From Geometry to a Digital Twin: Partially-Decoupled 3UPS-3UPRRR Parallel Robot for R-STEM Education

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***Abstract*– This paper present the design of a 3U<u>P</u>S-3U<u>P</u>RRR parallel robot . The robot have partially-decoupled motions, the position is driven by concurrent internal limbs in a tetrahedral structure, and the orientation is driven by external limbs. We selected the UPS limbs for orientation and 3UPRRR for position. We first create an inverse geometric model in GeoGebra and generate a digital model for simulation in MuJoCo. Finally we proposed a CAD design in SolidWorks..**

***Keywords*-- parallel robot; screw theory; geometric analysis; r-stem.**

I. Introduction

Parallel robots have gained attention because of their superior advantages over serial robots. A parallel robot consists of a platform attached to a base with several serial chains called limbs, some texts have been written foundational concepts of parallel mechanisms [1][2][3]. A complete general foundational book about parallel robots is [4]. The structural synthesis of parallel robots is in [5]. Parallel robots with constraint motions [6]. The dynamics of parallel robots is in [7].

In this work, we use the Screw Theory approach to study the kinematics [8] similarly as in [9].

Robotics is a complex field, which includes different areas such as geometry, electronics, programming, and control theory. Undergraduate robotic courses typically study mobile and serial robots. Nowadays, fundamental concepts in STEM (Science, Technology, Engineering, and Mathematics) education include Artificial Intelligence and Data Science.

Robotics-based STEM (R-STEM) is a growing field [11] [10][12]. In [13] we can take an historical overview for K-12 STEM Education.

In this work, we propose a parallel robot to understand the concepts of geometry through Screw Theory, using a reconfigurable six degrees of freedom parallel robot as a case study. This understanding is a valuable tool for STEM education.

Parallel robots have various applications like pick and place, 3D printers, computer numerical control machines, simulation platforms, robotic wrists, prosthesis, spines, backbones, and central mast platforms. Some examples of low-mobility parallel robots include delta, Shoenflies motion, spherical, and planar parallel mechanisms.

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The classification of parallel robots usually depends on number of degrees of freedom, symmetry, redundant actuators, planar or spatial mechanisms.

They are also classified as coupled, partially or totally decoupled or hybrid serial-parallel mechanisms.

The limbs kinematic chains are composed from revolute (R), prismatic (P), universal (U), and spherical (S) joints. When a joint is driven by a torque or force actuator, often bold and underlined letters (<u>**R**</u>, <u>**P**</u>) are used, prismatic linear actuators are preferred when space and high loads are needed.

In this work, we use a six degree of freedom robot with the central point of the platform driven by three linear actuators.

When the three central actuators are blocked, the orientation of the platform depends on the three external limbs. Then, we get three degrees of freedom. Blocking only two central actuators results in a rotation about the axis between the base points of the blocked actuators, resulting in 4 degrees of freedom.

If the three central actuators are in parallel, synchronized with the same position control, then the robot moves on the z-direction, also with 4 degrees of freedom.

We investigate the movements by blocking and controlling the central kinematic chains. We use a U<u>**P**</u>RRR concurrent configuration for the robot implementation.

A. Related Work

We searched for similar robots with central mast and partially decoupled motions. A novel partially-decoupled mechanism is in [14]. The structure design and kinematic analysis of a partially-decoupled 3T1R PM (Parallel Manipulator) is in [15]. A new approach to inverse kinematic solution for a partially-decoupled robot is in [16]. Early works on a three degrees of freedom partially decoupled robot with linear actuators are in [17], and a redundant partially decoupled robot [18]. Finally a XY-Theta positioning table with partially decoupled parallel kinematics is in [19].

B. Our Contribution

In this work, we designed a reconfigurable mechanism composed from a platform attached to a base through two groups of three actuators, an internal and an external group.

We reconfigure the degrees of freedom by blocking and paralleling the position control of the central actuators. We constrain the motion and show how the projection of redundant actuators act as an equivalent single actuator.

We start from the basics concept of simple kinematic chains, then we select an universal-prismatic-spherical (U**P**S) chain, is a compact and versatile configuration.

For simulation, we used GeoGebra 5 software, creating an interactive inverse geometric model.

II. Materials and Methods.

A. *Structure*

The main structure of the robot is composed of a platform and a base. The base is an equilateral triangle, which has three points B1, B2, and B3. The triangle center is in the coordinates' origin O. The base has three points located on the mean length from each side, the points are B4, B5, and B6. The Fig. 1. To illustrate the main structure, the platform is at an initial position and orientation with respect to the origin O.

