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Two-dimensional numerical study of the velocity profile in an incompressible laminar flow with solid particles on a flat plate

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Abstract– ***This article presents a two-dimensional numerical study of the velocity profile of an incompressible laminar flow with particles on a flat plate. The model was developed using COMSOL MULTYPHISICS 6.0 software. Various cases are presented varying parameters such as volumetric fraction of particles (α_p), relative particle size (Rsp) and Stokes number (Stk), entry velocity of particles (Vp) and particle entrance zone (H). This study focused on observing how these parameters affect the fluid flow profile velocity. The velocity profile of particle-laden flows was compared to the Blasius velocity profile (particle-free flow). The results show three zones, the first zone near the plate is an acceleration zone (when $Vp<V_f$) or deceleration zone (when $Vp<V_f$); a second zone that works as a transition to go from the affected zone to the non-affected zone; and a third zone away from the plate where no acceleration neither deacceleration are produce. This particle interfere with the development of velocity profiles increases proportionally with the volumetric fraction of particles (α_p), while it occurs inversely proportional to the relative particle size (Rsp). Increasing the Stokes number also generates variation in the flow. Additionally, it was found that the volume fraction of particles (α_p) and the relative particle size (Rsp) have a linear proportionality with the relative variation of the fluid velocity. For cases with low particle presence, the effects of the particle can be disregarded and considered the flow as particle-free flow, as well as for low Stokes numbers.***

Keywords--Velocity profile, flat plate, laminar regime, particles laden.

I. Introduction

Particles-laden fluids are a fundamental role in many aspects of dynamics physics fluids. For these fluids, one of the phases is continuous (liquid or gas) and the other phase is formed by particles. Many industrial processes involve flows with particles in the fields of aeronautics, oil, gas, etc. In addition to the friction generated between the continuous phase and the surface of the flat plate, the mere presence of particles could alter the behaviour or movement of the fluid, and thus its interaction with the flat plate.

There are few studies of laminar flows with particles on a flat plate that analyse the behaviour of the flow based on different parameters such as the Stokes number, the size of the particles, etc. Osiptsov, A.N. (1980) studied a laminar boundary layer of a disperse medium on a flat plate [1]. He carried out a numerical study of a two-phase steady flow composed of solid particles in a gas of low viscosity on a semi-infinite two-dimensional flat plate. The author studied the incompressible carrier phase with a small concentration of particles. He also considered that all the particles were spherical with the same radius and that the density of the particles was much greater than the density of the gas. The objective was to investigate how different applied velocities influenced the flow structure in the boundary layer for the different phases.

Stability of a dusty-gas laminar boundary layer on a flat plate was studied by Asmolov, E.S. and Manuilovich S.V. (1998) [2]. They considered the linear stability of the incompressible flow of the boundary layer of dusty gas over a semi-infinite flat plate, under the assumption that the particles were solely under the action of Stokes drag. The authors used particle concentrations (f) equal to 0.1 and 1. Additionally, they used a Reynolds number equal to 1000 to compare their velocity profile results with the Blasius velocity profile. From the results obtained, they concluded that the addition of fine powder increased the critical Reynolds number, while coarse powder decreased it. Qualitatively the effect was the same as in the case of plane parallel flows when the particles modified only the disturbance flow but did not influence the mean flow.

This article presents a study conducted to understand how the velocity profile is affected when there is interaction between a flat plate and an incompressible fluid carrying solid particles. Numerical simulations of the hydraulic phenomenon of a particle-laden flow over a flat plate was performed. The Navier-Stokes equations of the carrier fluid were coupled with the Newton's second Law utilized to track the particles. This study presents the unprecedented results of the velocity profile behavior at different locations on the flat plate as a function of varying Stokes number, volumetric fraction, the relative size of particles in the flow.

This paper is organized as follows: first, a section of the physical and numerical methodology, where the governing equations and the relevant equations are described, as well as the numerical domain. Then, the results and discussion section, where the results obtained from the COMSOL Multiphysics modeling software are presented, with their respective analyses.

Finally, the section of conclusions, which reflects the most outstanding observations of the research.

II. PHYSICAL AND NUMERICAL METHODOLOGY

2.1 Hypothesis

In order to develop the present study, the following hypotheses were established: 1) steady state, 2) incompressible Newtonian fluid with constant properties, 3) flat surface of length L in the x direction, negligible thickness and infinite extension in the z direction, 4) the fluid arrives in laminar flow parallel to the plate with a uniform velocity U, 5) two-dimensional flow where u and v are the velocity components along the x and y directions, while $w = 0$, 6) and negligible effects of gravity and no pressure gradient.

