, rather than derived from certain principles.

According to August Losch, a German economist, he was the predecessor of the theory of the central place established by Christaller, which proposes an economic development of territory [9]; which establishes that when an area has several overlapping hexagons surrounding a given center, a metropolitan city will be formed. In other words, it can be said that numerous hexagons or market areas of different products will grow around the core of a city [4].

This application of a model established from an assumption of centrality generates a perception of the forms as harmonic geometrized structurings that are based on the main trigonometric functions such as sine, cosine, tangent, cotangent, secant and cosecant [10] in order to develop the shape of the territory; This process is established as an iteration of forms that enact a complete and effective design of the conditions of the territory, generating use of complex forms of the terrain since it is based on a scalar order which can sub-establish partial geometrizations that fill the spaces with a new centrality.

Among the principles proposed in the model are:

- Considers an isotropic space, homogeneous in all directions.
- It assumes the existence of an urban center in which there are goods and services that are exchanged [7].
- The central place must produce or offer goods or services to a population spatially dispersed across the isotropic plain that surrounds it.
- There is only one form of transportation and transportation costs are proportional to distance.

The main purpose of this model is to show how products and services, especially tertiary functions, begin to organize territorially within the urban hierarchy [11].

B. *Application of geometrization in territorial planning*

Geometrization in territorial planning refers to the application of geometric principles and spatial analysis to organize, design and manage land use and development [12]. This approach takes advantage of mathematical and geometric concepts to create efficient, sustainable and aesthetically formalized spatial schematizations, which contributes to a spatial relationship of land use with topology using the properties of the figures that remain invariant, when said figures are folded, dilated, contracted or deformed to form various ordering processes of the form, so that ordered strategies of the form are generated [13]. After a brief historical review of the crucial facts in the evolution of topology, three topological theories are studied in a very intuitive way:

- Graph theory, insisting on two classic examples, the problem of the seven bridges of Könisberg and the four-color theorem, but which involve complicated mathematical theories in their resolution.
- Knot theory, with surprising applications in Molecular Biology, Physics.
- The theory of surfaces, a section developed with more mathematical rigor than the previous ones: Here it is about classifying all compact surfaces and classifying is the central object of the topology [14].

Graph Theory

Graph theory is linked to a set of points and vertices, some of which are linked to each other by means of lines, the edges. The geometric nature of these arcs is not important, only the way in which the vertices are connected counts to diagram the set as a knot [15], [16]. In the territorial structure it can be used on roads using a road organization such as the scheme of the 7 bridges of Könisberg, forming seven bridges (a, b, c, d, e, f and g in the figure) joining the four parts of the city (A, B, C and D) separated by water, and arranged as indicated:

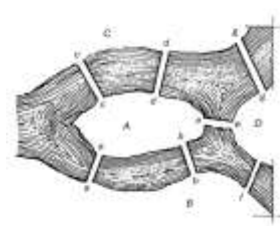

Fig. 1 Graph theory in spatial model.
Note. A combinatorial calculation of the Polynomial of Alexander (Soto, 2017).

This structure demonstrates how each point will be connected by bridges since water passages permeate the territory, so to reach each point a ring structure must be generated, from a correlation of a graph of points, being an initial form of perimeter organization. In the graph, it is translated by the fact that each vertex must be associated with an even number of edges. But the configuration of the Könisberg bridges does not obviously verify this condition, proven by Euler as necessary and sufficient to organize the territory but rather to find transition lines between centric points as conductive threads which do not intersect [17].

Knot Theory

The weaving technique requires the crossing and knotting of threads to transmit the organization of the shape through knotted or crossed unions that generate a knot. A knot is a continuous, closed curve without double points [18]. This curve is in a space of dimension three and it is accepted that it can be deformed, stretched, compressed, but it is prohibited to make cuts. When it is possible, through manipulations of this type (that is, through homeomorphism) to move from one node to another, they are said to be equivalent. In general, it is very difficult to decide when two knots are equivalent, and much of knot theory is dedicated to trying to resolve that question.

The knots are cataloged considering their complexity. A measure of complexity is the number of crossings, such as the number of double points in the simplest plane projection of the knot. The trivial knot has zero crossing numbers. The cloverleaf and the figure of eight are the only knots with crossing numbers three and four, respectively (*See figure 2*).

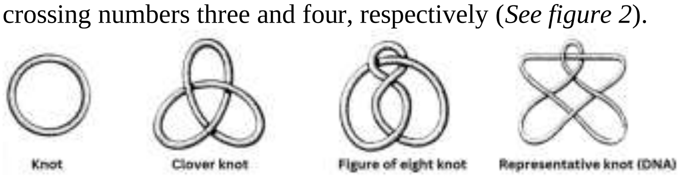

Fig. 2 Knot theory in spatial design.
Note. A combinatorial calculation of the Polynomial of Alexander (Soto, 2017).

Knots can be added, subtracted, multiplied and even divided (the algebra of knots). But when the knots become complicated, their simple description is not enough to distinguish them. Thus, starting from its shape (the geometry of the knot), formulas have been developed that work for all knots; there are topological invariants that are obtained by studying the complement of the knot.

