Energy transport problems in biophysics: BBM and Peyrad-Bishop models

Abstract-We develop analytical and numerical methods for calculating the solution of non-linear models of BBM system (3) ***and the DNA denaturation transition*** (4)***:***

$$\begin{aligned} &u_t - u_{xxt} - av_{xxt} + a_1 u_x + \\ &\quad + a_2 v^p v_x + u^p u_x + a_3 (u^p v)_x = f, \\ &v_t - v_{xxt} + a_1 v_x + a_2 u^p u_x + \\ &\quad + v^p v_x + a_3 (u v^p)_x = g, \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{\partial y^2}{\partial t^2} - \left[c_1 + 3c_2 \left(\frac{\partial y}{\partial x^2} \right)^2 \right] \frac{\partial y^2}{\partial x^2} \\ &\quad - D \left[\left(1 - \frac{b}{e^{\alpha y} + q} \right)^2 - 1 \right] = 0, \end{aligned} \tag{2}$$

The motivation of the problem is to predict energy transport of travelling waves in oceanographic engineering and biosystems such as transportation of electric energy in DNA or robotic genetics. Furthermore, also emphasize the importance of second order ordinary differential equations to obtain exact solution of coupled systems of partial differential equations.

Keywords: Coupled BBM equation, weighted Sobolev spaces, (G'/G)***-expansion method, DNA breathing.***

Energy transport problems in biophysics: BBM and Peyrad-Bishop models

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Abstract-We develop analytical and numerical methods for calculating the solution of non-linear models of BBM system* (3) *and the DNA denaturation transition* (4)*:

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$$\begin{aligned} &\frac{\partial y^2}{\partial t^2} - \left[c_1 + 3c_2\left(\frac{\partial y}{\partial x^2}\right)^2\right]\frac{\partial y^2}{\partial x^2} \\ &\quad - D\left[\left(1 - \frac{b}{e^{\alpha y} + q}\right)^2 - 1\right] = 0, \end{aligned} \tag{4}$$

The motivation of the problem is to predict energy transport of travelling waves in oceanographic engineering and biosystems such as transportation of electric energy in DNA or robotic genetics. Furthermore, also emphasize the importance of second order ordinary differential equations to obtain exact solution of coupled systems of partial differential equations.

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I. INTRODUCTION

Energy transport problems in mathematical physics and engineering lead to the analysis of systems of partial differential equations that are generally non-linear. In this article the travelling wave solutions that is caused in fluid mechanics is analyzed. We give a relatively simple proof to the local existence of the solutions of a coupled system of Benjamin-Bona-Mahony type in a weighted Sobolev spaces (wSs-BBM). T. R. Marchand emphasizes the asymptotic equivalence between e- BBM and e-KdV [1]. We have references about the interaction of the waves in Bisognin,E-Bisognin,V-Perla,G. [5], J. and Bona [2], Pereira, J. [14].

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We introduce the system for the Benjamin-Bona-Mahony equations (wSs-BBM):

$$\begin{aligned} &u_t - u_{xxt} - av_{xxt} + a_1u_x + \\ &\quad + a_2v^pv_x + u^pu_x + a_3(u^pv)_x = f \\ &v_t - v_{xxt} + a_1v_x + a_2u^pu_x + \\ &\quad + v^pv_x + a_3(uv^p)_x = g \\ &\text{In } -\infty < x < \infty\,,\ t > 0\,,\ p > 1 \end{aligned} \tag{5}$$

The unknowns are u, v defined in $\mathbb{R}^2_+ = (-\infty,\infty)\times[0,\infty)$. We consider the initial conditions

$$u(x,0) = u_0(x) \text{ and } v(x,0) = v_0(x) \tag{6}$$

for coupled system (5).

All these problems have the motivation in the ordinary differential systems:

$$X' = AX$$

$X = X(t)$ (n-coordinates), X =derivate of X, A =matrix $n\times n$.

