Students' difficulties interpreting kinematics graphs: A synthesis using the Root Cause Analysis method

Juan C. Morales, PhD[1]

[1]Universidad Ana G. Méndez – Recinto de Gurabo, Puerto Rico, jcmorales@uagm.edu

Abstract*– This investigation uses the Root Cause Analysis (RCA) method to take a fresh new look at synthesizing students' difficulties interpreting kinematics graphs. A list of fifty-one difficulties was generated and used as data for the RCA method. The difficulties were mostly obtained from the literature; therefore, they are not new in themselves. The novelties lie in the Fishbone diagram used to synthesize the difficulties into a coherent whole, in the robust Fault Tree Analysis diagram used to determine three root causes, and in the resulting clarity of the implications for instruction. The analysis strongly suggests that the vast majority of the difficulties are only symptoms of a deeper problem, the three root causes. These are: 1.) Student is unfamiliar with the three stages of reading graphs (identify visual features; relate visual features to concepts; apply judgment to interpret the graph), 2.) Student is unable to connect concepts to concrete, visible, and immediate experiences, and 3.) Student does not know or is unfamiliar with kinematics definitions. Instructional strategies should prioritize the eradication of the three root causes. Then, specific difficulties that linger can be addressed individually as a secondary priority. Implications for instruction are listed and demonstrated through a solved problem. The results of this investigation were used by the author as the basis to design an intervention to improve students' performance. Other researchers and instructors may find this synthesis equally useful. In addition, this article provides an example that may be followed by others on how to apply the RCA method to disentangle a complex problem.

Keywords—Rout Cause Analysis, Fishbone Diagram, Fault Tree Analysis, Kinematics Graphs.

I. Introduction

Graph interpretation plays a vital role in science and engineering. Beichner [1] compiled the following reasons: "graphs are an integral part of experimentation, the heart of science; graphs allow a glimpse of trends which cannot be easily recognized in a table of the same data; there is no other statistical tool as powerful for facilitating pattern recognition in complex data; graphs summarize large amounts of information while still allowing the details to be resolved; widespread use of graphs as a teaching tool; graphs are such efficient packages of data, they are used almost as a language by physics teachers". "Unfortunately," Beichner adds, "this study indicates that students do not share the vocabulary".

In the specific case of kinematics graphs, the vocabulary also includes calculus-level skills because the two fundamental formulas of kinematics require derivatives. These formulas state that velocity equals the derivative of position with respect to time ($v = dx/dt$), and acceleration equals the derivative of velocity with respect to time ($a = dv/dt$). Furthermore, separating variables and integrating, the formulas may also be stated in integral form: position equals the integral of velocity with respect to time ($x = \int v dt$), and velocity equals the integral of acceleration with respect to time ($v = \int a dt$). Rather than solving the equations analytically, the interpretation of kinematics graphs relies on applying the geometric meaning of the derivative ("slope of the line tangent to the function at any point") and the integral ("area under the curve"). The topic of kinematics graphs is typically taught in Physics 1 and in Engineering Mechanics: Dynamics. The author has more than 10 years of experience as an instructor of the latter course.

Figure 1 illustrates a kinematics graph problem of the type addressed in this article. The vertical axis plots a kinematics variable - velocity in this case - while the horizontal axis plots time. The student must interpret the plotted function to answer a kinematics question, in this case, "when is the acceleration the most negative?". The solution requires an analysis of the slope of the function ($a = dv/dt$); however, Beichner [1] reported that students sometimes confuse slope with the height of the axis value, i.e., some students erroneously select the lowest point in the graph as "the most negative height" (answer C in Fig. 1) instead of selecting the steepest negative slope (the correct answer is E).

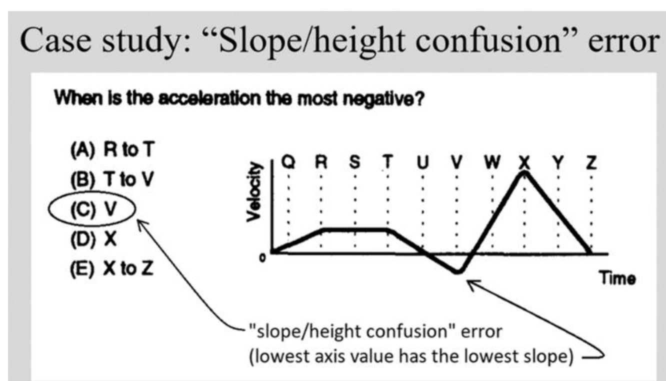

Fig. 1 Kinematics graph problem of the type addressed in this article (Source: [1], the comments were added to the original figure.) Some students erroneously select the lowest axis value as the "most negative height" (answer C) instead of the steepest negative slope (answer E is the correct answer).

