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A Multivariate EWMA Control Chart with fuzzy approach for multinomial variables

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Abstract*– The aim of this paper is to improve the response of the Multivariate Multinomial Fuzzy T2 control chart to detect small shifts by the implementation of a MEWMA control chart. In this research a multivariate control chart based on the MEWMA chart is proposed to improve the sensitivity to detect small shifts in the vector of means of the T2 chart with fuzzy approach presented in the literature, to deal with p correlated multinomial multivariate variables. The parameters for the chart are obtained from historical data and the control limits by simulation. The performance of the new chart proposed is compared with the performance of the T2 control chart through the Average Run Length (ARL) value. The performance of this new chart can be obtained through a program developed by the authors.

***Keywords*-- Fuzzy Theory, Hotelling T^2, MEWMA, Control charts, ARL.**

I. Introduction

Control charts are one of the most widely used tools in statistical process control and have become a highly developed research field. Wang and Raz propose in [1] a fuzzy approach for a situation in which the quality characteristics can be evaluated through linguistic variables that can be classified in terms like "perfect", "good", " average "," poor "or" bad ", depending on its ability to meet specifications. Base on this publication various works were proposed, especially for the univariate case (see [2], [3]¸ [4], [5] and [6]. Those works include examples of application in manufacturing of different types of industry such as textiles or food.

Usually, the quality of a product is measured by the combination of several quality characteristics, so, the design of control charts for multivariate multinomial variables is important. When the articles are classified successively with respect to each of the quality characteristics, [7] and [8] suggest two approaches to the construction of control charts, the first is based on fuzzy theory and the second uses a probabilistic approach. If each element is controlled simultaneously with respect to all quality characteristics; [9] proposes two approaches: The first is based on the theory of probability and following the [10] work, and the second approach is based on the fuzzy theory in which each linguistic variable has several linguistic terms to describe the quality of the process. In [11] Kumar and Mohapatra, design control charts for quality characteristics of multiple attributes estimated through a ranking, the quality of the global sample is calculated by the weighted interactive addition of the fuzzy values assigned to each quality characteristic, the control charts are drawn using the measures of possibility and necessity.

In [12] the authors propose a control chart for correlated multivariate multinomial variables based on the T^2 control chart using Fuzzy Theory. To deal with situations in which the quality is defined by linguistic variables and is determined by two or more related quality characteristics avoiding the complexity of multinomial distribution by using a fuzzy approach. Although the T^2 control charts have been widely used, they show a decrease in their performance when the shift in the vector of means is small or medium, in this situation a proposal is the Multivariate Exponentially Weighted Moving Average (MEWMA) control chart, developed by [13]. Several versions of MEWMA type charts have been developed for discrete variables: for Poisson variables, there are some works such as those developed by [14] and [15]; while, for binomial variables [16] and for multinomial variables [17].

This paper proposes a MEWMA control chart for multinomial variables using a fuzzy approach. Taking as a starting point the work carried out by [12]. In addition, a method is presented to evaluate the performance of these charts through simulations as is done in the development of classic control charts.

II. Basic definitions

A. Average Run Length

The ARL is the most common measure of the control chart's response sensitivity, and much of its popularity is due to its intuitively appealing interpretation. A large value of in-control ARL is generally chosen to avoid frequent false alarm signals; while it is desirable that, the out-of-control ARL would be small, in order to detect quickly the process changes. In the case that the statistics used are independent and identically distributed and control limits are fixed, the chart's RL follows a geometric distribution, and the in-control ARL can be easily calculated from:

$$ARL = \frac{1}{\alpha} \tag{1}$$

where α is the probability that any point exceeds the control limits [18].

A typical $\bar{x}$ control chart with three-sigma limits (α = 0.0027), based on known parameters, has an in-control ARL of 370 under the assumption of normally distributed data.

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The out-of-control ARL is a measure of how quickly an out-of-control signal is detected. If the RL of a chart is a geometric random variable, the ARL control chart to detect a size d change is calculated as:

$$ARL_d = \frac{1}{1-\beta} \quad (2)$$

where β is the probability of Type II error, that is, the probability of not detecting that the process is out-of-control [18].