Let us first introduce some notation. Let $X_r = H^2_r(\mathbb{R})\times H^2_r(\mathbb{R})$. Where $H^2_r(\mathbb{R})$ is a Hilbert space with the inner product $(u,v)_{r,2} = (M_r\Lambda^2u, M_r\Lambda^2v)_2$. Here $(\cdot,\cdot)_2$ is the inner product in $L^2(\mathbb{R})$. For $r\in\mathbb{R}^+$, let Λ^2 be the Pseudo-differential operator defined by $\Lambda^2 = (I-\partial_x^2)$. Furthermore, we set $M_rf(x) = (1+|x|^2)^{r/2}f(x)$. In order to apply semi-group theory, we must first transform the coupled system (5) to a first-order system.

For this purpose, We set

$$A = \begin{pmatrix} \Lambda^2 & -a\partial_x^2 \\ -a\partial_x^2 & \Lambda^2 \end{pmatrix}\ ,\ B = \begin{pmatrix} a_1 & 0 \\ 0 & a_1 \end{pmatrix}\ ,\ w = \begin{pmatrix} u \\ v \end{pmatrix}$$

$$(Fw)_x = \begin{pmatrix} \left(a_2\frac{v^{p+1}}{p+1} + \frac{u^{p+1}}{p+1} + a_3(u^p v)_x\right)_x \\ \left(a_2\frac{u^{p+1}}{p+1} + \frac{v^{p+1}}{p+1} + \ a_3(v^p u)_x\right)_x \end{pmatrix},$$

and we rewrite (5) and (6) as

$$\begin{cases} Aw_t + Bw_x + (Fw)_x = 0 & , \ x \in \mathbb{R}, \ t > 0 \\ w(x,0) = w_0(x) & , \ x \in \mathbb{R} \end{cases} \tag{7}$$

where $w_0 = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$.

This work with two equations is very important as they help to understand more complex systems. We can propose that this work serve as a model for the analysis of DNA vibrations with a lower number of base pairs. For more analysis with less pair of bases we have the references from [8] to [13] on these discussions.

II. ANALYSIS OF THE TRAVELLING WAVE SOLUTIONS FOR SOME COUPLED EQUATIONS

A. Theorems for the BBM equations

Before proceeding to the proof of the main result, we establish some preliminary lemmas.

Lemma II.1. *If A is as above, and let $\eta > 0$, $\delta > 0$, $r \geq 0$ such that*

$$0 < a < \text{mín}\left\{1, \frac{\delta}{\eta\delta + C_r}, \frac{\eta}{\eta\delta + C_r}\right\},$$

for some $C_r > 0$, then A is invertible and bounded below.

Proof. Let $w = (u, v) \in X_r$. We have

$$\begin{aligned} |Aw|^2_{L^2_r \times L^2_r} &= |u|^2_{r,2} + |v|^2_{r,2} + a^2(|v_{xx}|^2_{r,0} + |u_{xx}|^2_{r,0}) - \\ &\quad - 2a\int_{\mathbb{R}} M_r\Lambda^2 uv_{xx}dx - \int_{\mathbb{R}} M_r\Lambda^2 uu_{xx}dx \\ &\geq \ |u|^2_{r,2} + |v|^2_{r,2} + a^2(|v_{xx}|^2_{r,0} + |u_{xx}|^2_{r,0}) - \\ &\quad - 2a|u|_{r,2}|v_{xx}|_{r,0} - 2a|v|_{r,2}|u_{xx}|_{r,0} \end{aligned}$$

Since also $\eta > 0$ and $\delta > 0$, it follows that

$$\begin{aligned} |Aw|^2_{L^2_r \times L^2_r} \geq |u|^2_{r,2} + |v|^2_{r,2} + a^2(|v_{xx}|^2_{r,0} + |u_{xx}|^2_{r,0}) - \\ - a\eta|u|^2_{r,2} - \frac{a}{\eta}|v_{xx}|^2_{r,0} - a\eta|v|^2_{r,2} - \\ - \frac{a}{\eta}|u_{xx}|^2_{r,0} \end{aligned}$$

