

Analysis of Bounded Variation Function Space and Critical Points in 1-Laplacian Operator Problems: A Systematic Review

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Abstract– This paper presents a systematic review on the space of bounded variation functions and critical points in problems related to the 1-Laplacian operator. Bounded variation is a fundamental concept in functional analysis and theory of differential equations, relevant to understanding the behavior of solutions in various variational problems. The 1-Laplacian operator, which arises in contexts such as image theory and shape analysis, poses unique challenges due to its nonlinearity and the singular properties of its solutions. That is why this research focuses on the role those various nations, researchers and institutions play in underlining international collaboration, establishing connections and partnerships with research centers around the world, which allows access to up-to-date knowledge on the subject, such as the exchange of ideas and the development of international scientific capabilities. This systematic review allows for a compact examination of the current state of the art in the analysis of bounded variation function space and critical points in problems related to the 1-Laplacian operator, through the analysis of 129 articles collected from the SCOPUS database and 125 from WoS, highlighting the leadership of countries such as China, Italy, and France, as major players, emphasizing their significant contributions to the advancement and research in this field. The study shows that international collaboration has been fundamental for scientists and institutions to build strategic alliances and global research centers. Major challenges have been identified, such as investment in talent training and development in this field due to the limited data found. This review provides a comprehensive view on the subject highlighting the achievements and challenges for the scientific advancement of this field.

Keywords– Bounded Variation, Critical Points, 1-Laplacian, Functional Analysis and Variational Problem.

I. INTRODUCTION

The theory of the space of bounded variation (BV) functions has proven to be a fundamental area of study in functional analysis and in the theory of partial derivative equations (PDEs). These functions, characterized by having a derivative in a generalized sense that is a Borel measure with finite total variation, play a crucial role in a variety of applications, including image processing, geometric measurement theory and solid mechanics. [1-5].

Within this context, the 1-Laplacian operator emerges as a powerful and complex tool for the study of optimization and equilibrium problems in various fields of mathematics and physics [4-11]. This operator, which is a limiting case of the p -Laplacian when $(p \rightarrow 1)$, presents unique challenges due to its

nonlinear and non-differentiable nature, requiring specialized methods and approaches for its analysis [10,12-17].

The study of critical points in problems involving the 1-Laplacian operator is particularly relevant because of their connection to minimum energy solutions and steady states in physical and mathematical systems [1,18-20]. The identification and characterization of these critical points not only provide a deeper understanding of the intrinsic properties of the system, but also offer valuable insights for the design and implementation of problem-solving algorithms in applied areas [21-24].

Given the exponential growth of the literature in these fields, a bibliometric review becomes an essential tool to map and analyze the development of research, identify emerging trends and highlight key contributions. This paper sets out to perform a comprehensive bibliometric analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems. Through this review, it is intended to provide a comprehensive view of the advances, challenges and future opportunities in this area of research.

In structuring this analysis, bibliometric techniques and tools such as VOSviewer [25-27] and Bibliometrix (in R) [28-30], R software and RStudio [31-32] will be used to evaluate scientific production, collaboration between researchers and institutions, as well as the temporal evolution of research topics. It is hoped that the findings of this review will not only facilitate a better understanding of the current state of knowledge, but also inspire new lines of research and interdisciplinary collaboration.

II. METHODOLOGY

The strategies employed to perform a bibliometric analysis on the analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems are based on statistical techniques. This analysis aims to evaluate the scientific contributions and the scope of publications in various research areas. Articles were collected from recognized databases such as SCOPUS and WoS, which were analyzed in detail and graphically represented through tables, graphs and figures to understand the intellectual and conceptual evolution in this field. This process included the study of elements such as citations, keywords and authorship [33]. In addition, metrics such as citations per article,

contributions of authors and/or institutions per country were used to determine the influence of the published literature on the research, as well as to evaluate the specific impact of authors, journals and institutions within each area of study [34-36].

Taking into account the significant scientific and technological advances of the 21st century, especially in the analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems, we have decided to focus our study on publications between the years 2002 and July 2024 [37]. This period was selected for its historical relevance and statistical significance, encompassing the most current and relevant trends in this study. Regarding exclusion criteria, we discarded papers in languages other than those specified for inclusion, as well as publications prior to 2002, as detailed in Figure 1 and Table 1.