According to [19] in the case of control schemes based on charts with memory effect MEWMA-type, ARL values are obtained through simulation, Markov chains or integral equations. In this research the ARLs will be calculated by simulations.

B. Definitions from Fuzzy Theory

Linguistic variables: Their values are not numbers but words or phrases in any language. A linguistic variable L is characterized by its set of terms *T(L).*

Set of terms: It is the set of all possible values (or categories) that L can take. Every term in *T(L)* is an adjective or a combination of adjectives, and is associated with a fuzzy subset F, which has a variable numerical basis.

Membership function: The fuzzy subset F is characterized by a membership function $\mu_F(x)$, which associates to each x value of the standardized variable basis *(in fuzzy terms),* a number in the interval [0, 1]. This number represents the degree to which the x value belongs to the fuzzy subset F.

To perform arithmetic operations on linguistic variables represented as fuzzy sets, the membership function should be *normal* (meaning that there is at least one value of x whose degree of membership is equal to 1) and *convex*. To ensure compliance with those conditions, triangular membership functions were used and, therefore, a triangular fuzzy number will represent each element of the set of terms.

Triangular Fuzzy Number: Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. It is a fuzzy number represented with three points as follows: $F = (a, b, c)$.

Thus, its corresponding $\mu_F(x)$ membership functions would be defined as:

$$\mu_F(x) = \begin{cases} 0, & if\ \ x < a\ \ or\ x > c \\ \frac{1}{b-a}(x-a), & if\ \ a \le x \le b \\ \frac{1}{b-c}(x-c), & if\ \ b \le x \le c \end{cases}$$

Representative Values: To facilitate the plotting of observations on the chart, it is necessary to convert the fuzzy sets associated with linguistic values into scalars, which will be referred to as representative values. This may be done in several ways, if the result is intuitively representative of the range of the base variable included in the fuzzy set as in [1]. Four ways, which are similar in principle to the measures of central tendency used in descriptive statistics, are: *fuzzy mode*, *α-level fuzzy midrange*, *fuzzy median*, and *fuzzy average*. However, there is no theoretical basis supporting any specific transformation method [8]. The choice between them should be based primarily on ease of calculation or user preference [4].

The defuzzification method used to derive the representative value in this document is the fuzzy average f_{avg} based on [20], defined by

$$f_{avg} = Av(x\colon F) = \frac{\int_0^1 x\mu_F(x)dx}{\int_0^1 \mu_F(x)dx} \quad (3)$$

Considering triangular membership function $\mu_F(x)$ corresponding to the triangular fuzzy number $F = (a, b, c)$, the representative value R would be obtained by:

$$R = \frac{\int_0^1 x\mu_F(x)dx}{\int_0^1 \mu_F(x)dx} = \frac{\int_a^b \frac{x(x-a)}{b-a}dx + \int_b^c \frac{x(x-c)}{b-c}dx}{\frac{c-a}{2}} = \frac{a+b+c}{3} \quad (4)$$

Example:

Let *L=Appearance*, a linguistic variable. Then its set of terms may be given by:

T (L) = {good, medium, poor}

The corresponding representative values (vr) using the fuzzy average method are:

$vr_{good} = 0.0833$, $vr_{medium} = 0.5$, $vr_{poor} = 0.8333$

III. The Fuzzy Approach Multivariate Multinomial T^2 Control Chart By [12]

Pastuizaca, Carrión y Ruiz proposed in [12] a control chart to monitor jointly p correlated multivariate multinomial variables $Q_1, Q_2, \ldots, Q_p$. Each one with s_j categories, q_{jk} with $j = 1, \ldots, p$ and $k = 1, \ldots, s_j$. Discrete multinomial variables with their categories can be viewed as linguistic variables with linguistic categories viewed as sets of terms that are associated with fuzzy subsets F_{jk} described by the corresponding membership functions μ_{ijk}.