2.2 Computational domain and boundary conditions

In this study, a water flow with a density of $\rho = 999\ ^{kg}/_{m^3}$ and dynamic viscosity $\mu = 0.00112$ Pa.s , was considered and a domain of 0.04 m high and 0.7 m in length was used. The length was divided into 2 zones, the first is a sliding zone of 0.1 m in length and the second non-sliding zone is of 0.6 m in length. The boundary conditions in the computational domain are shown in fig. 2.1.

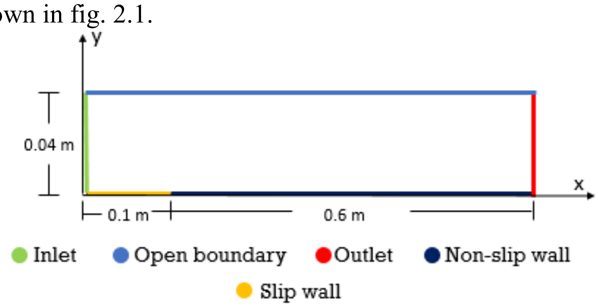

Figure 2.1. Boundary conditions in the computational domain.

2.3 Construction of the mesh

A structured mesh with square elements was used. To refine the mesh, we use a mesh factor (MF) to equally divide the shorter side of the domain (the height or the inlet/outlet sides) in small elements. Fig. 2.2 shows the mesh factor in the geometric model. We increased MF to refine the mesh.

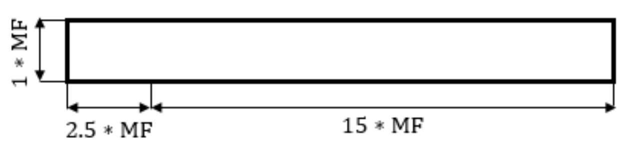

Figure 2.2. Meshing factor in the geometric model.

Square elements were established for the mesh structure, in order to efficiently control the size of all elements in the computational domain, ensuring that the mesh elements are larger than any solid particles in the flow.

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2.4 Sensitivity of meshing

To perform the sensitivity of the mesh the following parameters were used: Reynolds local number of 20,000, a Stokes local number of 10, the relative size of 0.04 particles and volumetric fraction of 0.1 particles. The position on the plate for local calculations was set at 0.4 m. For the mesh sensitivity we varied the mesh factor (MF) in increments of 30 as shown in table 1.

Table 1. Sensitivity of meshing

Mesh	Meshing Factor (MF)	#Elements	Size of elements (m)
Extra coarse	30	15750	1.33E-3
Very coarse	60	64111	6.67E-4
Coarse	90	143416	4.44E-4
Normal	120	254221	3.33E-4
Fine	150	396526	2.67E-4
Very fine	180	567000	2.22E-4
Extra fine	300	1575000	1.33E-4

To calculate the relative difference of the normalized velocity components with respect to the Extra fine mesh, the following equation (1) were used:

$$\%\text{Dif. } f'_M = \frac{f'_{\text{Mesh Extra fine}} - f'_{\text{Mesh to evaluate}}}{f'_{\text{Mesh Extra fine}}} .100 \quad (1)$$

The sensitivity of the mesh was performed for both a flow without particules, and with particles following the parameters presented above (see figure 2.3). This exercise let us to determine that the "very fine" mesh would yield optimum results because the difference on the velocity profile results from this mesh compare to the results from a more refined mesh was less than 0.3%.

2.5 Governing equations

Fluid movement

The incompressible laminar flow in this study is characterized by the continuity equation (2) and momentum conservation equations (3):

$$\rho \nabla . \mathbf{u} = 0 \quad (2)$$

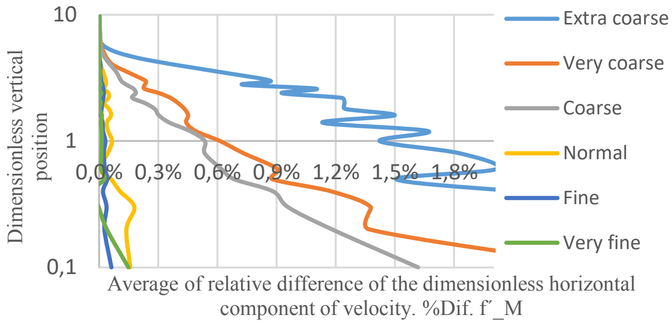

Figure 2.3. Variation of the relative percentage of the difference of the normalized horizontal component of the flow velocity of each mesh with the "Extra fine" mesh.

$$\rho\frac{\partial u}{\partial t}+\rho(\mathbf{u}.\nabla)\mathbf{u} = \nabla.[-pI+K]+F \qquad (3)$$

where **u**=(u, v) is the fluid velocity vector, $\frac{\partial u}{\partial t}$ represents the variation of the fluid velocity with respect to time, ρ represents the fluid density, p represents the fluid pressure, I represents the identity matrix, $K=\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)$ is the viscosity term, and F represents the feedback force in the control volume due to the presence of particles.