Figure 1 illustrates the research plan for this study, which includes a search strategy in Scopus and WoS, incorporating the keywords mentioned in Table 1. A search comprising titles, abstracts and keywords was conducted to identify relevant articles. This search yielded 129 articles published in journals indexed in Scopus and 125 in WoS. A manual check of the relevance of each article was carried out, resulting in the exclusion of 72 duplicate articles, leaving a total of 150 articles for bibliometric analysis.

The analysis was carried out in two main phases: in the first phase, contributions were identified using the Scopus and WoS databases, while in the second phase, a bibliometric analysis was carried out using bibliometric software in RStudio and the Biblioshiny and VOSviewer tools.

TABLE I
SEARCH STRATEGY COMPONENTS FOR BIBLIOMETRIC ANALYSIS

| Component | Description | Terms / Operators |
|-------------------|---|--|
| Keywords Group 1 | Terms relates to 1-Laplacian Operator Problems | "1-Laplacian", "1-Laplace", "Laplacian Operator", "Laplace Operator", "Laplacian", "First-Order Laplacian", "First-Order Laplace". |
| Keywords Group 2 | Terms relates to Bounded Variation Function space | "BV", "BV Function", "BV Space", "BV-Class", "BV-Space", "Bounded Variation" |
| Excluded Keywords | Terms to be Excluded | "Other Languages", "Other Years", "Other Terms" |
| Boolean Operators | Connectors for combining terms | OR (within each group), AND (between groups), AND NOT (to exclude terms) |
| Search Fields | Specific areas of the database | Title, Abstract, Keywords (TITLE-ABS-KEY) |

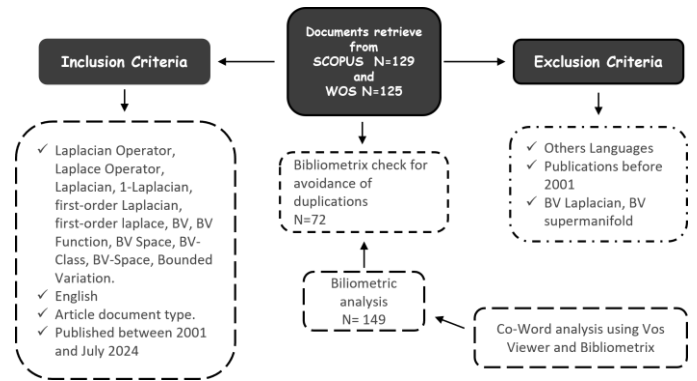


Fig. 1 - Search Process flowchart for the Bibliometric Review.

III. ANALYSIS AND DISCUSSION

A. Annual scientific production

Figure 2 shows the annual scientific production in terms of the number of articles published from 2002 to 2024. This is framed in the context of a bibliometric review on the analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems.

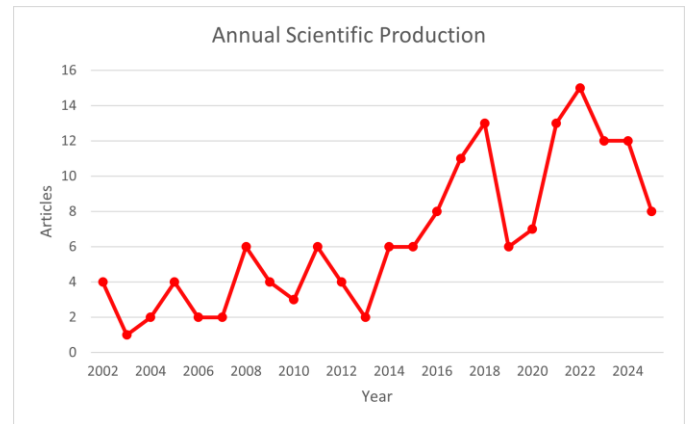


Fig. 2 - Annual production of articles.