To construct the control chart, the authors developed a method using fuzzy theory. The method can be described by the following procedure:

1. A size n sample, A_i, is obtained, on which p qualitative (multinomial) variables, Q_j, are measured, each one with s_j categories $T(Q_j) = \left\{q_{j1}, \ldots, q_{jk}, \ldots q_{js_j}\right\}$.

2. A fuzzy subset Fjk is assigned to each category qjk which is described by a membership function $\mu_{jk}(x)$, defined by a triangular fuzzy number.

3. Sample i items $i = 1, \ldots, m$ are classified, according to the category k of the variable j, q_{jk}, $j = 1, \ldots, p$, obtaining in that way the n_{ijk} for each sample.

4. Each n_{ijk} is associated with its corresponding fuzzy subset, given by a triangular fuzzy number $F_{jk} = (a_{jk}, b_{jk}, c_{jk})$. Its corresponding *representative value* is calculated by the defuzzification method, fuzzy-average, $vr_{jk} = \frac{a_{jk}+b_{jk}+c_{jk}}{3}$

5. The corresponding representative values vector, to the sample i is obtained, $\boldsymbol{R_i} = (R_{i1}, R_{i2}, ..., R_{ip})$ where

$$R_{ij} = \frac{1}{n}\sum_{k=1}^{s_j} n_{ijk} vr_{jk}$$

The above procedure is repeated for each sample. When the conversion process is made, they propose the use of a T^2 control chart for one observation to monitor the resultant variable $\boldsymbol{R_i}$. When the in-control mean vector μ_R and covariance matrix σ_R are known; the statistical control chart at time $i = 1, \ldots, m$ is given by:

$$T_i^2 = (\boldsymbol{R_i} - \boldsymbol{\mu_R})' \Sigma_R^{-1} (\boldsymbol{R_i} - \boldsymbol{\mu_R}) \quad (5)$$

If the mean vector $\boldsymbol{\mu_R}$ and the covariance matrix σ_R are unknown; they are replaced by their estimators, $\bar{\boldsymbol{R}}$ and $\mathbf{S}$ respectively, calculated from m preliminary samples taken when the process is supposed to be in-control, according to the following expression:

$$\bar{\boldsymbol{R}} = (\bar{R}_1, ..., \bar{R}_j, ..., \bar{R}_p) \quad (6)$$

where $\bar{R}_j = \frac{1}{m}\sum_{i=1}^{m} R_{ij}$ and $R_{ij} = \frac{1}{n}\sum_{k=1}^{s_j} n_{ijk} vr_{jk}$.

The estimator chosen for the covariance matrix was

$$\boldsymbol{S} = \begin{pmatrix} S_1^2 & S_{12} & \cdots & S_{1P} \\ S_{21} & S_2^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ S_{P1} & S_{P2} & \cdots & S_P^2 \end{pmatrix} \quad (7)$$

Where $S_j^2 = \frac{1}{2(m-1)}\sum_{i=1}^{m} (R_{i+1\mathrm{j}} - R_{ij})^2$

and

$$S_{jh} = \frac{1}{2(m-1)}\sum_{i=1}^{m} (R_{i+1\mathrm{j}} - R_{ij})\,(R_{i+1\mathrm{h}} - R_{ih}), for \quad j \neq h$$

The statistical chart given in equation (3) is replaced with:

$$T_i^2 = (\boldsymbol{R_i} - \bar{\boldsymbol{R}})' \boldsymbol{S^{-1}} (\boldsymbol{R_i} - \bar{\boldsymbol{R}}); \quad i = 1,2, ..., m \quad (8)$$

Control Limits: unlike what was proposed by [12], the upper control limit for this control chart will be obtained through simulations for a desired ARL_0 value since comparisons with the proposed chart in this work are required under similar conditions. Then, its performance will be assessed for established shifts.