Equations of particles

For the study, all particles were considered as solid spheres of uniform size. Their motion equation follows the Newton's 2nd law (4):

$$\frac{d(m_p v)}{dt} = F_t \qquad (4)$$

where m_p represents the mass of the particle, v represents the particle velocity, and F_t represents the forces acting upon the particle. The only force on the particles considered in this study is the drag force. Due to the small size of the particle, this force follows the Stokes' law (5):

$$F_D = \frac{1}{T_p} m_p(\mathbf{u}\text{-}\mathbf{v}) \qquad (5)$$

$$T_p = \frac{\rho_p d_p^2}{18\mu} \qquad (6)$$

where T_p represents the particle response time, ρ_p represents the particle density, d_p represents the particle diameter and μ the dynamic viscosity of the fluid surrounding the particle.

Fluid-particle interaction

Two-way interaction is considered in this study. The fluid acts upon the particle following equation 4, while the fluid feels the presence of the particle through the feedback force F in equation 2. This feedback force follows these rules:

When a particle enters an element of the domain (7):

$$F_{v.new} = F_v - \frac{F_D}{meshvol} \qquad (7)$$

When a particle exits an element of the domain (8):

$$F_{v.new} = F_v + \frac{F_D}{meshvol} \qquad (8)$$

Local Reynolds number

For these flat plate studies, the fluid behaviour was tracked by the local Reynolds number (9):

$$Re_x = \frac{\rho u x}{\mu} \qquad (9)$$

where ρ represents the fluid density, u represents the fluid velocity, x represents the position on the flat plate, μ represents the dynamic viscosity of the fluid. The position on the plate for initial calculations of these parameters was arbitrarily set to 0.4 m.

Stokes number

The Stokes number characterizes the behaviour of a particle suspended in a flow stream, and is computed as follows (10):

$$Stk = \frac{T_P}{T_f} \qquad (10)$$

where T_p represents the characteristic time of the particle and Tf represents the characteristic time of the fluid. Both are determined as (11) and (12) respectively:

$$T_P = \frac{\rho_p {d_p}^2}{18\rho v} \qquad (11)$$

$$T_f = \frac{x}{u} \qquad (12)$$

The position on the plate for initial calculations of these parameters was set to 0.4 m. It is important to note that a low Stokes number implies that the particle tend to follow the fluid flow without major resistance, while a larger Stokes number implies that the particle does not follow the flow and it might perturb it considerably.

Relative particle size

The relative particle size is considered as the ratio of the particle diameter to the thickness of the hydrodynamic boundary layer, expressed as (13):

$$Rsp = \frac{d_p}{\delta} \qquad (13)$$

$$\delta = \sqrt{\frac{vx}{u}} \qquad (14)$$

where δ represents the thickness of the hydrodynamic boundary layer and x represents the position on the plate. The position on the plate for initial calculations of these parameters was set to 0.4 m.

Volume fraction

The volume fraction is the volume of the average total number of particles divided by the domain volume, and it was calculated as (15):

$$\alpha_p = N_p \frac{\pi/_6\, d_p^3}{H\, d_p\, l_p} \quad (15)$$

where N_p represents the number of particles being released into the domain by the inlet at a particular time, H represents the inlet height and l_p represents the average distance between particles. It was assumed that the domain has a thickness of d_p. The average distance between particles is calculated following equation (16):

$$l_p = \left(\frac{\pi}{6\,.\alpha_p}\right)^{1/3} .\, d_p \quad (16)$$

Moreover, the particle release time is calculated following expression (17):

$$t_{release} = \frac{l_p}{|V_f\text{-}V_p|/_2} \quad (17)$$

where V_f represent fluid inlet velocity and V_p particle inlet velocity.

III. Results and Discussion

3.1 Velocity profiles

The present study analyses the fluid-particle interaction of a flow over a flat plate. Because the magnitude of the vertical component of the velocity is extremely small $v \ll u$, only the horizontal component of the fluid will be analysed.