On the other hand, we know that $|u_{xx}|_{r,0} \leq C_r|u|_{r,2}$ for some constant $C_r > 0$. Where thereby

$$\begin{aligned} |Aw|^2_{L^2_r \times L^2_r} \geq \left[1 - a\left(\eta + \frac{C_r}{\delta}\right)\right]|u|^2_{r,2} \\ + \left[1 - a\left(\delta + \frac{C_r}{\eta}\right)\right]|v|^2_{r,2} \end{aligned}$$

And so, for some constant $C > 0$ which depend only on a and r, we obtain $|Aw|_{X_r} \geq C|w|_{X_r}$. This completes the proof. ■

Lemma II.2. *Let $g = (g_1, g_2) \in X_r$, and consider the assumptions of Lemma II.1. Then, the following holds.*

i) $K * \frac{\partial g}{\partial x} \in X_r$, *and there exists* $C > 0$ *such that* $\left|K * \frac{\partial g}{\partial x}\right|_{X_r} \leq C|g|_{X_r}$

ii) $A^{-1}g = K * g$, *where* $K = (K_{mn})_{1 \leq m,n \leq 2}$ *and each of the* $K'_{mn}s$ *satisfies* $K_{mn} = \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} e^{ixy}a_{mn}(y)dy$, $i = \sqrt{-1}$ *and we set*

$$\widehat{A^{-1}g}(y) = (a_{mn}(y))\hat{g}(y) \ , \ \hat{g}(y) = \begin{pmatrix} \hat{g}_1(y) \\ \hat{g}_2(y) \end{pmatrix}$$

Proof.

(i) Let $g = (g_1, g_2)$ be defined in the domain $D(A^{-1}) \subset X_r$. It follows easily from lemma II.1 that $\left|A^{-1}\frac{\partial g}{\partial x}\right|_{X_r} \leq C^{-1}|g|_{X_r}$

(ii) Note that $\widehat{K_{mn}}(y) = a_{mn}(y)$, therefore $\widehat{A^{-1}g}(y) = \widehat{K * g}(y)$.

According to part (i), $A^{-1}\frac{\partial g}{\partial x}$ is the infinitesimal generator of a C_0-semigroup in X_r, therefore the domain $D\left(A^{-1}\frac{\partial g}{\partial x}\right) = R(A)$ is dense in X_r furthermore, $R(A)$ is closed, and so $D\left(A^{-1}\frac{\partial g}{\partial x}\right) = X_r$.

That is, $A^{-1}g = K * g$ for all $g \in X_r$. ■

Theorem II.3. *Let $w_0 = (u_0, v_0) \in X_r$ be as above and suppose the assumptions from Lemma 2.1 hold. Then there exists a unique solution of the coupled system (5) and (6) satisfying $w \in C^0([0, T_0]; H^2_r(\mathbb{R})) \times C^0([0, T_0]; H^2_r(\mathbb{R}))$, for some $T_0 > 0$. In addition, we have $w_t = (u_t, v_t) \in C^0([0, T_0]; H^2_r(\mathbb{R})) \times C^0([0, T_0]; H^2_r(\mathbb{R}))$.*

Proof of Theorem II.3

Proof. Consider the space $X(T) = C^0([0,T];H_r^2(\mathbb{R})) \times C^0([0,T];H_r^2(\mathbb{R}))$. $X(T)$ is a Banach space when equipped with the norm $|w|_{X(T)} = \sup_{t\in[0,T]} |x|_{X_r}$. Let $R > 0$, and consider the set $Y_R(T) = \{w \in X(T) | |w(\cdot,t) - w_0|_{X_r} \leq R, w(x,0) = w_0\}$ with the induced norm by $X(T)$. We see that $Y_R(T)$ is a closed set. Furthermore, for every $t \in [0,T]$ the function $\Phi w(t)$ defined by

$$\Phi w(t) = w_0(x) - \int_0^t K(x) * \frac{\partial}{\partial x}(B\ w(s) + F\ w(s))ds$$

lie in $Y_R(T)$. In fact, it follows from Lemma II.2 that

$$|\Phi w(t) - w_0(x)|_{X_r} \leq C\int_0^t |B\ w(s) + F\ w(s)|_{X_r} ds.$$

On the other hand, from the definition of B, F and since $H_r^2(\mathbb{R})$ is a multiplicative Algebra. We see that