IV. The Proposed Fuzzy Mewma Control Chart

Taking as a starting point the work of [12] and using the chart proposed by [13]. We define a new control chart, that improves the response to small and medium changes in the means vector, called Fuzzy Mewma control chart. The statistic of control for this chart is defined by:

$$Z_i = \lambda R_i + (1-\lambda) Z_{i-1} \qquad i = 1,2, \ldots \quad (9)$$

where $\mathbf{0} \leq \boldsymbol{\lambda} \leq \mathbf{1}$ y $\mathbf{Z_0} = \bar{\boldsymbol{R}}$. The quantity plotted on the control chart is:

$$\boldsymbol{E_i^2} = \boldsymbol{Z_i^T \Sigma_z^{-1} Z_i} \quad (10)$$

The vector $\boldsymbol{Z_i}$ has mean $\boldsymbol{\mu}_R$ when the process is in-control and covariance matrix

$$\boldsymbol{\Sigma_z} = \frac{\boldsymbol{\lambda[1-(1-\lambda)^{2i}]}}{\boldsymbol{2-\lambda}} \mathbf{S} \quad (11)$$

This chart requires an upper control limit that is obtained by simulation for a desired ARL0 value.

A. *Proposed Methodology to measure the ARL.*

An important factor to consider in the design of control charts, especially when comparisons with existing proposals are required, is their performance. One of the most used performance measures, for the simplicity of its application, is the Average run length (ARL).

According to the reviews made by [21] and [22] research on multivariate multinomial variables with a fuzzy approach is almost non-existent. [21] shows that only 50% of the fuzzy control charts proposed in the literature make a performance study. In this work the ARLs will be calculated by simulations. To obtain the corresponding ARL we apply the next algorithms:

Algorithm 1

Problem: *Generate m samples of p correlated multinomial variables each one of size n, with a predetermined vector's proportions and a correlation matrix given.*

To solve the problem a program in R Studio was built, with the following algorithm:

1. Set the values of *m, n, p, q,* the proportions vector, the mean vector $\boldsymbol{\mu}$ and the correlation matrix to use $\boldsymbol{\Sigma}$.
2. Generate a multivariate normal sample with *p* variables and size *n*, using the *mvrnorm* function contained in the MASS package of statistical software R. The number of variables is determined by the dimensions of the mean vector and the correlation matrix used.
3. Transform the generated normal variables in multinomial variables using proportions' vectors given. Use the components of these vectors as the corresponding quantiles and proceed, in this way, to discretize each one of thenormal variables, counting the number of observations in each quantile, which will be the corresponding n_{ijk} values.
4. Repeat the process, *m* times to obtain the desired number of samples.
5. For each sample calculate $R_i = (R_{i1}, R_{i2}, ..., R_{ip})$

6. Estimate the vector of means $\overline{\boldsymbol{R}}$ and the matrix of variances and covariances $\boldsymbol{S}$.

Algorithm 2.

Problem: *Determine the average run length, ARL, for a given type I error (α).*

As before, to solve this problem a program in R Studio was built with the following algorithm:

1. Determine the upper control limit (UCL) for the desired combination of *p, m, λ* and *α* using simulation.
2. Start a counter to measure the run length.
3. Generate a random sample of multinomial variables using algorithm 1 to represent the new process information at time *k*.
4. Transform the sample generated in the corresponding representative vector values. Calculate $\boldsymbol{Z_k} = \lambda\, \boldsymbol{R_k} + (\mathbf{1} - \lambda\,)\boldsymbol{Z_{k-1}}$, with initial value $\mathbf{Z_0} = \overline{\boldsymbol{R}}$
5. Calculate $\boldsymbol{E_k^2} = \boldsymbol{Z_k^T}\boldsymbol{\Sigma_z^{-1}}\boldsymbol{Z_k}$, con $\boldsymbol{\Sigma_z} = \frac{\lambda\,[\mathbf{1}-(\mathbf{1}-\lambda\,)^{\mathbf{2}\boldsymbol{k}}]}{\mathbf{2}-\lambda}\mathbf{S}$
6. Compare E_K^2 with the upper control limit. If $E_K^2 < UCL$, increment the counter and go to step 3; if not, go to step 7.
7. Register a run length, measured on the counter, in a vector.
8. Repeat steps 2-7 until you have completed the desired number of repetitions.
9. Calculate the average of the values registered in step 7.