The relative difference of the normalized (following Blasius) velocity components was calculated with respect to the normalized components of the Blasius velocity. The following equation was used:

$$\%Dif.\ f' = \frac{f' - f'_{Blasius}}{f'_{Blasius}} .\, 100 \quad (18)$$

where f' represents the dimensionless horizontal component of the velocity of the fluid with particles and $f'_{Blasius}$ the dimensionless horizontal component of the velocity from the Blasius solution.

The dimensionless horizontal component of velocity is defined by the following equation:

$$f' = \frac{u}{U} \quad (19)$$

where u represent the velocity component in the x direction and U free current velocity.

Additionally, a similarity variable η was used, as Blasius did in his research, and is expressed as:

$$\eta = y\sqrt{\frac{U}{\nu\, .x}} \quad (20)$$

where y represent the position perpendicular to the plate.

In the next few sections, the flow behavior was analyzed with $V_p = V_f$. This is the particles were released with the same velocity of the fluid at the inlet.

3.1.1 Volume fraction effects

The particles interact with the flow exchanging of kinetic energy. Fig. 3.1 shows how the presence of a greater number of particles generates a greater exchange of energy with the fluid for the horizontal component of the velocity, causing it to accelerate in the areas closest to the plate, increasing the flow speed up to 0.33 times, when the presence of particles is α_p=0.1. In the area sufficiently far from the plate, stability is achieved with free current flow because particles and flow move at the same speed from the inlet. The parameters set for these results were Rsp= 0.02 and $Stk = 1$ (at the point $x = 0.4\ m$).

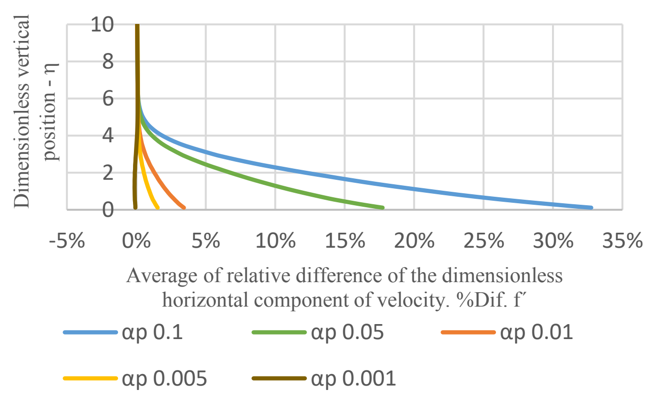

Figure 3.1. Variation of the relative percentage of the difference of the normalized horizontal component of the flow velocity for different α_p.

The velocity differences observed indicate that cases with $\alpha_p = 0.001$ and even $\alpha_p = 0.005$ have negligible impact and can be considered as flows without particles. On the other hand, a proportional increase is observed in the normalized horizontal component of the flow velocity with respect to the volumetric fraction. To test this proportionality, we plotted the results by normalizing them as follows:

$$(\%Dif.\ f') / \alpha_p \quad (21)$$

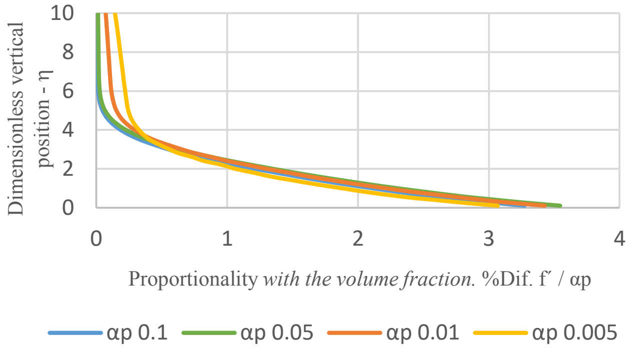

Figure 3.2. Proportionality of the normalized horizontal component of the flow velocity with the volume fraction.

Figure 3.2 shows the results with the new normalization. The superposition of the curves shows the proportionality evidence of the relative difference of the normalized horizontal component of velocity with the volumetric fraction. The case $\alpha_p = 0.001$ was removed because the relative differences were negligible.

3.1.2 Relative particle size effects

Fig. 3.3 shows the relative difference in the normalized horizontal components of the velocity for different particle relative size. It is shown that the relative size of the particles also impacts the fluid-particle interaction. When the particles are smaller in size, the horizontal velocity of the flow increases in the areas close to the plate. With smaller particles, for a constant particle volume fraction, the number of particles in the flow is greater, increasing the flow speed between 0.07 and 0.09 times by reducing one unit of Rsp. The parameters set for these results were $\alpha_p = 0.1$ and Stk = 1 (at the point $x = 0.4\, m$).