$$\begin{aligned} |B\ w(s) + F\ w(s)|_{X_r} &\leq C(1 + |u|_{r,2}^p + |v|_{r,2}^p) \\ &\quad (|u|_{r,2} + |v|_{r,2}) \\ &\leq C(1 + |w(s)|_{X_r}^p)|w(s)|_{X_r}, \end{aligned}$$

for some constant $C > 0$. Thus, using the inequality $|w(s)|_{X_r} \leq R + |w_0|_{X_r}$, we see from the above inequality that

$$\begin{aligned} |\Phi w(t) - w_0(x)|_{X_r} &\leq C\int_0^t [1 + (R + |w_0|_{X_r})^p] \\ &\quad (R + |w_0|_{X_r})ds \\ &\leq C[1 + (R + |w_0|_{X_r})^p] \\ &\quad (R + |w_0|_{X_r})T \end{aligned}$$

Hence, we choose $T > 0$, such that

$$C[1 + (R + |w_0|_{X_r})^p](R + |w_0|_{X_r})T \leq R$$

and being $\Phi w(\cdot, 0) = w_0$. We see that Φ maps the closed ball $Y_R(T)$ to itself. Similarly, we show that the operator Φ is a contraction. To see this, let $w_1,\ w_2 \in Y_R(T)$ and consider $w_1 = (u_1, v_1)$, $w_2 = (u_2, v_2)$. Then

$$\begin{aligned} |\Phi w_1(t) - \Phi w_2(t)|_{X_r} \leq \int_0^t &|K(x) * \\ &* \frac{\partial}{\partial x}[B\ (w_1(s) - w_2(s)) + \\ &+ F w_1(s) - F w_2(s)]|_{X_r} ds \end{aligned}$$

By using $|w_i(s)|_{X_r} \leq R + |w_0|_{X_r} = \alpha$, $i = 1, 24$. We find $|u_i|_{r,2} \leq \alpha$, $|v_i|_{r,2} \leq \alpha$. Where thereby we obtain

$$\begin{aligned} &|u_1^{p+1} - u_2^{p+1}|_{r,2} \leq \alpha^p(p+1)|u_1 - u_2|_{r,2} \\ &|u_1^{p+1} - u_2^{p+1}|_{r,2} \leq \alpha^p(p+1)|v_1 - v_2|_{r,2} \\ &|u_1^p v_1 - u_2^p v_2|_{r,2} \leq p\alpha^p|u_1 - u_2|_{r,2} + \alpha^p|v_1 - v_2|_{r,2} \\ &|v_1^p u_1 - v_2^p u_2|_{r,2} \leq p\alpha^p|v_1 - v_2|_{r,2} + \alpha^p|u_1 - u_2|_{r,2} \end{aligned}$$

Therefore, by using the above inequalities, we find that

$$\begin{aligned} &|B\ (w_1(s) - w_2(s)) + F w_1(s) - F w_2(s)|_{X_r}^2 = \\ &= |a_1|^2[|u_1 - u_2|_{r,2}^2 + |v_1 - v_2|_{r,2}^2] + \\ &\quad + \frac{a_2^2 + 1}{(p+1)^2}\left[|u_1^{p+1} - u_2^{p+1}|_{r,2}^2 + |v_1^{p+1} - v_2^{p+1}|_{r,2}^2\right] + \\ &\quad + a_3^2[|u_1^p v_1 - u_2^p v_2|_{r,2}^2 + |u_1 v_1^p - u_2 v_2^p|_{r,2}^2] \\ &\leq |a_1|^2|w_1 - w_2|_{X_r}^2 + (a_2^2 + 1)\alpha^{2p}|w_1 - w_2|_{X_r}^2 + \\ &\quad + a_3^2 2p^2\alpha^{2p}|w_1 - w_2|_{X_r}^2 + 2\alpha^{2p}|w_1 - w_2|_{X_r}^2 \\ &= C|w_1 - w_2|_{X_r}^2, \end{aligned}$$

where $C = C(\alpha, p)$. Hence, it follows from Lemma II.2 that

$$\begin{aligned} |\Phi w_1(t) - \Phi w_2(t)|_{X_r} &\leq C\int_0^t |w_1(s) - w_2(s)|_{X_r} ds \\ &C\,|w1 - w2|_{X(T)}\int_0^t ds. \end{aligned}$$