B. Application example.

To illustrate the proposed Fuzzy MEWMA control chart, and to make a comparative analysis, the numerical example proposed by [12] will be used. This example was originally provided by [8] in which three quality characteristics of a food processing industry are jointly measured: appearance (Q1), colour (Q2), and flavor (Q3), with the corresponding sets linguistic terms:

$$T(Q_1) = \{q_{11}, q_{12}, q_{13}\} = \{good, medium, poor\}$$
$$T(Q_2) = \{q_{21}, q_{22}, q_{23}\} = \{standard, acceptable, rejected\}$$
$$T(Q_3) = \{q_{31}, q_{32}, q_{33}, q_{34}\} = \{perfect, good, medium, poor\}$$

The corresponding fuzzy triangular numbers are:

$F_{11} = (0{,}0{,}0.25)$ $F_{12} = (0{,}0.25{,}0{,}75)$ $F_{13} = (0.25{,}1{,}1)$

$F_{21} = (0{,}0{,}0.5)$ $F_{22} = (0{,}0.5{,}0{,}75)$ $F_{23} = (0.5{,}1{,}1)$

$F_{31} = (0{,}0{,}0.25)$ $F_{32} = (0{,}0.25{,}0{,}75)$

$F_{33} = (0.25{,}0.75{,}1)$$F_{34} = (0.75{,}1{,}1)$

To determine the parameters of the control chart, 50 initial samples of size *n=220* taken when the process is supposed to be in-control are used. Instead of the samples provided by [8] these samples are obtained by simulation, using the algorithm 1 described above, which was later implemented in R. This algorithm uses a Monte Carlo simulation to generate *m* samples of *p* random correlated multinomial variables of size *n*, each one with s_j categories $j = 1, \ldots, p$. For the simulation, the mean vector used is null vector, the correlation matrix is an initial matrix given, and the proportions vector is given by [8] when the process is in-control. Table I shows the first 24 samples of the generate data.

TABLE I
GENERATED DATA WITH ALGORITHM

s	n11	n12	n13	n21	n22	n23	n31	n32	n33	n34
1	207	9	4	202	13	5	176	40	2	2
2	210	4	6	209	7	4	179	38	3	0
3	214	5	1	199	11	10	166	43	10	1
4	205	11	4	203	9	8	167	46	6	1
5	206	9	5	203	7	10	176	39	5	0
6	200	16	4	199	11	10	178	32	10	0
7	210	6	4	206	10	4	180	37	3	0
8	209	6	5	204	11	5	167	46	5	2
9	210	5	5	202	13	5	173	46	1	0
10	212	4	4	212	4	4	164	53	3	0
11	211	7	2	207	9	4	175	42	2	1
12	212	4	4	214	4	2	176	39	4	1
13	203	11	6	200	10	10	173	42	3	2
14	209	10	1	208	5	7	171	45	3	1
15	210	6	4	197	15	8	167	48	4	1
16	206	9	5	205	10	5	178	39	3	0
17	205	5	10	208	6	6	163	51	2	4
18	208	5	7	211	5	4	170	44	4	2
19	207	7	6	204	8	8	167	48	4	1
20	209	10	1	200	13	7	167	51	2	0
21	206	6	8	201	12	7	181	37	2	0
22	208	5	7	204	4	12	169	45	4	2
23	209	8	3	206	7	7	161	52	6	1
24	204	8	8	200	12	8	165	46	8	1

From the data generated and using the membership functions, the representative values vectors R_i for each sample *k* are calculated by the procedure provided in section III. Table II shows the R_i values with its corresponding T^2 and the Z_i values with the corresponding E^2 for λ=0.05, for the first 24 samples. The upper control limits obtained by simulation for an ARL in-control of 370, were 13.505 for the T^2 control chart and 34651 for the E^2 control chart.