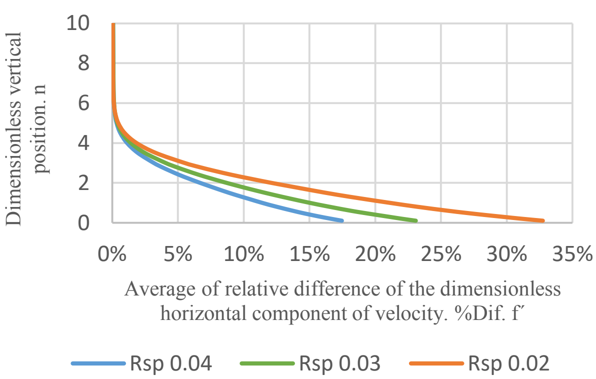

Figure 3.3. Variation of the relative percentage of the difference of the normalized horizontal component of the flow velocity for different Rsp.

Similarly, to particle volume fraction, there seems to be a proportionality of the relative difference in the normalized horizontal components of the velocity with respect to the inverse of relative particle size. Figure 3.4 confirms the proportionality relationship that also exists for the relative particle size, following the following expression:

$$(\%Dif.\ f') \ .\ Rsp \quad (22)$$

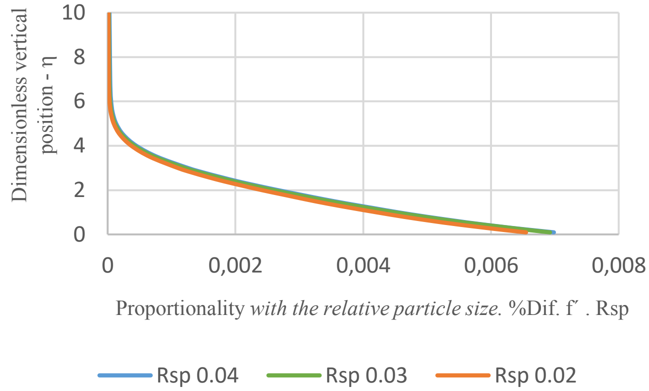

Figure 3.4. Proportionality of the normalized horizontal component of flow velocity with relative particle size.

It is observed that all the curves overlap. By combining the proportionality of both parameters:

$$(\%Dif.\ f') \ .\ Rsp / \alpha_p \quad (23)$$

It is possible to superimpose all the curves as shown in Figure 3.5.

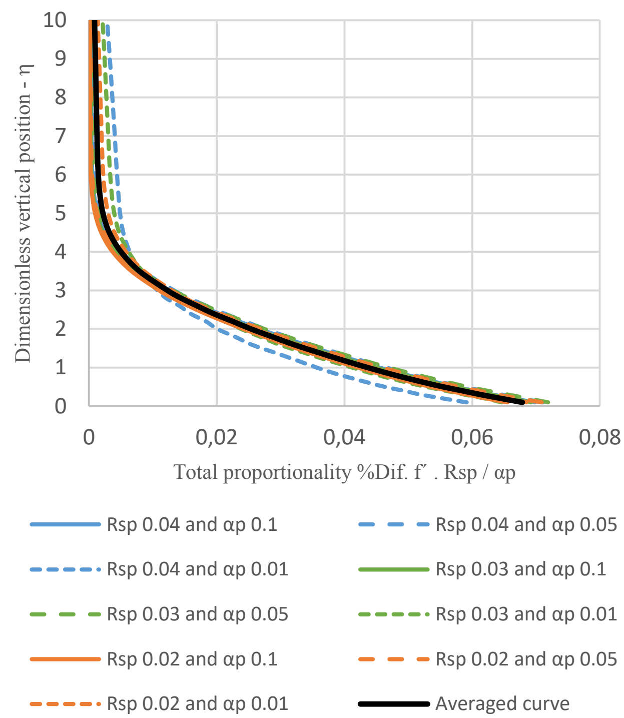

Figure 3.5. Averaged curve of the proportionality of the normalized horizontal component of the flow velocity with the relative size and volume fraction of particles.

3.1.3 Stokes number effects

The Stokes number also has a significant impact on the fluid-particle interaction due to the inertia that the particles possess.