The red line graph shows the number of articles published per year over a period of almost 25 years. There is a general trend of increasing scientific production, with certain fluctuations over the years. In the period from 2002 to 2008 we see a relative stability in scientific publications with small changes between 2 and 5 articles per year, considering this first stage as an initial phase, with moderate growth and significant trends of increase. The period from 2009 to 2014 presents a gradual increase in scientific production reaching a peak of 6 articles in 2011. In 2012 there is a notable drop of 2 articles, followed by a steady recovery until 2014, reaching 6 articles again. A very important period is from 2015 to 2021, which is characterized by accelerated growth and significant production peaks. In 2017, production reached 11 items, marking a remarkable increase. Despite a decrease in 2018, the overall trend is upward, reaching a peak of 13 items in 2021. The last period from 2022 to July 2024, the most recent

fluctuations are presented, observing considerable variability, after reaching 15 articles in 2022, production decreases in the following years. In 2024 production dropped to 6 articles, since it is influenced by external factors such as changes in research policies, financing, or global events, but it is important to emphasize that this last year the articles published up to July were considered.

The fluctuations analyzed are influenced by several factors such as: the introduction of new research methodologies and technologies, which facilitate an increase in scientific production, especially visible in periods of significant growth; the increase in international collaborations and the formation of research networks, which contribute to the increase in the number of publications; more favorable research policies and increased funding for scientific projects, which positively influence the production peaks observed; and events such as pandemics, economic crises, or political changes affect scientific production, which explains the recent fluctuations.

It is important to mention that there were 4 articles published between 1981 and 2002, talking about the existence

of solutions in $BV(\Omega)$ for certain nonlinear partial differential equations involving the 1-Laplacian operator and the critical Sobolev exponent, and that has not been considered in this study because we seek to analyze the most current trends and relevant statistics.

B. Countries with the highest scientific production

Figure 3 shows the countries of the corresponding authors and their participation in scientific publications on the analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems. The data are broken down into two collaboration categories: Single Country Publications (SCP, in blue) and Multi-Country Publications (MCP, in red), where the horizontal bar chart shows the total number of papers published by corresponding authors from multiple countries. Each bar is divided into two sections to reflect national collaborations (SCP) and international collaborations (MCP).