The slope/height confusion error depicted in Fig. 1 is just one of many difficulties reported in the literature. The large number of difficulties, their wide variety, and the fact that some difficulties seemed more profound than others, led the author to apply the Root Cause Analysis (RCA) method. The objectives were to synthesize the difficulties into a coherent whole, to determine which of the difficulties were the root causes (the more profound difficulties), and to generate a clear set of

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instructional priorities. The motivation was to achieve a very clear understanding of the problem before designing an intervention to improve students' skills.

The difficulties were mostly obtained from the literature; therefore, they are not new in themselves. The novelty lies in taking a fresh new look at the problem with the RCA method to achieve a high degree of clarity of the problem. The synthesis provided in this article fills a gap in the literature and may perhaps be as valuable to other researchers and instructors as it was for the author.

The next section provides the methods used in the study, including a brief description of the RCA method and the three relevant RCA techniques that were applied. The literature review section provides an overview of the state of research in "kinematics graphs" and synthesizes the difficulties into a coherent whole by applying the RCA technique known as the Fishbone diagram. Next, the case study shown in Fig. 1 is used to determine the root causes by applying the Fault Tree Analysis diagram and the "5 Whys" techniques of RCA. The root causes are then generalized to the entire data set. Precise implications for instruction are listed and demonstrated through a solved problem. The article finalizes with a section on limitations and the conclusions.

II. METHODS

The literature review to determine students' difficulties was conducted based on the saturation criterion which states that the review concludes when "no new information seems to emerge during coding" [2].

The difficulties were synthesized using the well-established, straightforward, and systematic RCA method [3]. RCA is taught around the world as part of the Total Quality Management approach to continuous improvement. It is mostly used in engineering, but an inspection of the literature shows that it has been successfully adapted to problems in health care, business transactions, service delivery, and education, among others.

The following three techniques of RCA were used:

1. Fishbone diagram [4]. A cause-and-effect diagram named for its resemblance to a fish skeleton. The head identifies the effect while the bones identify the potential causes. The slanted bones indicate major categories while the horizontal bones list specific causes within each category. It offers a convenient structure to concisely identify, classify, and visualize the many potential causes of a problem.

2. Fault Tree Analysis (FTA) diagram [5]. This is another type of cause-and-effect diagram but focuses on revealing root causes. It addresses a specific "fault condition", in this case, the "slope/height confusion" error shown in Fig. 1. It works backward by priority levels to uncover the "failure chain" that leads to the root causes. It is arranged vertically, in top-down fashion (like a tree), so the potential root causes conveniently accumulate at the bottom of the diagram (the roots of the tree). FTA diagrams use logic gates of the "OR" and "AND" type which provide a robust rationale to the analysis. The output of "OR" logic gates occurs if at least one input is present while the output of "AND" logic gates occurs if all the inputs are present. The fishbone diagram is used as the basis to discriminate between relevant and irrelevant difficulties that lead to the "fault condition". The relevant causes are then treated as pieces of a puzzle that must be fitted together in the correct order as inputs or outputs (priority levels) of the logic gates.

3. The "5 Whys" [6]. As its name implies, the "5 Whys" is a technique that asks "Why?" five times or until one cannot ask "Why?" any longer because the root cause has been determined. Five is a good rule of thumb but it may take more (or less) queries to reach the root cause. The "5 Whys" is used to disentangle the priority levels between the contributing causes during construction of the FTA diagram. It assists in revealing the correct order of the "failure chain" that leads to the root causes.

III. LITERATURE REVIEW

The origin of "kinematics graphs" research was traced to two articles by Trowbridge and McDermott from 1980 [7] and 1981 [8]. These two articles addressed conceptual understanding difficulties regarding velocity and acceleration, respectively. In 1987, McDermott, Rosenquist, and van Zee [9], directly studied the difficulty of connecting graphs to physics. They uncovered that the nature of the difficulties is the same across all populations of university-level physics students, including honors students, although there are differences in severity. In 1990, Arons [10] published a guide that included suggestions for teaching the topic of kinematics graphs. This entire foundational research effort was conducted by the Physics Education Group that originated in the early 1970's at the University of Washington [11].

In 1994, Beichner [1] published the development, content, and results of a Test of Understanding Graphs in Kinematics (TUG-K). The TUG-K became a benchmark test and ignited a surge in research on the topic of "kinematics graphs" that has grown – and continues to grow - at an approximately linear rate of nearly two publications per year since 1994 (Fig. 2).