Then, $|\phi w_1 - \phi w_2|_{X(T)} \leq CT\,|w_1 - w_2|_{X(T)}$. Next, take $T > 0$ such that $CT < 1$. If we choose T as being

$$T = T_0 < min\left\{\frac{1}{C[1+(R+|w_0|_{X_r})^p](R+|w_0|_{X_r})}, \frac{1}{C}\right\}$$

We thus end up with a new mapping $P : Y_R(T_0) \longrightarrow Y_R(T_0)$, hence P is a contraction. It follows from Banach's fixed point theorem that P has a unique fixed point $u \in Y_R(T_0)$, which solves the following equation

$$w(t) = w_0(x) - \int_0^t K(x) * \frac{\partial}{\partial x}(B\ w(s) + F\ w(s))ds$$

In addition, $w(x,0) = w_0(x)$.

We shall now prove from the integral equation that the derivative w_t exists and belongs to $X(T)$.

We define $z(x,t) = \int_0^t \frac{d}{dx}K(x) * H\ w(s)ds$, where we have set $Hw(t) = Bw(t) + Fw(t)$, for all $t \in [0, T_0]$. We choose for any $h > 0$ such that $t + h \in [0, T_0]$. Hence we find

$$\frac{z(x,t+h) - z(x,t)}{h} = \frac{1}{h}\int_t^{t+h} \frac{d}{dx}K(x) * H\ w(s)ds.$$

Using the mean value theorem for Bochner's integrals, we obtain the following identity

$$\frac{z(x,t+h) - z(x,t)}{h} = \frac{d}{dx}K(x) * H\ w(t_1),$$

or some $t_1 \in [t, t+h]$. Letting, $h \to 0^+$. We find

$$\frac{\partial^+ z}{\partial t} = \frac{d}{dx}K(x) * H\ w(t).$$

By substituting $t-h$ for t in the identity above, we also obtain the existence of the left derivative

$$\frac{\partial^- z}{\partial t} = \frac{d}{dx} K(x) * H \ w(t).$$

In summary, we find that

$$\frac{\partial z}{\partial t} = \frac{d}{dx} K(x) * H \ w(t), \text{ for all } t \in [0, T_0].$$

We now turn to the integral equation, from which we find

$$\begin{aligned} w_t &= -\frac{\partial z}{\partial t} = -\frac{d}{dx} K(x) * [Bw(t) + Fw(t)] \\ &= K * \frac{\partial}{\partial x}[B \ w(t) + F \ w(t)] \\ &= -A^{-1}\frac{\partial}{\partial x}[B \ w(t) + F \ w(t)] \\ &= -A^{-1}[B \ w_x + (F \ w)_x]. \end{aligned}$$

Which can be written as $Aw_t = -(Bw_x + (Fw)_x)$. And so, we achieve to solve the initial value problem

$$\begin{cases} Aw_t + Bw_x + (Fw)_x = 0 \\ w(x,0) = w_0(x) \end{cases} \tag{8}$$

Furthermore, we see that

$$\begin{aligned} w_t &= (u_t, v_t) \in X(T_0) \\ &= C^0([0,T_0]; H_r^2(\mathbb{R})) \times C^0([0,T_0]; H_r^2(\mathbb{R})) \end{aligned}$$

Since

$$w_t = \frac{\partial}{\partial t}\Phi w = -A^{-1}\frac{\partial}{\partial x}[B \ w(t) + F \ w(t)],$$

which is a continuos function on $[0, T_0]$ with values in $H_r^2(\mathbb{R})$. Whereas the uniqueness of solution, it follows from Gronwall's Lemma, Renardy,M. [6].