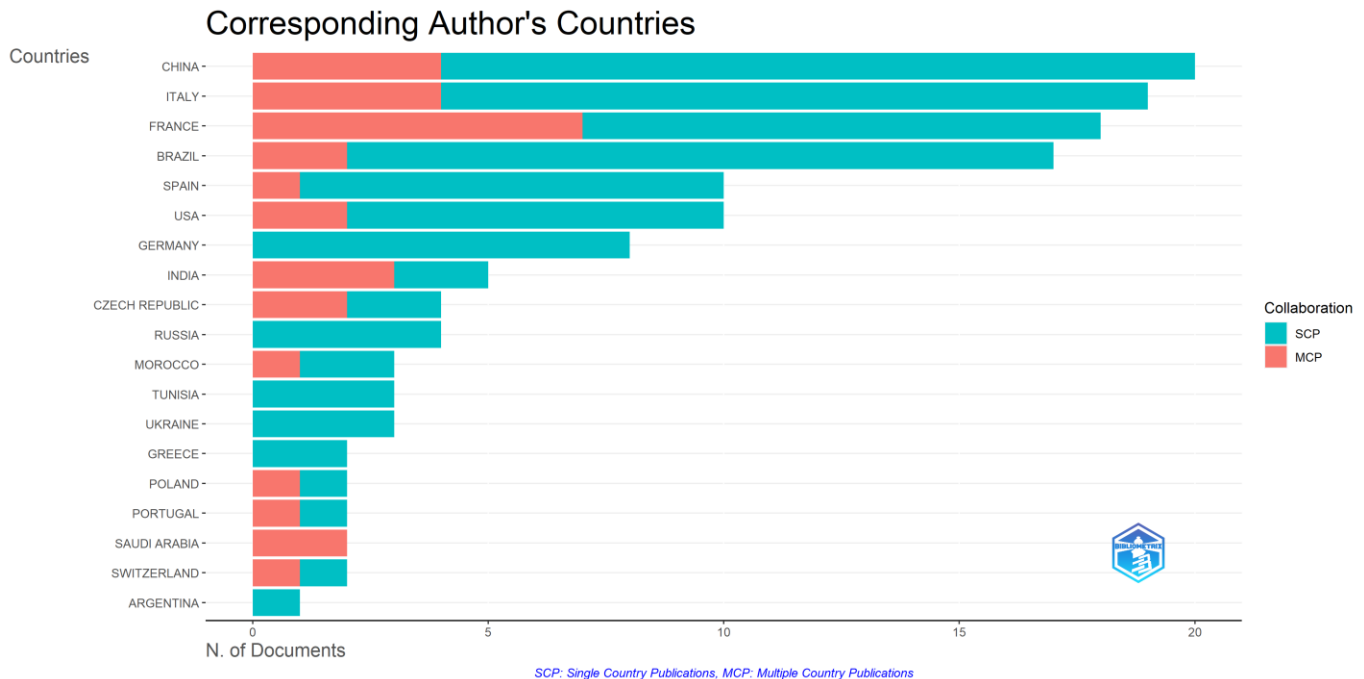


Fig. 3 - Distribution of scientific publications by country

In this graph we can see that China leads in scientific production with the highest number of total publications, most of these publications are SCP, indicating a strong domestic scientific production. Italy also shows a high number of publications, with a combination of SCP and MCP, suggesting that Italian researchers are involved in both national and international collaborations. France shows a remarkable balance between SCP and MCP, indicating significant collaboration with other countries, the proportion of MCP is

high, suggesting a trend towards international collaborative research. Brazil has a strong presence in SCP, similar to China, however, it also participates in CCM, although to a lesser extent. Spain and the United States have a considerable number of publications, with a higher proportion of SCP, reflecting a strong domestic research capacity. Germany has a high number of publications, with a predominance of SCP. India shows a balance between SCP and MCP, similar to France. We continue with the Czech Republic, which has a

higher proportion of MCPs, indicating a trend towards international collaboration. Russia, on the other hand, shows a balance similar to that of Spain and the United States. Morocco and Tunisia have a notable presence in CCM, suggesting significant international collaboration, the amount of SCP is comparatively lower. Ukraine, Greece and Poland have a moderate scientific production, with a balance between SCP and MCP, indicating participation in both national and international research. Poland, Portugal and Switzerland show a balance between SCP and MCP, despite having a lower number of total publications, the diversity of collaboration is remarkable.

Analysis of the distribution of publications by country and type of collaboration offers several important insights: Countries such as France, India, and Czech Republic show a strong inclination toward international collaborations (MCP). This suggests a global network active in research on bounded variation functions and 1-Laplacian operator problems. China, Brazil, and Germany stand out for their high domestic scientific production (SCP), indicating a strong domestic research capability. These nations lead technological and methodological advances in this field. The balance between SCP and MCP allows us to see diversity in collaboration between countries such as Italy and Spain, and suggests a flexible research environment, capable of operating both nationally and internationally. Countries such as Morocco, Tunisia and Saudi Arabia are actively participating in international collaborations, indicating a growth in their research capacity and a deeper integration into the global scientific community.

Figure 4 shows scientific production by country on a world map. The countries are colored in different shades of blue, with darker shades indicating higher scientific output and lighter shades indicating lower scientific output. China, Italy, France and Brazil: These countries are colored in darker shades of blue, indicating that they are the world leaders in scientific production. Countries such as Germany, France and the United Kingdom also have a high scientific output. Brazil stands out with a dark shade of blue, suggesting that it is the leader in scientific production in South America. Japan, South Korea and Australia have significant scientific output, but not as high as the United States and China. Most countries in Africa and the Middle East are colored in very light or gray tones, indicating low scientific output or lack of data. Central America and the Caribbean also show limited scientific production.

The relevance of the study presented in this map is given by: Identifying research centers, as the scientific output by country indicates where significant research on the 1-Laplacian operator is being conducted. The United States, China and European countries are probably the main contributors. International collaborations, as knowing the scientific output by country helps to identify potential international collaborators and establish research networks. And access to resources and data, since countries with high scientific production tend to have better resources and access to data, which is crucial for the analysis of complex problems such as those of the 1-Laplacian operator.