The 575 relevant articles published up to 2019 had different focus areas, including 257 articles (45%) on the use of digital tools to improve student performance (computer-based laboratories, simulations, virtual reality), 183 articles (32%) on general research (such as transfer between math and physics and how students deal with the concept of "rate" across different disciplines), 119 articles (20%) on teaching methods (peer instruction, didactic sequence, problem-based learning), and 16 articles (3%) on eye-tracking (to discover where students focus their eyes while they solve a kinematics graphs problem).

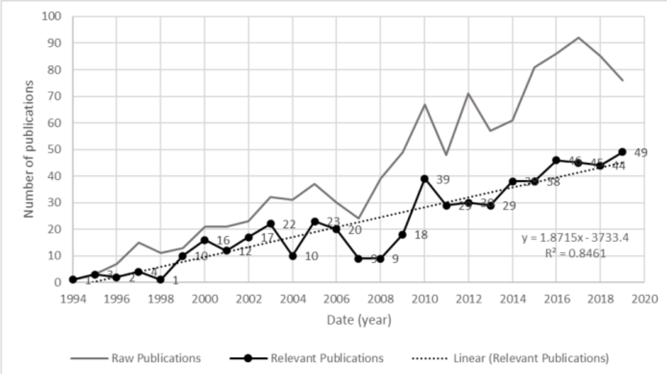

Fig. 2 The top graph includes the raw Google Scholar results for the number of "kinematics graphs" articles published between 1994-2019 (1,121 total publications). The bottom graph only includes the publications deemed relevant by the author, based on a closer inspection of the titles and abstracts (575 relevant articles). The growth rate since 1994 is approximately linear, with a slope of 1.87 publications per year. The 1994 onset for the growth coincides with the publication of Beichner's TUG-K paper [1]. Note: date filtering was used to obtain the yearly data from Google Scholar.

The Fishbone diagram shown in Fig. 3 summarizes the results of the review. It contains 51 difficulties (placed in the horizontal bones) that are organized into six categories (slanted bones). The difficulties and the categories were coded into self-explanatory short phrases to fit the fishbone diagram.

Table 1 briefly describes the six categories in more detail and provides the references for the 51 difficulties displayed in Fig. 3. The references are listed from top to bottom within each category of the diagram. Only one reference is listed for each difficulty because the objective was to list the difficulty but not the number of times that it was repeated.

The review reached saturation after reviewing 64 publications in three weeks (approximately 120 hours).

TABLE I
DEFINITION OF CATEGORIES USED IN THE FISHBONE DIAGRAM (FIG. 3).

Category (Slanted bones)	**Description**	**References [a]**
Area Related	Specific difficulties related to understanding and determining the area between a function and the time axis in a kinematics graph. Related to $\mathbf{x} = \int \mathbf{v}dt$ and $\mathbf{v} = \int \mathbf{a}dt$.	[12], [13], [14], [13], [1]
Slope Related	Specific difficulties related to understanding and determining the slope of a function plotted in a kinematics graph. Related to $\mathbf{v} = d\mathbf{x}/dt$ and $\mathbf{a} = d\mathbf{v}/dt$.	[13], [1], [9], [13], [1], [9], [8]
Vocabulary	Difficulties with the language of kinematics, including definitions and concepts, plus additional vocabulary-related issues.	<u>Left bones:</u> [9], [15], [13], [16], [12], [12] <u>Right bones:</u> [7], [7], [7], [1], [12], [9], [13], [1], [13]
Units	Specific difficulties related to units.	[12], [12], [13], [13], [13], [13]
Connecting Motion to Graphs	Difficulties relating physical ideas and the real world of motion with the abstract representation of motion in kinematics graphs.	[7], [17], [1], [13], [12], [18], [18], [9]
Graph Features	Difficulties with the general requirements of reading graphs fluently.	<u>Left bones</u>: A, [9], A, A, A <u>Right bones</u>: [14], [19], [19], [19], [19], [12]

[a]The references are given in the same order that they are listed in Fig. 3, from top to bottom (horizontal bones), within each category of the diagram (slanted bones). References listed as "A" are difficulties that were added by the author based on his own experience.