Figure 3.6 presents the variation of the relative difference of the normalized components of the velocity for different Stokes numbers. It is observed that as the Stokes number increases (which is evidence of particle inertia increase), the particles drive even further the fluid in the horizontal direction to a greater extent, causing a single order of magnitude increase in Stk to double the interaction of the particles with the fluid. Additionally, particles with low inertia do not affect the fluid in any of its components, as is the case of Stk = 0.1, so they can be considered as flows without particles. The parameters set for these results were $Rsp = 0.04$ and $\alpha_p = 0.1$ (at the point x=0.4 m).

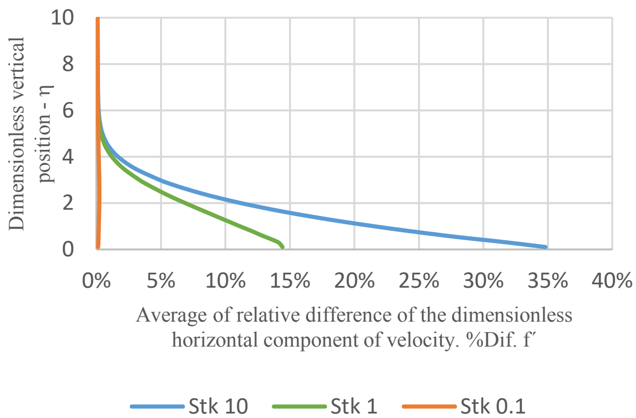

Figure 3.6. Variation of the relative percentage of the difference in the horizontal component of the flow velocity for different Stk.

3.2 Velocity profiles for different V_p

In this section, two cases were studied; a case where the particles enter the domain with half the velocity of the fluid ($V_p = 0.5V_f$), another case where the particles enter the domain without velocity ($V_p = 0$). For all cases, the Rsp and α_p parameters were varied, setting $Stk = 1$ at the evaluation point $x = 0.4\,m$.

3.2.1 Volume fraction effects for different V_p

The results indicate that when the particles enter at a lower velocity than the fluid, as expected, an exchange of energy from the fluid to the particles occurs, which causes a deceleration of the fluid, as shown in figure 3.7 (dotted blue curves). For a low particle volume fraction, this effect vanishes. The parameters set for these results were $Rsp = 0.02$ and $Stk = 1$ (at the point $x = 0.4\,m$).

3.2.2 Relative particle size effects for different V_p

As expected, smaller relative particle sizes generate a greater impact on the flow velocity components, as shown in Figure 3.8. The relative difference in the normalized component of the velocity between flows with lower Rsp increases with respect to a flow with higher Rsp. Likewise, a reduction in the speed of the fluid is noted due to the exchange of energy with the slower particles. The parameters set for these results were $\alpha_p = 0.1$ and $Stk = 1$ (at the point $x = 0.4\,m$).

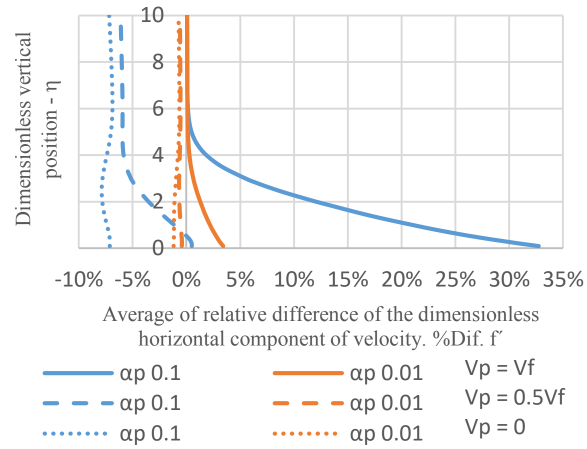

Figure 3.7. Variation of the relative percentage of the difference of the normalized horizontal component of the flow velocity for different α_p.

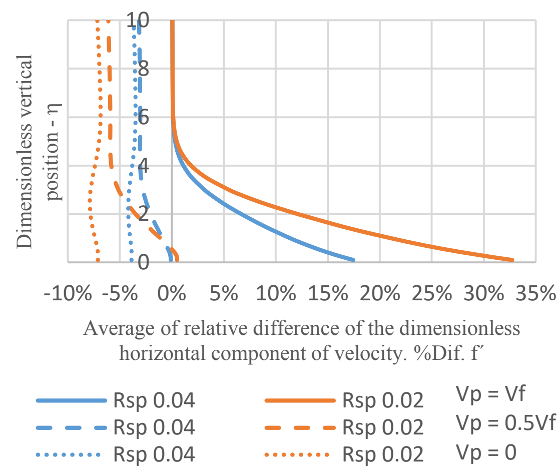

Figure 3.8. Variation of the relative percentage of the difference of the normalized horizontal component of the flow velocity for different Rsp.