Country Scientific Production

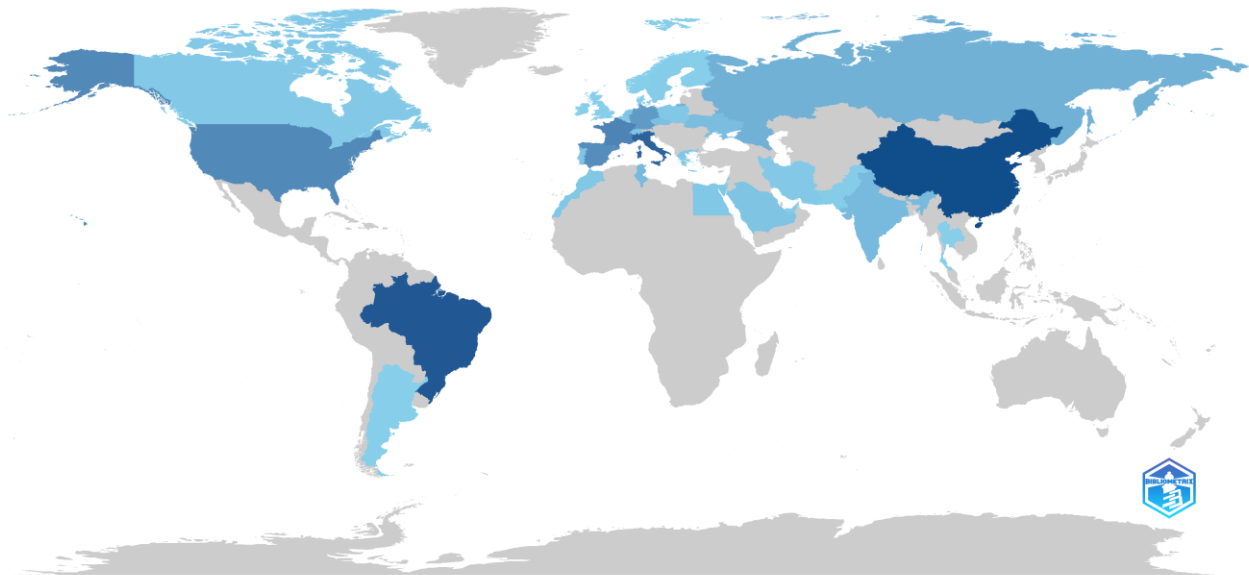


Fig. 4 - Country scientific production.

C. Most frequent keyword.

The graph in Figure 5 shows us a map of co-occurrence networks of author keywords, this type of visualization is

particularly useful in our bibliometric review study on the analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems.