Surface Theory

This theory is applied to land use directly as a territorial planning strategy and may be suitable for a formal development with mathematical variants [19], waiting for the following resolutions that start from a mathematical function that manages surfaces as structures of space from an environment of a point a in a space X is a set of points close to x. The base formula is applied, if a = (a1, . . ., an) e Rn , a neighborhood of a is a set N Rn such that there exists « > 0 and Bn (a,«) = {(x1, . . ., xn) e Rn : (a1 - x1) 2 + . . . + (an - xn) 2 < «2} N.

A manifold of dimension n is a space in which each point has a neighborhood homeomorphic to Bn = {(x1, . . ., xn) e Rn : x1 2 + . . . + xn 2 < 1 } : the open unit ball in the n-dimensional Euclidean space, Rn. Next, surface structures made from the base formulation are presented (*See figure 3*).

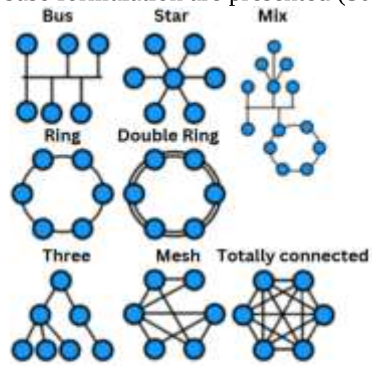

Fig. 3 Graph theory in spatial model.
Note. A combinatorial calculation of the Polynomial of Alexander (Soto, 2017).

This application of classical theories is dynamized to the contemporary environment thanks to its applicability in traditional territorial planning of spaces, which is convenient for an innovative study that improves the design structure of the territory managed as a separate spatial particle that can form new cells together dependent on the organizational form of the conducting module and its alignments such as bus or ring type that correlate its organization through conducting threads [20], [21].

The algorithm in the analysis model consists of the following prospective development, from the samples of x, the dependence of the predicted shape indicator N with respect to the shape characteristics is constructed, denoting the following formula for the model:

N = a1i xi + a2 j x j . (1)

III. Results and Discussion

A. *Initial Planning*

From graph theory, the first dynamics of the territory are proposed, with sports meeting points being a determining factor and the crossings will be established by permeable and non-permeable paths, forming a connected graph, which exists by a path linking each pair of vertices which will be called Eulerian, passing through each edge exactly once (*See figure 4*).

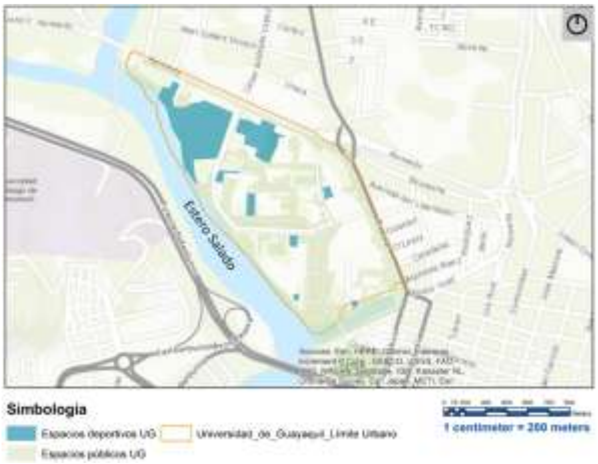

Fig. 4 Structure of sports spaces – full and empty UG.
Note. Own elaboration

B. Geometrized planned model - Level 1

From graph theory, the first dynamics of the territory are proposed, with sports meeting points being a determining factor and the crossings will be established by permeable and non-permeable paths, forming a connected graph, which exists by a path linking each pair of vertices which will be called Eulerian, passing through each edge exactly once (See Figure 5).

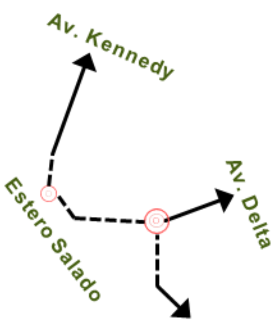

Fig. 5 Base graph of FAU-UG sports spaces.
Note. Own elaboration

There are two closed circuits according to the structure of the graph, which constitute a round-trip circulation that is not convenient for the proposal since it encloses the dynamics of the sporting activity and does not interrelate them.

As a territorial scheme, it can be established that the graph theory can be made dynamic through the internal subdivision of the points, generating a matrix structure that will allow the dynamic counting of spaces and their arrangement around the development of the activity, which supports the concept of paths and distances in accordance with the first of the 7 Könisberg bridges that comprise a closed structure that proposes equidistant distances and much more connective proximities to avoid leaving the point of convergence as a structure within the resulting form. (closed).

C. Geometrized planned model - Level 2

More interrelational planning is evoked for the territorial design process as a dynamic of sharing and connecting uses that can be provoked or attended by the relationship of permissible nodes and crossings. The node scheme used is based on a central node with 3 exits, this organization contributes to the conduction of flows and outlines a form of central node tied to its center (See figure 6).