The corresponding control charts obtained are showed in the Figure 1.

TABLE II
R_i VALUES, THE CORRESPONDING T^2 AND THE Z_i VALUES THE CORRESPONDING E^2 FOR Λ=0.05.

								Lambda=0.05
sample	R_1	R_2	R_3	T^2	Z_1	Z_2	Z_3	E^2
1	0,106	0,197	0,142	0,529	0,107	0,197	0,148	33964,57
2	0,106	0,187	0,134	3,511	0,107	0,197	0,147	33797,09
3	0,092	0,209	0,163	8,626	0,106	0,197	0,148	33851,09
4	0,108	0,201	0,155	0,739	0,106	0,197	0,148	33878,55
5	0,109	0,205	0,141	1,264	0,106	0,198	0,148	33880,77
6	0,114	0,209	0,146	2,274	0,107	0,198	0,148	34125,39
7	0,102	0,190	0,133	3,462	0,107	0,198	0,147	34116,90
8	0,105	0,194	0,156	0,798	0,106	0,198	0,148	34091,40
9	0,104	0,197	0,138	1,282	0,106	0,198	0,147	34101,20
10	0,100	0,183	0,152	2,851	0,106	0,197	0,147	33852,25
11	0,097	0,189	0,140	3,01	0,106	0,197	0,147	33831,84
12	0,100	0,177	0,142	5,726	0,105	0,196	0,147	33710,68
13	0,114	0,208	0,147	2,042	0,106	0,196	0,147	33856,59
14	0,098	0,194	0,146	1,623	0,105	0,196	0,147	33860,58
15	0,102	0,208	0,152	2,234	0,105	0,197	0,147	33918,23
16	0,109	0,193	0,136	2,026	0,105	0,197	0,146	33957,66
17	0,119	0,192	0,162	5,446	0,106	0,196	0,147	33951,08
18	0,110	0,184	0,152	2,453	0,106	0,196	0,147	33838,56
19	0,109	0,200	0,152	0,357	0,106	0,196	0,148	33699,09
20	0,098	0,203	0,147	2,178	0,106	0,196	0,148	33753,1
21	0,114	0,202	0,131	4,145	0,106	0,197	0,147	33780,53
22	0,110	0,208	0,153	1,613	0,107	0,197	0,147	34018,88
23	0,102	0,196	0,162	2,439	0,106	0,197	0,148	33874,8
24	0,117	0,205	0,161	3,829	0,107	0,197	0,148	34139,09

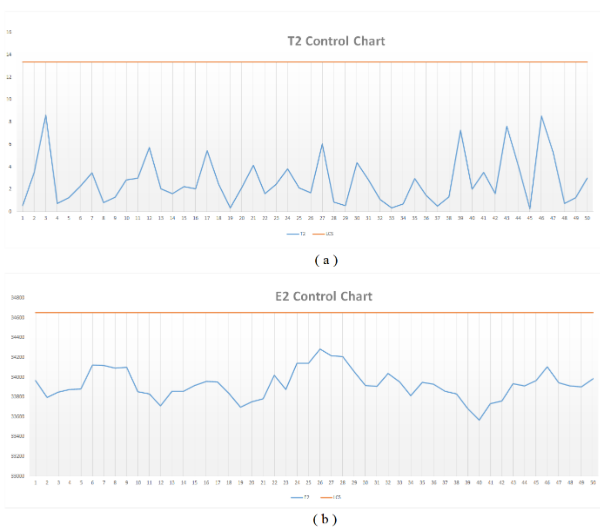

Fig. 1 (a) T^2 Control Chart. (b) E^2 Control Chart

C. *Performance comparative Analysis*

To measure the performance of the proposed control chart we use the ARL calculated by simulation. For the simulation we considered the generation of 10.000 *RLs* using the second algorithm and then calculate the average of them.

To calculate the *RL* values for the out-of-control state, the shifts in the proportion's vectors of the multinomial variables proposed by [8] are used. These shifts, the corresponding Mahalanobis distance (displacement) for the T^2 statistic are shown in Table III.