As in the previous section, the proportionality relationship of the present parameters (α_p and Rsp) was sought, to superimpose the different curves. Figure 3.9 shows the averaged curves for the studied cases of reduction of the particle entry velocity ($V_p = 0.5V_f$ and $V_p = 0$) together with the average curve of the case with the particle entry velocity equal at the velocity of the fluid ($V_p = V_f$). In this figure, the deceleration of the fluid speed is evident due to the decrease in the entry speed of the particles; and it can be observed that the lower the particle entry velocity, the lower the fluid velocity in areas close to the plate and in the area of free current.

3.2.3 Stokes number effects for different V_p

Reducing the entry velocity of the particles allows the inertia of the particles to affect the movement of the fluid even more, slowing it down, as shown in Fig. 3.10. The deceleration of the fluid is shown to be greater when $Stk = 10$ (for Vp<Vf), due to the higher particle inertia. The parameters set for these results were $Rsp = 0.02$ and $\alpha_p = 0.1$ (at a distance $x = 0.4\ m$).

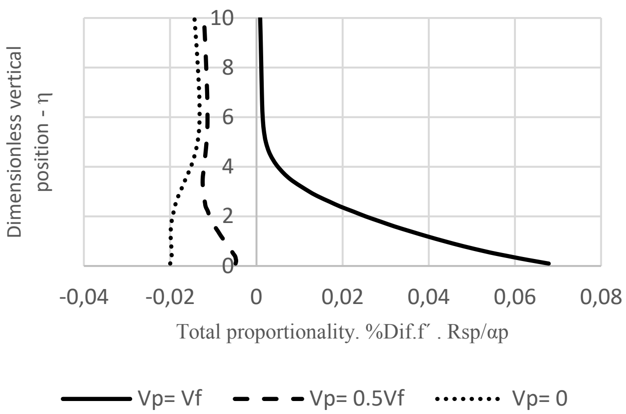

Figure 3.9. Averaged curves of the proportionality of the relative size and volume fraction of particles in the horizontal component of the flow.

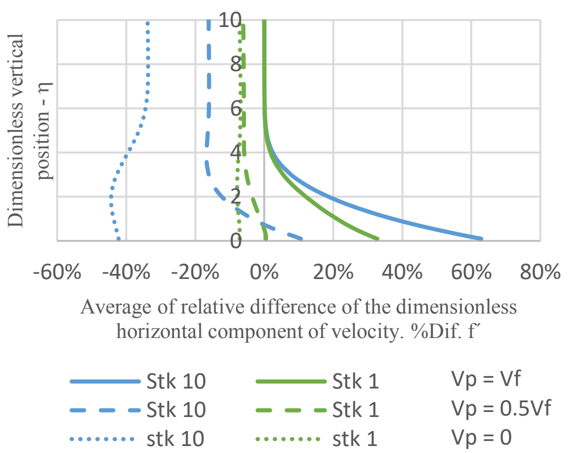

Figure 3.10. Variation of the relative percentage of the difference in the horizontal component of the flow velocity for different Stk.

It is important to point out that regardless of the changed parameter, if a variation of the relative difference of the normalized component of the velocity is observed, there are 3 zones. The first one near the wall were a deceleration or acceleration of the fluid is present. Another zone away from the wall where the particle presence has no effect on the fluid velocity. And an intermediate buffer zone that works as a transition between the other two described zones.

3.3 Velocity profiles for different H

In this section, the particle entry zone was divided into two equal parts. Particles were then released either through the upper half or the lower half (see figure 3.11). For these studies, the particle entrance velocity is equal to that of the fluid, $V_p = V_f$.

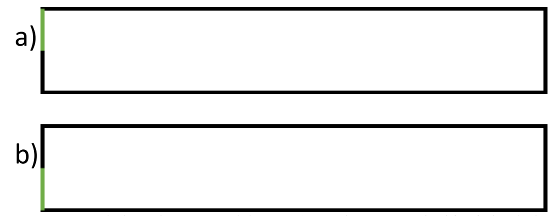

Figure 3.11. Particle release position: (a) In the upper half of the inlet, and (b) In the lower half of the inlet.