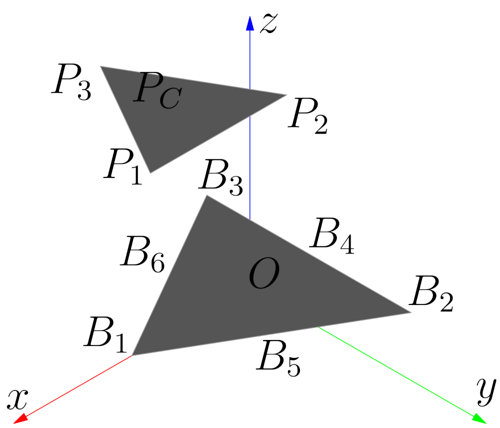

Fig. 1. Base and platform representation.

B. *Basic Limbs*

It is necessary to attach the platform to the base with different limbs, which are simple kinematic chains with an active joint. The Fig. 2 shows some basic kinematic chains with an active joint. Planar examples, from the left to the right, are **P**PR, **R**RR, R**P**R and **P**RR.

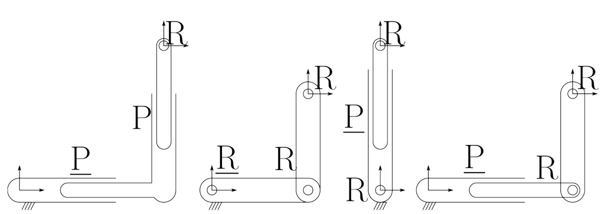

Fig. 2. Planar kinematic chains.

From the kinematic chains in the Fig. 2, the R**P**R have the most compact size.

An analog compact configuration in three dimensions is the universal-prismatic-spherical (U**P**S) kinematic chain. In this work, we use the U**P**S kinematic chains for the external limbs.

C. *Closed Loops*

Parallel mechanisms have closed loops, also they have the same types of kinematic chains.

In the Fig. 3, we show some closed loops using **P**PR, **R**RR, R**P**R and **P**RR limbs.

From the top-down and left-right order, the Fig. 3 illustrates 2-**R**RR, 2-R**P**R, 2-**P**PR, and 2-**P**RR closed loop mechanisms, respectively. The dotted circles and lines are the free trajectories for the passive joints when the active joints are blocked. The passive revolute joints intersections are in two points. When {P} have multiple solutions, they are called assembly modes.

The frames {A} and {B} are fixed and the frame {P} represents the end effector position.

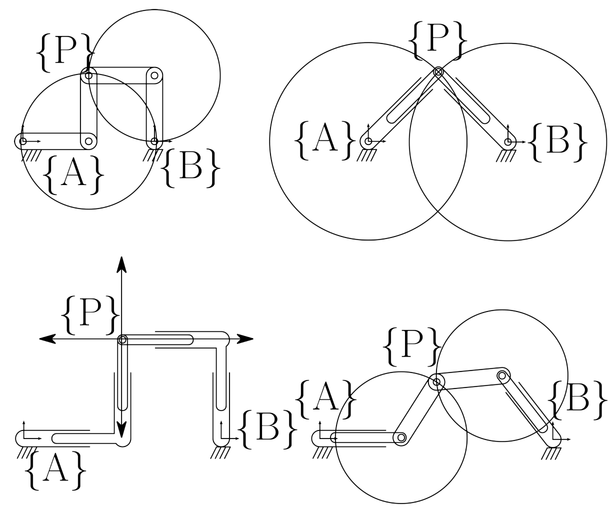

Fig. 3. Closed loops.

The 2-R**P**R closed loop is compact and conforms a triangle. In three dimensions a similar compact configuration is 3-U**P**S, which conforms a tetrahedron.

D. *Basic Screw Representation*

The screw theory representation is useful for the kinematics analysis in robotics.

In a serial chain, a rotation and a translation for the end effector can be represented by the product of exponential matrices. It is the concept of a spatial transformation, a rigid body movement can be represented with a rotation and a translation in a instantaneous screw axis \$, similar to a screw movement with the right-hand rule.

The Chasles' theorem establishes that a rotation $\boldsymbol{\theta}_1$ about an screw axis $\$_1$, followed by a translation $\boldsymbol{\theta}_2$ along $\$_2$ is equivalent to a $\boldsymbol{\theta}_T$ rotation about a total screw axis $\$_T$.

The exponential expressions are 4x4 homogeneous transformation matrices. From an initial configuration matrix A_0 to a final configuration matrix A, we can apply an homogeneous transformation matrix by left multiplying the initial configuration. This procedure is a matrix product application to homogeneous transformations in spatial geometry, very useful in mathematics and physics.