We have used an infinitesimal generator of a strongly continuos semigroup(see Pazy,A.[7]). The solutions of a coupled system of wSs-BBM equations with initial data, it was considered the domain of the infinitesimal generator as a Banach space equipped with a suitable norm and so, we have obtained a contraction mapping Φ which it defined in the closed R-ball $(Y_R(T))$ of the space $X(T) = C^0([0,T]; H_r^2(\mathbb{R})) \times C^0([0,T]; H_r^2(\mathbb{R}))$ into itself. The unique fixed point w(t) for the operator Φ is in fact differentiable and solve the initial value problem

$$\begin{cases} Aw_t + Bw_x + (Fw)_x = 0 \\ w(x,0) = w_0(x) \end{cases}$$

■

III. EXACT SOLUTIONS

A. BBM equations

Case 1: Small-amplitude long waves on the surface of water in a channel: BBM equation for $u = u(x,t)$:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x} - \frac{\partial^2}{\partial x^2}\frac{\partial u}{\partial t} = 0$$

Substituting the solution $u(x,t) = u(\zeta)$, $\zeta = x - ct$ we obtain the *ordinary differential equation*

$$-c\frac{du}{d\zeta} + \frac{du}{d\zeta} + u\frac{du}{d\zeta} - c\frac{d^3u}{d\zeta^3} = 0$$

The solution is the form

$$u(x,t) = 3(c-1)\text{sech}^2\left(\frac{x-ct}{2}\sqrt{\frac{c-1}{c}}\right)$$

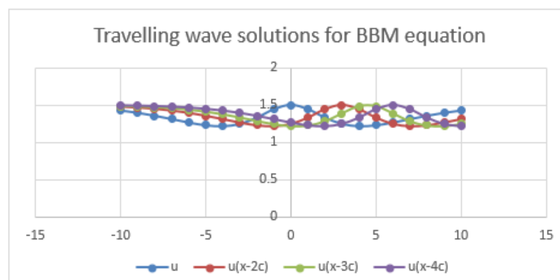

Fig. 1: Travelling wave solutions for BBM equations traveling to the right

Case 2: Small-amplitude long waves on the surface of water in a channel with interactions (u, v):

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial v}{\partial x} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} - \frac{\partial^2}{\partial x^2}\frac{\partial u}{\partial t} &= 0 \\ \frac{\partial v}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial (uv)}{\partial x} - \frac{\partial^2}{\partial x^2}\frac{\partial v}{\partial t} &= 0 \end{aligned}$$

Substituting the solution $u(x,t) = u(\zeta)$ and $v(x,t) = v(\zeta)$, $\zeta = x - ct$ we obtain one system of the *ordinary differential equations*

$$-c\frac{du}{d\zeta} + \frac{dv}{d\zeta} + \frac{d\,(uv)}{d\zeta} + c\frac{d^3u}{d\zeta^3} = 0$$

$$-c\frac{dv}{d\zeta}+\frac{du}{d\zeta}+\left(\frac{1}{2}\right)\frac{d(u^2)}{d\zeta}+\left(\frac{1}{2}\right)\frac{d(v^2)}{d\zeta}+c\frac{d^3v}{d\zeta^3}=0$$

The (G'/G)–expansion method help to solve this type of equations [13].

Particular solutions:

$$u=6\frac{1}{(x+t)^2} \qquad , \qquad v=-u$$

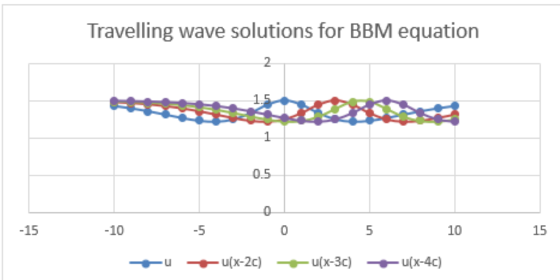

Fig. 2: Travelling wave solutions for coupled BBM equations

B. DNA denaturation transition $(q=0)$

We recall that the expansión method G'/G for the type

$$F(u,u_t,u_x,u_{xx},u_{tt})=0 \tag{9}$$

with

$$u(x,t)=U(\xi)=U(x-vt) \tag{10}$$

Give the relation

$$F(U,-vU',U',-vU'',U'',v^2U'',\dots)=0 \tag{11}$$

The solution of (11) is

$$U(\xi)=\sum_{n=0}^{m}a_n(G'/G)^n \tag{12}$$

where a_n, $n=0,1,2,\dots,m$ and $G(\xi)$

$$G''+\lambda G'\mu G=0 \tag{13}$$

Can see general solution in (21) and (22) also

$$U'^{(\xi)}=\sum_{n=1}^{m}na_n\left(\frac{G'}{G}\right)^{n-1}\left[\frac{G''(\xi)}{G(\xi)}-\frac{G'(\xi)^2}{G(\xi)^2}\right], \tag{14}$$

On the other hand (13)

$$\frac{G''(\xi)}{G(\xi)}=-\lambda\frac{G'(\xi)}{G(\xi)}-\mu,$$

gives the relations:

$$U'(\xi)=\sum_{n=1}^{m}na_n\left(\frac{G'}{G}\right)^{n-1}\left[-\lambda\frac{G'(\xi)}{G(\xi)}-\mu-\frac{G'(\xi)^2}{G(\xi)^2}\right] \tag{15}$$

$$U'(\xi)=-\sum_{n=1}^{m}na_n\left[\left(\frac{G'(\xi)}{G(\xi)}\right)^{n+1}+\lambda\left(\frac{G'(\xi)}{G(\xi)}\right)^{n}+\mu\left(\frac{G'(\xi)}{G(\xi)}\right)^{n-1}\right] \tag{16}$$

$$\begin{aligned}U''(\xi)=\sum_{n=1}^{m}na_n\Bigg[&(n+1)\left(\frac{G'(\xi)}{G(\xi)}\right)^{n+2}+\\&+(2n+1)\lambda\left(\frac{G'(\xi)}{G(\xi)}\right)^{n+1}+\\&+n(\lambda^2+2\mu)\left(\frac{G'(\xi)}{G(\xi)}\right)^{n}+\\&+(2n-1)\lambda\mu\left(\frac{G'(\xi)}{G(\xi)}\right)^{n-1}+\\&(n-1)\mu^2\left(\frac{G'(\xi)}{G(\xi)}\right)^{n-2}\Bigg]\end{aligned} \tag{17}$$

For the study of DNA breathing:

$$\begin{aligned}&u_{tt}-(c_1+3c_2u_x^2)u_{xx}-\\&-2aDe^{-au}(e^{-au}-1)=0\end{aligned} \tag{18}$$

Using the relation $u(x,t)=U(\xi)=U(x-vt)$ we obtain the differential equation

$$\begin{aligned}&v^2U''-(c_1+3c_2(U')^2)U''-\\&-2aDe^{-aU}(e^{-aU}-1)=0\end{aligned} \tag{19}$$

We find the solution

$$\begin{aligned}u(x,t)&=-\frac{1}{a}\ln(\varphi(\xi))\\&=-\frac{1}{a}\ln\left(\pm\frac{2\sqrt{3}}{a^2}\sqrt{\frac{c_2}{D}}\left(\frac{G'(\xi)}{G(\xi)}\right)^2\pm\frac{D+\sqrt{D^2-DC}}{D}\right)\end{aligned} \tag{20}$$

When $\lambda^2-4\mu>0 \Rightarrow -\mu>0$

$$\frac{G'(\xi)}{G(\xi)}=\sqrt{-\mu}\left(\frac{C_1\text{senh}(\sqrt{-\mu}\xi)+C_2\text{cosh}(\sqrt{-\mu}\xi)}{C_1\text{cosh}(\sqrt{-\mu}\xi)+C_2\text{senh}(\sqrt{-\mu}\xi)}\right)$$

And the solution is (23), if $C_1^2>C_2^2$ we obtain (24).