TABLE III
SHIFTS IN THE PROPORTIONS'S VECTOR ([8]) AND MAHALANOBIS DISTANCE

	C_{11}	C_{12}	C_{13}	C_{21}	C_{22}	C_{23}	C_{31}	C_{32}	C_{33}	C_{34}	D^2
V. P. I-C	0,942	0,035	0,023	0,925	0,045	0,03	0,774	0,206	0,016	0,004	0,0
S1	0,942	0,035	0,023	0,905	0,065	0,03	0,754	0,226	0,016	0,004	0,6
S2	0,934	0,035	0,043	0,925	0,045	0,03	0,774	0,206	0,016	0,004	3,3
S3	0,942	0,035	0,023	0,925	0,045	0,03	0,674	0,306	0,016	0,004	6,3
S4	0,892	0,035	0,073	0,925	0,045	0,03	0,774	0,206	0,016	0,004	19,9

V.P.I-C *: vector of proportions in-control

Observations regarding the proposed shifts, made by [8]: S1 is considered as a small shift, in effect, the proportions of low quality are not affected. In S2, the proportion of "poor" (c_{13}) increases by 0.02, and the proportion of (c_{11}) decreases by the same amount, although it is also considered as a small shift is more important than S1 because it affects the "poor" quality. S3 and S4 can be considered as medium and high, respectively.

The ARL is calculated for different values of lambda and compared with the T^2 proposed by [12]. The corresponding ARLs for the T^2 and E^2 are shown in Table IV.

TABLE IV
ARL VALUES FOR THE CORRESPONDING Λ, COMPARED WITH T^2

	Lambda	0.05	0.10	0.15	0.20	0.25	0.5	0.75	0.80	0.85	0.90	0.95
	ARL T^2	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**	**ARL E^2**
IC	**371,8**	370,9	369,0	373,71	369,76	371,27	370,8	371,9	370,7	371,3	370,3	372,5
S1	**113,1**	13,9	13,5	13,76	14,74	16,03	25,7	40,2	42,5	47,7	50,6	57,1
S2	**8,4**	12,9	12,0	11,84	12,18	12,67	17,3	25,1	27,1	28,7	30,8	33,6
S3	**5,4**	5,79	4,99	4,61	4,39	4,30	4,69	6,49	6,90	7,72	8,35	9,57
S4	**1,4**	4,77	4,03	3,63	3,40	3,29	3,05	3,27	3,31	3,41	3,61	3,78

Table IV shows that the control chart presents a better performance with small lambda values, as expected, it is very sensitive in detecting small shifts in the vector of means, having an improvement of 87% for the first shift (S1) and working with a lambda of 0.05. On the other hand, when the shift is larger, the optimal lambda begins to increase, as in the case of S2, the best performance of the E^2 chart is obtained with the lambda of 0.15.

The best performances are obtained with lambda values between 0.05 and 0.25. While the T^2 is better when you want to control large shifts in the vector of means.

V. CONCLUSIONS

One interesting and relevant initial observation is that, both control charts, the proposed in this work and the proposed by [12] shows operative coherence in the sense that as greater are shifts in quality variables, greater is the probability of detecting these shifts with the respective control chart. This is the case even after complex fuzzy transformations and defuzzification process are performed. In effect, shifts in the mean's vector of the representative values, corresponding to the shifts in the proportions's vector of the multinomial variables, are proportional to the severity of the shift. i.e., if the shift in the proportion affected is a threat to the quality of the product, this will be reflected in higher mean vector shift in the control chart and hence will be detected earlier.

In the application example, although the shape of the control chart is different, the result, that is the detection of the control outputs is similar for the data provided. On the other hand, in the comparative analysis of the performance of the control charts, for all the lambda values generated, the control chart proposed in this work presents a greater sensitivity for small shifts as expected. Another important result is that the best performance for all types of shifts is obtained with the MEWMA control chart when λ is small.

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