In essence, it is seen that knot diagramming is concerned with understanding the properties and characteristics of knots and links in mathematical and physical contexts. A knot can be considered a loop in space, while a link is a collection of such loops. In this case, the formula was applied that considers that in each movement there will be different cases, the orientation and the mirror image of the movement generating an analogous development.

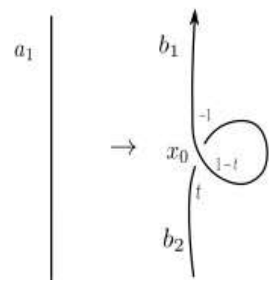

Fig. 6 Knot geometrization in FAU-UG sports spaces
Note. Own elaboration

We obtain a new crossing x0 and from arc a1 two new arcs b1 and b2 are born, like this:

$$\begin{matrix} & a_1 & & & \\ x_1 & \left[\begin{matrix} a_{11} & * & \cdots & * \\ * & & & \\ \vdots & & Q & \\ * & & & \end{matrix}\right] \end{matrix} \longrightarrow \begin{matrix} & b_1 & b_2 & & & \\ x_0 & \left[\begin{matrix} (1-t)-1 & t & 0 & \cdots & 0 \\ * & * & * & \cdots & * \\ * & * & & & \\ \vdots & \vdots & & Q & \\ * & * & & & \end{matrix}\right] \\ x_1 & \end{matrix}$$

Therefore, the base form of the Alexander Polynomial that develops in b1 + b2-1 – 1, which is developed with the following parametric equation:

$$f(t) = \cos t + 2\cos 2t$$
$$g(t) - \sin t - 2\sin 2t$$
$$h(t) = -\sin 3t$$

D. Geometrized planned model - Level 3

A surface is constructed from the combination of two known basic structures through the process of connected addition: given M1 and M2 two compact (mesh) and connected (bus) surfaces, a small open ball is removed from each of the surfaces and the resulting spaces are glued across the border circumferences; the resulting space M1 # M2 is the connected sum of M1 and M2. Various spatial organizational configurations can be visualized below based on this base formulation (*See figure 7*).

Fig. 7 Diagram of bus and mesh surfaces of the FAU-UG sports spaces
Note. Own elaboration

With the main central place, the Christaller and Losch Model is built where the shape used is the hexagon where sports services are offered at its exit vertices. But each vertex place belongs to two adjacent hexagons: therefore, each center place has three minor center places inscribed on it. 1/3 of the student population passes through the central place marked with 3 red knot rings, which makes the pattern dissipate towards its three bus-type flows (2 towards Kennedy Avenue and 1 Delta Avenue). Generating the following mathematical graph: $n= 1 + (6*1/3) = 3$.

Among the assumptions considered is isotropic space, homogeneous in all directions. It assumes the existence of an urban center in which there are goods and services that are exchanged. The central place must produce or offer goods or services to a population spatially dispersed across the isotropic plain that surrounds it. There is only one form of transportation and transportation costs are proportional to distance.

E. Discussion

It is defined that a poor spatial distribution of central points and topological geometrizations leads to the isolation of services and a low territorial and hierarchical functionality, which explains the behavior of the distribution in this area. However, the latter would be less likely, since the model favorably explains specialization in the rest of the city. It is also important to note that the rank 4 hexagon (total coverage area) does not encompass all the locations in the city, this means that the model does not completely adjust to its distribution of the coverage area.

Finally, at the Delta Campus of the University of Guayaquil, Christaller's model fits excellently to explain the distribution of sports centers and their services (equipment, furniture and transition areas), as demonstrated in Figure 7, without competing among them and with total coverage generating transition flows.

II. CONCLUSIONS

The Theory of Central Places by Christaller and Losch demonstrated in almost all cases that the spatial distribution from the process of graphs to surfaces connoted a strong central distribution that converged towards exit and entry flows (being characteristic convergences of a central model), they correctly fit the hexagon model proposed by Christaller by generating convergences between centralities and subcentralities [22] from the bus and mesh surface configurations, so, thank you To the topology, a technical analysis is feasible to understand and improve the location in the territory of this type of (sports) activities.

The Christaller model is a tool that helps not only the understanding of spatial territorial dynamics, but also a fundamental piece for the user's understanding by monitoring flows, which it attributes to a decision-making tendency.

Using topological theory, it was determined that the proposed model contemplates a dynamic territorial model analyzed from the behavior of the spatial distribution in its flows. Despite the results, which were mostly satisfactory, certain limitations of the model established by the areas not designated for sports activity were observed since their natural and physical infrastructure does not allow it (natural edge of the estuary), in terms of the level of detail that allows knowing the coverage of the service areas at larger scales, in addition to the lack of information regarding the exact location of the providers within each hexagon, with which the coverage ranges in each hexagon could be better defined and so on. obtain a complete fit of the model without leaving out empty spaces.

The present study provides a technical proposal for the explanation of planned geography under the geometrization approach in terms of the distribution, location and interrelation of a network of subcentralities aligned to a node centrality. The main purpose of this model is to show how products and services, especially tertiary functions, begin to organize territorially within the urban hierarchy [23].

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