3.3.1 Velocity profiles with release of particles in the upper half of the inlet

For this case, $Stk = 10$ was set at $x = 0.4\ m$. Figure 3.12 shows the variations of the relative differences in the normalized horizontal component of the flow velocity when the particles are released through the upper part of the inlet. With a $Stk = 10$, it is expected to see a major impact on the fluid velocity. However, the relative difference remained less than 0.4%, which is negligible. It can be concluded that releasing the particles thorugh the upper half of the inlet do not affect the flow velocity profile.

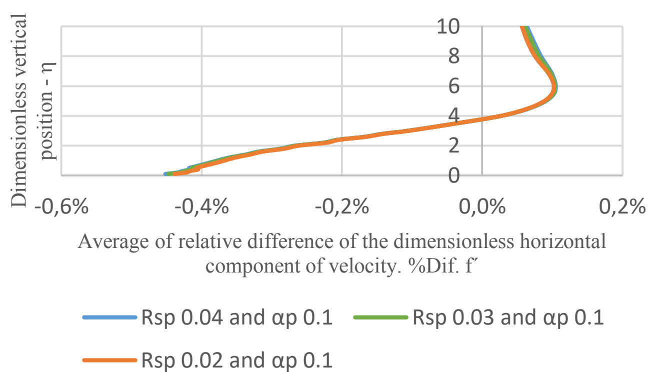

Figure 3.12. Variation of the relative percentage of the difference in the normalized horizontal component of the velocity for the release of particles at the upper height.

This is because for this case the particles move almost entirely horizontally, so they will not descend to the area near the plate as shown in Figure 3.13, so they do not affect the flow velocity profile near the flat plate, not affecting the boundary layer.

Figure 3.13. Tracking particle release in the upper half of the domain entrance.

3.3.2 Velocity profiles with release of particles in the lower half of the inlet

The relative differences in the normalized horizontal component of velocity obtained in section 3.1, where full height particle inlet release was used, were compared with the results obtained using the lower half for particle release. The comparison is shown in Figure 3.14 for different α_p and using $Rsp = 0.02$ and $Stk = 10$ (at $x = 0.4\ m$). When analysing this figure, it can be observed that the results were the same as those obtained in section 3.1. This finding complements the finding in section 3.3.1, the particles released through the upper inlet section does not affect the boundary layer.

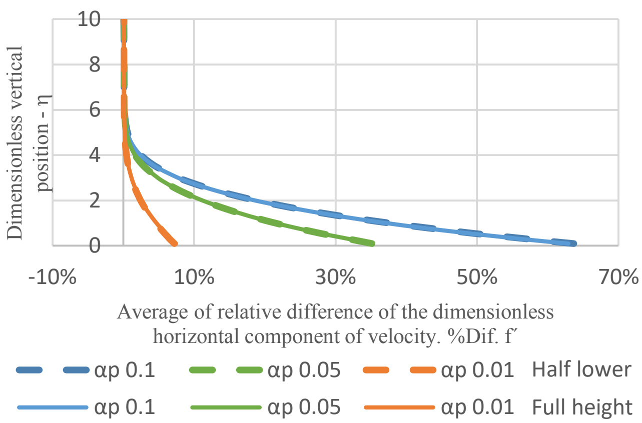

Figure 3.14. Comparison of the relative percentage of the difference of the normalized horizontal component of the velocity with the release of the particles in the full height and in the lower half for different α_p with $Stk = 10$ at $x = 0.4\ m$.

IV. Conclusion

This research studied the velocity profile in an incompressible laminar flow with solid particles on a flat plate. By varying the volumetric fraction, the relative size of particles and the entry velocity of the particles, it was determined that the number of particles present in the flow significantly influences the velocity profiles. The greater the number of particles in the flow, the greater the energy exchanges with the fluid. The number of particles increases with increasing volume fraction and decreasing relative size. Therefore, the volume fraction acts directly proportional, while the relative size of the particles is inversely proportional in the fluid-particle interaction.

The inertia of the particles, as expected, influences the flow behavior. The higher the Stokes number of the particles, the greater the energy exchange with the fluid. Very small Stokes numbers, such as $Stk = 0.01$, do not affect the flow.

By reducing the entry velocity of the particles, the fluid slows down because the slow particles decelerate the fluid. Again, a linear proportionality was observed with respect to relative particle size and volume fraction.

When particles enter the flow in an area away from the surface of the flat plate, regardless of the quantity, size, or inertia of the particles, the fluid is not affected by them. This is because the particles do not interact with the boundary layer located near the wall. On the other hand, by releasing the particles in the area close to the flat plate, this interaction occurs, affecting the velocity profile. This implies that the fluid is affected only by particles that are close to the flat plate.

References

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