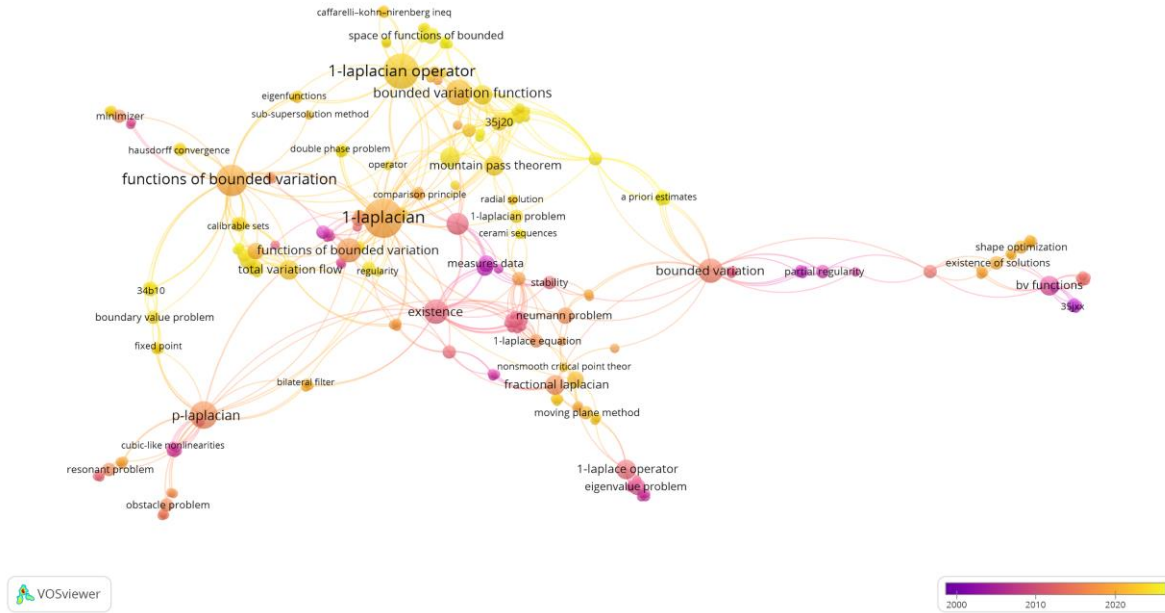


Fig. 5 - Thematic network visualization of keywords

The nodes represent the keyword used by the authors in their publications. The size of the node indicates the frequency of use of that keyword. Connections are the lines connecting the nodes represent the co-occurrence of keywords in the same articles. The closeness between nodes indicates the close relationship between topics. The color gradients vary from purple (least recent) to yellow (most recent). This reflects the temporal evolution of the investigated topics. The cluster in the center of the graph groups the nodes as "1-laplacian", "functions of bounded variation", and "bounded variation functions", which are highly interconnected, indicating that these are the most central and frequently co-occurring themes in the literature. Secondary clusters, around the central cluster, include related topics such as "mountain pass theorem", "existence", "p-laplacian", "neumann problem", and "fractional laplacian". These represent important subtopics in the investigation of the 1-Laplacian operator and bounded variation functions.

We identify "1-laplacian", "functions of bounded variation", and "bounded variation functions" as the main themes, suggesting that these are the key concepts structuring research in this field. Also, topics such as "mountain pass theorem", "existence", and "fractional laplacian" are also significant but are more specialized. We visualize temporal evolution as the colors closer to yellow, indicating more recent

research trends. Topics such as "shape optimization" and "existence of solutions" show increasing interest in the recent literature. The purple nodes represent topics that were more relevant in the past, such as "boundary value problem" and "fixed point". It also shows us the interconnections, as the density of connections in the center of the graph shows areas of high thematic interconnectedness, suggesting that research on "1-laplacian" and "bounded variation" is highly integrated, and more isolated topics, such as "resonant problem" and "obstacle problem", indicate less integrated but still relevant areas within the field.

D. Authors and their collaborative networks.

The graph in Figure 6 shows the collaborative and authorship networks, showing the connections and collaborations between authors in the field of bounded variation functions and critical points in 1-Laplacian operator problems.

Each node represents an author, and connections between nodes indicate collaborations in scientific publications. This type of analysis is crucial to understand the structure of the research network and the dynamics of collaboration in this field.