And if $C_1^2<C_2^2$ and $\theta=\tanh^{-1}(C_1/C_2)$, we obtain the relations

$$\frac{G'(\xi)}{G(\xi)}=\sqrt{\mu}\left(\frac{-C_1\text{sen}(\sqrt{\mu}\xi)+C_2\text{cos}(\sqrt{\mu}\xi)}{C_1\text{cos}(\sqrt{\mu}\xi)+C_2\text{sen}(\sqrt{\mu}\xi)}\right)$$

and (25). If $C_1^2>C_2^2$ we obtain (26).

$$\left(\frac{G'(\xi)}{G(\xi)}\right)=\begin{cases}-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\text{senh}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\text{cosh}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\text{cosh}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\text{senh}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)\\ \text{if } \lambda^2-4\mu>0\end{cases} \tag{21}$$

$$\left(\frac{G'(\xi)}{G(\xi)}\right)=\begin{cases}-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{-C_1\text{sen}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}{C_1\cos\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\text{sen}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)}\right)\\ \text{if } \lambda^2-4\mu<0\end{cases} \tag{22}$$

$$u(x,t)=-\frac{1}{a}\ln\left(\frac{D+\sqrt{D^2-DC}}{D\text{cosh}^2\left(-\frac{\sqrt{6}}{6}\sqrt{-\frac{\sqrt{3}a^2}{c_2}\sqrt{\frac{c_2}{D}}(D+\sqrt{D^2-DC})}\xi+\theta\right)}\right) \tag{23}$$

$$u(x,t)=-\frac{1}{a}ln\left(\frac{-D-\sqrt{D^2-DC}}{D\text{cosh}^2\left(-\frac{\sqrt{6}}{6}\sqrt{-\frac{\sqrt{3}a^2}{c_2}\sqrt{\frac{c_2}{D}}\left(D+\sqrt{D^2-DC}\right)}\xi+\theta\right)-1}\right) \tag{24}$$

$$u=\frac{-1}{a}\ln\left(\frac{D+\sqrt{D^2-DC}}{D}\left(\tan^2\left(\frac{\sqrt{6}}{6}\sqrt{\frac{\sqrt{3}a^2}{c_2}\sqrt{\frac{c_2}{D}}(D+\sqrt{D^2-DC})}\xi+\theta\right)+1\right)\right) \tag{25}$$

$$u=\frac{-1}{a0}\ln\left(\frac{D+\sqrt{D^2-DC}}{D}\left(\cot^2\left(\frac{\sqrt{6}}{6}\sqrt{\frac{\sqrt{3}a^2}{L_2}\sqrt{\frac{c_2}{D}}(D+\sqrt{D^2-DC})}\xi+\theta\right)+1\right)\right) \tag{26}$$

IV. RESULTS AND DISCUSSIONS

We can complete the existence of solutions in Sobolev spaces with the problems of energy location and control. Likewise, the expansion method to solve the partial differential equation as an ordinary second order differential equation allows one to conjecture that in particular cases analyzed there will be an energetic decay (Fig. 3).

The numerical solution of DNA breathing is given in [9]. We do not use the genetic sequence information in the partial differential equation.

V. CONCLUSIONS

The traveling waves can be considered as pulses that travel according to the dynamics of the ordinary differential equations of the second order. Also its existence of generalized solutions can be given in weighted Sobolev spaces. We can conjecture that in particular cases analyzed there will be an energetic decay for the solutions of coupled BBM equations.

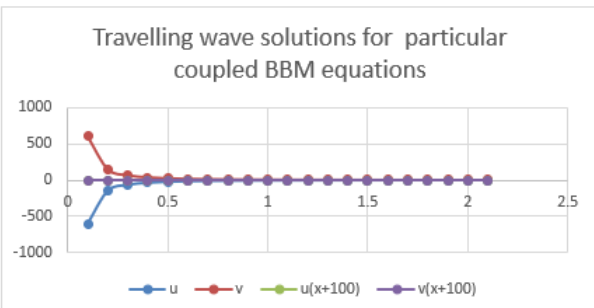

Fig. 3: Travelling wave solutions for coupled BBM equations for $t = 0$ and $t = 100$

From our analysis it becomes clear that the (G'/G)–expansion method leads to particular solutions with energy decay with time of the traveling waves.

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