In the Fig. 4, we show the screw theory basic concept for a rotation followed by a translation.

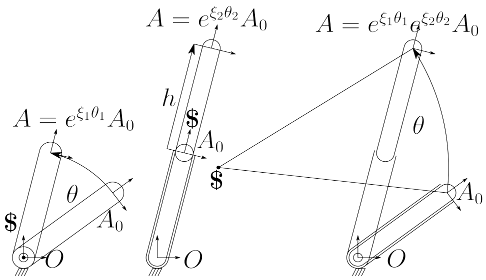

Fig. 4. Screw representation for a rotation followed by a translation.

E. Schematic Design

For the central limbs we use U**P**RRR kinematic chains, and for the external limbs, U**P**S chains.

We placed the central limbs U joint to the points B_4, B_5, and B_6 on the base, and end RRR joint to the P_C point on the platform. They form a tetrahedral structure, which have good structural properties.

The external limbs join the platform to the base from B_1 to P_1, B_2 to P_2, and B_3 to P_3, respectively.

The Fig. 5 shows the parallel robot's schematic representation. The red, green and blue circles represents the number of degrees of freedom in each attaching point.

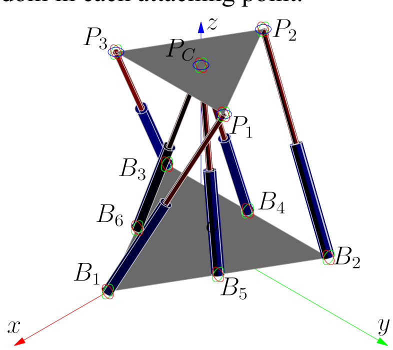

Fig. 5. Schematic representation of the six degrees of freedom partially decoupled robot.

B. Geometric Analysis.

The geometric analysis of the robot consists on forward and inverse analysis. The inverse analysis is about of computing the limbs lengths from the platform position and orientation.

The inverse geometric analysis have graphical and analytical methods.

In this work we use GeoGebra 5 to solve graphically the inverse geometric analysis.

For the simulation, we start from the base radius, by creating a slider r. Then, we create an equilateral triangle representing the base. For the position, we create three sliders for each coordinate.

The platform orientation is in angle-axis representation. We place a slide for the angle, and the axis is in the z-positive direction.

The limbs lengths are computed from the position of the platform.

The Fig. 6 shows the detailed 2D screen for the inverse geometric model.

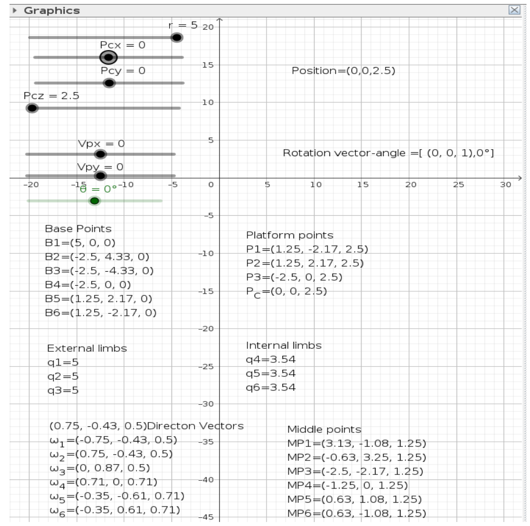

Fig. 6. Detail of the sliders and the base in the GeoGebra 5 2D graphics view.

The 3D figure view in GeoGebra 5 let us change the point controlling the axis.

We show a capture for the example position and orientation in the Fig. 7.

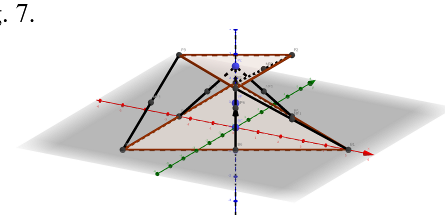

Fig. 7. 3D graphics view in GeoGebra for the inverse geometric model.

From the GeoGebra geometric model, we translate the structure to an XML file for MuJoCo simulation. We first define the options, default and asset sections. The Fig. 8 shows the XML sections.

```xml
<mujoco model="parallel_robot">
  <option>
    <flag eulerdamp ="enable" gravity="enable" contact="enable"/>
  </option>
  <default>
    <geom rgba=".8 .6 .4 1"/>

  <default class="translation">
      <position ctrllimited="true" ctrlrange="0 2.5" kp="100000"/>
    </default>
  </default>

  <asset>
    <texture type="skybox" builtin="gradient" rgb1="1 1 1" rgb2=".6 .8 1" width="256" height="256"/>
    <mesh file="glass.stl" refpos="0 0 0" refquat="1 0 0 0" />
  </asset>
```

Fig. 8. Options, defaults and assets for MuJoCo.