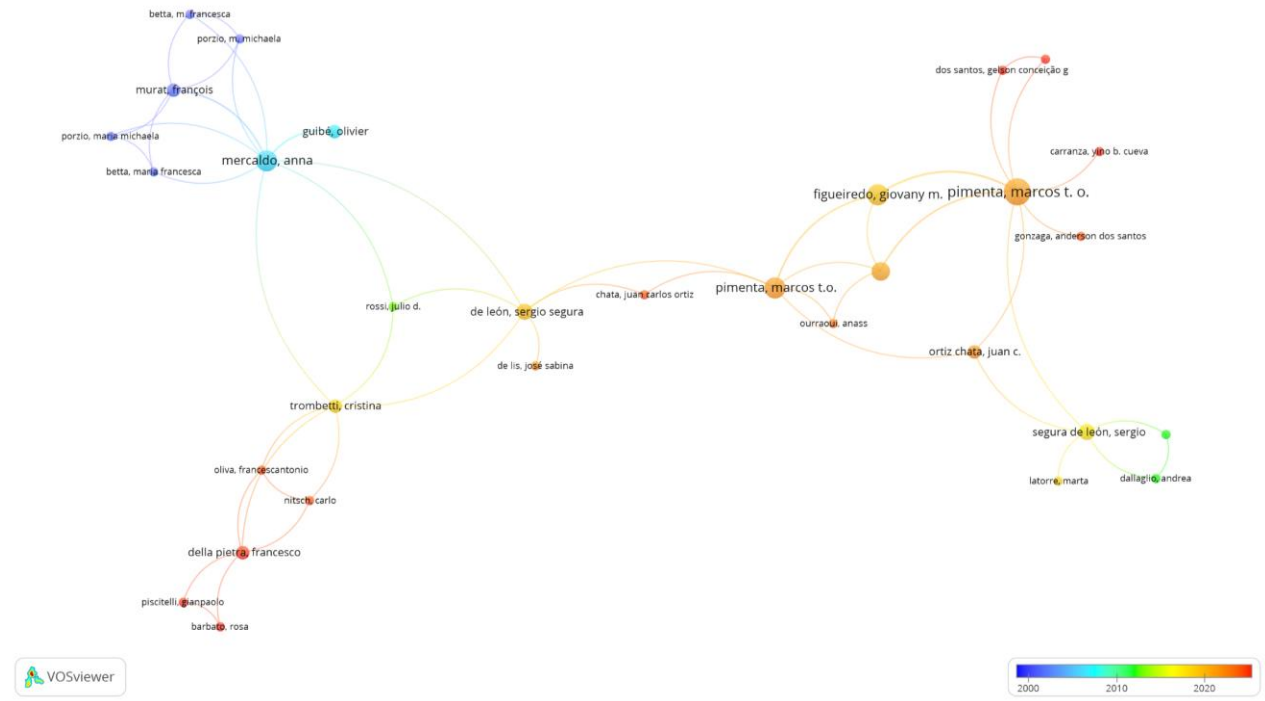


Fig. 6 - Map of authors with the greatest scientific production and their collaborative networks.

The nodes represent authors who have contributed to the literature on constrained variational functions and the 1-Laplacian operator. The size of the node indicates the number of publications by that author. Connections are the lines between nodes and represent collaborations between authors in scientific publications. The number of lines or the thickness of the lines indicate the frequency of these collaborations. The colors vary from blue (older collaborations) to red (more recent collaborations). This makes it possible to observe how collaborations have evolved over time. Regarding collaborations, the group led by authors such as Mercado, Anna and Murat, François, indicates a group of collaborators who have worked together in previous publications (light blue cluster). Another group led by Pimenta, Marcos T.O. and Figueiredo, Giovany M., this group seems to have a significant number of recent collaborations (yellow/orange cluster). Another group highlighted by Segura de León, Sergio, shows another area of active collaboration, although more recent compared to other clusters (green cluster). And the other clusters, include authors such as Della Pietra, Francesco and Trombetti, Cristina, who have also collaborated significantly within their clusters.

We identified authors with larger and more centralized nodes, such as Pimenta, Marcos T.O., Mercado, Anna, and Segura de León, Sergio, who can be identified as leaders in the field due to their high number of publications and collaborations. The graph shows a network of collaborations that includes authors from different countries, which is indicative of the international nature of research in this field.

The colors allow us to observe how the collaborations have evolved over time. The most recent clusters indicate emerging areas of research and new collaborations. The density of connections in certain clusters indicates particularly active areas of research. The cluster led by Pimenta, Marcos T.O. shows a high density of recent collaborations.

E. Thematic map.

Figure 7 shows us a thematic map, which classifies various research topics according to their centrality and density. This type of graph is crucial for our study, since it provides a clear view of the themes that are emerging, declining, or consolidating in the field of analysis of the space of bounded variation functions and critical points in 1-Laplacian operator problems

The X-axis shows centrality, as it measures the importance of a topic within the field. A high centrality value indicates that the topic is relevant and well connected to other topics. The Y-axis shows density, as it measures the degree of internal development of a topic. A high density value suggests that the theme is mature, with a well-defined internal structure. The upper right quadrant shows us driving themes, themes with high centrality and high density. They are well developed and central to the discipline. The lower right quadrant shows us basic themes with high centrality, but low density. They are important to the discipline, but may need further development. The upper left quadrant shows us specialized topics with low centrality and high density. They are well developed internally

but are not as connected to other topics. And the lower left quadrant shows us emerging or declining topics having low

centrality and low density. They are emerging or losing relevance.