Then, we defined "*worldbody*", "*floor*", the platform and a glass representation as we show in the Fig. 9.

```xml
<worldbody>
<!-- defining the base -->
  <geom pos="0 0 -0.1" type="plane" size="5 5 .01"/>
  <light pos="0 0 20"/>
<site name="bc" pos="0 0 0" size="0.1"/>
<!-- platform -->
<body name = "platform">
  <freejoint/>
  <geom type="cylinder" fromto="0 0 2.6 0 0 2.65" size="2.5" mass="1"/>
  <site name="p1" pos="1.25 -2.17 2.5" size="0.1"/>
  <site name="p2" pos="1.25 2.17 2.5" size="0.1"/>
  <site name="p3" pos="-2.5 0 2.5" size="0.1"/>
  <site name="pc" pos="0 0 2.5" size="0.1"/>
</body>
<body>
 <freejoint/>
 <geom name="glass" type="mesh" mesh="glass" pos="0 0 2.63" rgba=".85 .79 .72 0.7" density="1"/>
</body>
```

Fig. 9. "Worldbody", "floor", platform and glass in XML for MuJoCo simulator.

The six limbs have similar structure, we show the limb1 representation. We show the limb 1 representation in the Fig. 10.

```xml
<!-- limb 1 -->
<body name = "ujoint1">
  <site name="b1" pos="5 0 0" size="0.1"/>
  <geom type="sphere" size="0.01" pos="5 0 0"  density="0.01"/>
  <joint name="u1a" type="hinge" axis = "1 0 0" pos="5 0 0"/>
  <joint name="u1b" type="hinge" axis = "0 1 0" pos="5 0 0"/>
  <geom type="cylinder" size="0.1" fromto="5 0 0 3.13 -1.08 1.25" density="0.01"/>
  <body name ="cylplat1" >
    <site name="bc1" pos="5 0 0" size="0.1"/>
     <geom type="cylinder" fromto="5 0 0 1.25 -2.17 2.5 " size="0.05" />
    <joint name= "q1" type="slide" axis="-0.75 -0.43 0.5" limited="true" range="0 4"/>
  </body>
</body>
```

Fig. 10. Structure for each limb.

Finally, we defined connections, sensors, and actuators. This is an easy way to define closed loops in MuJoCo. We show this in the Fig. 11.

```xml
<equality>
 <connect name="cyl1"  body1="cylplat1" body2="platform" anchor="1.25 -2.17 2.5"/>
 <connect name="cyl2"  body1="cylplat2" body2="platform" anchor="1.25 2.17 2.5"/>
 <connect name="cyl3"  body1="cylplat3" body2="platform" anchor="-2.5 0 2.5"/>
 <connect name="cyl4"  body1="cylplat4" body2="platform" anchor="0 0 2.5"/>
 <connect name="cyl5"  body1="cylplat5" body2="platform" anchor="0 0 2.5"/>
 <connect name="cyl6"  body1="cylplat6" body2="platform" anchor="0 0 2.5"/>
</equality>

<sensor>
 <framepos name ="plpos" objtype="site" objname="pc"/>
</sensor>

<actuator>
 <position name="act1"  site="b1" refsite="bc1" gear="0.75 0.43 -0.5 0 0 0" class="translation"/>
 <position name="act2"  site="b2" refsite="bc2" gear="-0.75 0.43 -0.5 0 0 0" class="translation"/>
 <position name="act3"  site="b3" refsite="bc3" gear="0 -0.87 -0.5 0 0 0" class="translation"/>
 <position name="act4"  site="b4" refsite="bc4" gear="-0.71 0 -0.71 0 0 0" class="translation"/>
 <position name="act5"  site="b5" refsite="bc5" gear="0.35 0.61 -0.71 0 0 0" class="translation"/>
 <position name="act6"  site="b6" refsite="bc6" gear="0.35 -0.61 -0.71 0 0 0" class="translation"/>
</actuator>
```

Fig. 11. Connections, sensors, and actuators.

We used OpenScad to create the glass mesh. This model will be placed on the platform to demonstrate rotation and translation.

III. RESULTS

First, we experimented with the minimum and maximum limb values in GeoGebra 5. We summarized the minimum 2.5 and maximum 5 z values with a radius r=5. We show the minimum values in Fig. 12.