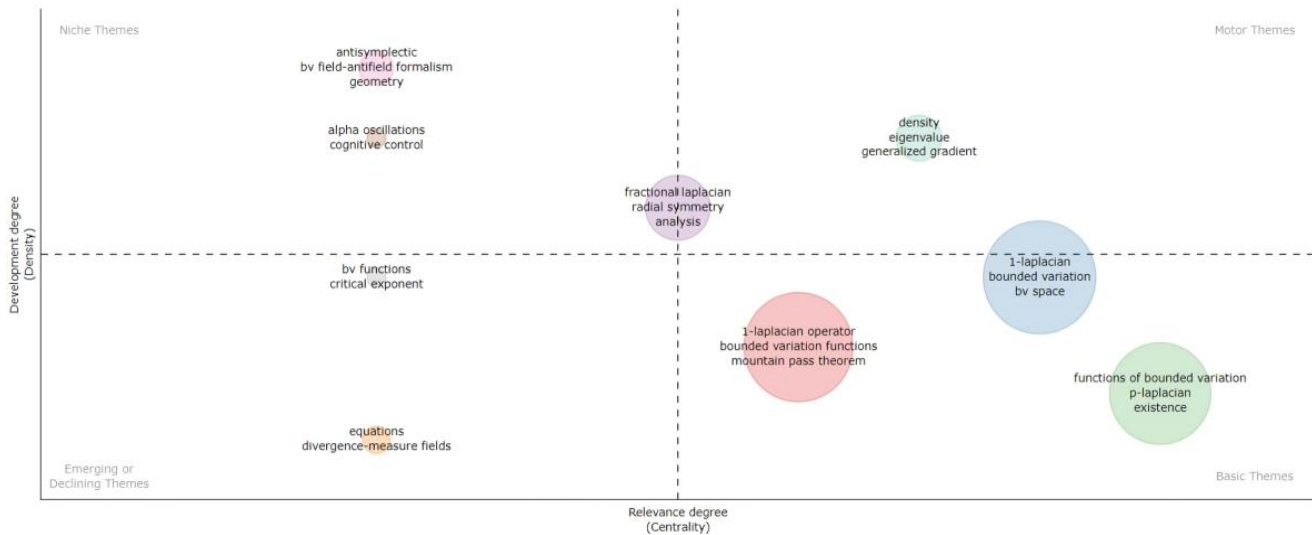


Fig. 7 - Strategic diagram of research theme

We identified the driving themes, such as "density", "eigenvalue", "generalized gradient" as crucial and well-integrated themes in the field, suggesting that they are key areas of current research with a high degree of development and connection to other themes. Basic topics such as "1-laplacian", "bounded variation", "bv space", essential to the field of 1-Laplacian operator and bounded variation functions. Although central, their development may benefit from further research to strengthen their internal structure. specialized topics such as "antisymplectic", "bv field-antifield formalism", "geometry", as highly developed areas but with less impact on the overall research of the field. They indicate very specific sub-areas that are important for certain research niches. And emerging or declining topics such as "equations", "divergence-measure fields", which represent areas that are beginning to gain attention or are in decline. These topics may offer opportunities for new research or indicate a shift in research priorities.

IV. CONCLUSIONS

In this article, we have carried out a systematic review of the space of bounded variation functions and critical points in 1-Laplacian operator problems, analyzing the bibliometric trends and conceptual evolution in this field. Throughout the period analyzed (2002-2024), a significant increase in scientific production related to the analysis of bounded variation functions and problems with the 1-Laplacian operator is observed. This increase is influenced by factors such as the introduction of new research methodologies, the strengthening of international collaborations, and the increase in funding for scientific projects. China, Italy, France and Brazil emerge as the leading countries in scientific production in this field, followed by European nations such as Italy,

France and Germany. These results highlight the internal research capacity in some countries, while others show a trend towards international collaborations, underscoring the global nature of this area of study. Keyword co-occurrence analysis reveals that terms such as "1-laplacian", "functions of bounded variation", and "bounded variation functions" are central to the literature. In addition, important subtopics such as the "mountain pass theorem", "existence", and "fractional Laplace operator" were identified, indicating a diversification in the areas of research interest within the field. The collaborative networks show that there are several groups of researchers actively working in this field, with authors such as Marcos T.O. Pimenta, Anna Mercado, and Sergio Segura de León emerging as key figures. These collaborations are fundamental for the advancement of knowledge in the analysis of bounded variation functions and the 1-Laplacian operator, allowing the integration of different approaches and the consolidation of new lines of research. Finally, the study identifies emerging areas such as shape optimization and solution existence as recent research trends, suggesting that these topics could gain more prominence soon. In addition, the importance of continuing to explore the intersections between the analysis of functions of bounded variation and other areas of applied mathematics is recognized, which could open new avenues of research and interdisciplinary collaboration.

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