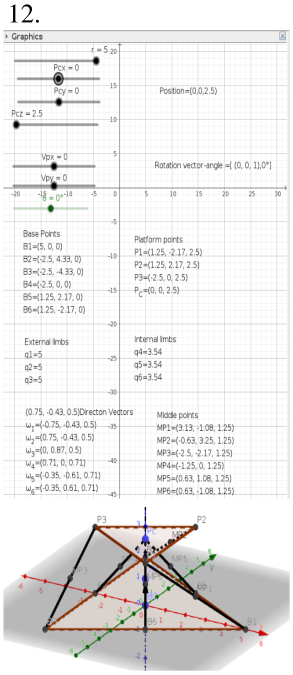

Fig. 12. GeoGebra 5 with minimum z value.

For the maximum values, the results are in Figure 13.

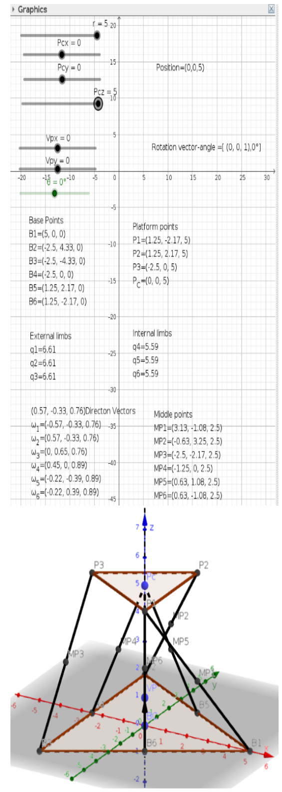

Figure 13. Resulting maximum values.

The reader can play with the model in [20].

The resulting model for the minimum values of the robot's limbs is in the Fig. 14.

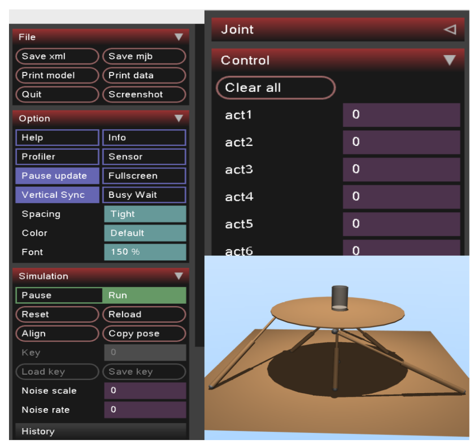

Fig. 14. Resulting simulation in MuJoCo 3.1.1.

We used SolidWorks 2022 and designed the platform in such way that can be easily exported to the simulators.

In the Fig. 15, $\$_1$, $\$_2$, and $\$_3$ are the screws representing the motion direction for the external limbs prismatic actuators, and P is the end effector central point. Also we show the detailed view of the central RRR joint.

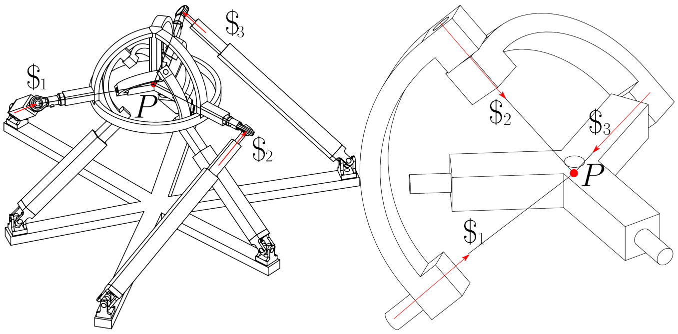

Fig. 15. Parallel robot with 6 degrees of freedom.

IV. CONCLUSIONS

In this work, we proposed a partially-decoupled parallel robot using UPS for external limbs and UPRRR kinematic chains for internal limbs.

The geometric analysis of a robot is a good application example to understand spatial geometry concepts, like rigid body position and orientation.

Linear algebra and spatial geometry are important fields of study in basic STEM education.

Engineering application in robotics as multidisciplinary nature helps to connect fields like electronics, informatics, mechanics and arts.

A six degrees of freedom parallel robot have real applications in platforms for simulators, gaming, antennas, solar panels, vibration, computed numerical control machines, rehabilitation, robotic heads, pan-tilt cameras, etc.

Partially-decoupled motion simplifies the kinematics analysis and control in parallel mechanisms.

The final prototype can be imported to several simulators like Gazebo, CoppeliaSim, Webots and MuJoCo.

The model can be 3D printed in a desktop printer.

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