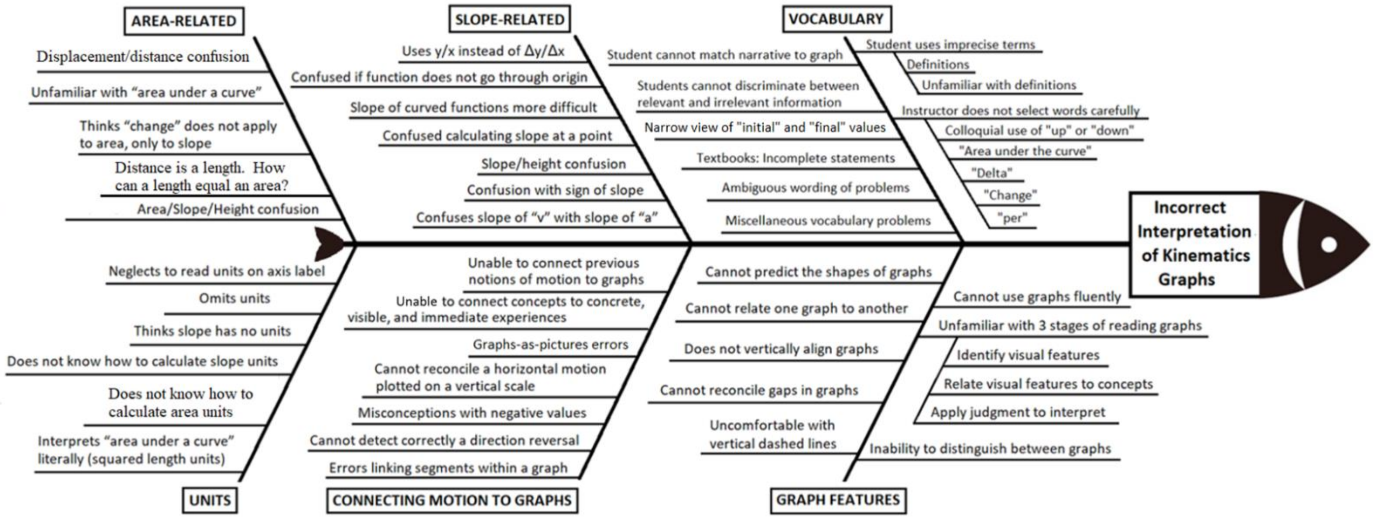

Fig. 3 Fishbone diagram. The six categories are designated within rectangles at the end of the six slanted bones. The 51 horizontal bones list the specific difficulties encountered while interpreting kinematics graphs.

IV. Root Causes

The Fishbone diagram (Fig. 3) was very successful at organizing and synthesizing the difficulties into a coherent whole. However, the root causes remain hidden within the diagram. One way to extract the root causes is to analyze each of the 51 difficulties in Fig. 3, one by one, and attempt to rank each one of them in terms of severity levels.

Another approach is to apply the Fault Tree Analysis (FTA) technique of RCA which provides a systematic way of conducting an analysis for root causes. As explained in section II, the FTA works backward from a "fault condition" and establishes priority levels that uncover the "failure chain" that leads to the root causes. The FTA uses the "5 Whys" technique to assist in establishing the priority levels.

For this project, the chosen "fault condition" is the "slope/height confusion error" shown in Fig. 1. The question becomes, what are the root causes that could potentially explain – at the deepest level – why a student is making this mistake?

The procedure to establish the "failure chain" that extracts the root causes with FTA is given in Table II. Three priority levels were expected in this project, L1, L2 and L3, as defined in Table II. Note that in other contexts (accident reconstruction or machinery failure analysis, for example), the FTA may result in four, five, or even more levels. In this case of kinematics graphs, only three levels were required to reach the root causes.

Figure 4 shows the results of the Fault Tree Analysis for the "slope/height confusion" difficulty.

TABLE II
PROCEDURE AND EXPECTED PRIORITY LEVELS IN THE FAULT TREE ANALYSIS (FTA) THAT UNCOVERS ROOT CAUSES.

Priority Level	The "5 Whys"	Output
L1	1st Why? Why does the difficulty occur? Examine each of the six categories and determine which ones could potentially explain the difficulty.	Select one or more of the six categories (slanted bones).
L2	2nd Why? Why does each category occur? Examine each of the difficulties listed within each of the L1 categories and determine which ones could potentially explain the difficulty.	Select one or more difficulties (horizontal bones) within each category.
L3	3rd Why? Why do students have issues with each of the L2 difficulties? The question may be slightly modified to, is there an L2 difficulty that is more deeply rooted than the others? Examine the L2 difficulties and disentangle the priority levels. Place the more profound difficulty at L3. The remaining difficulties stay at level L2.	The L3 difficulties become the root causes of the problem.

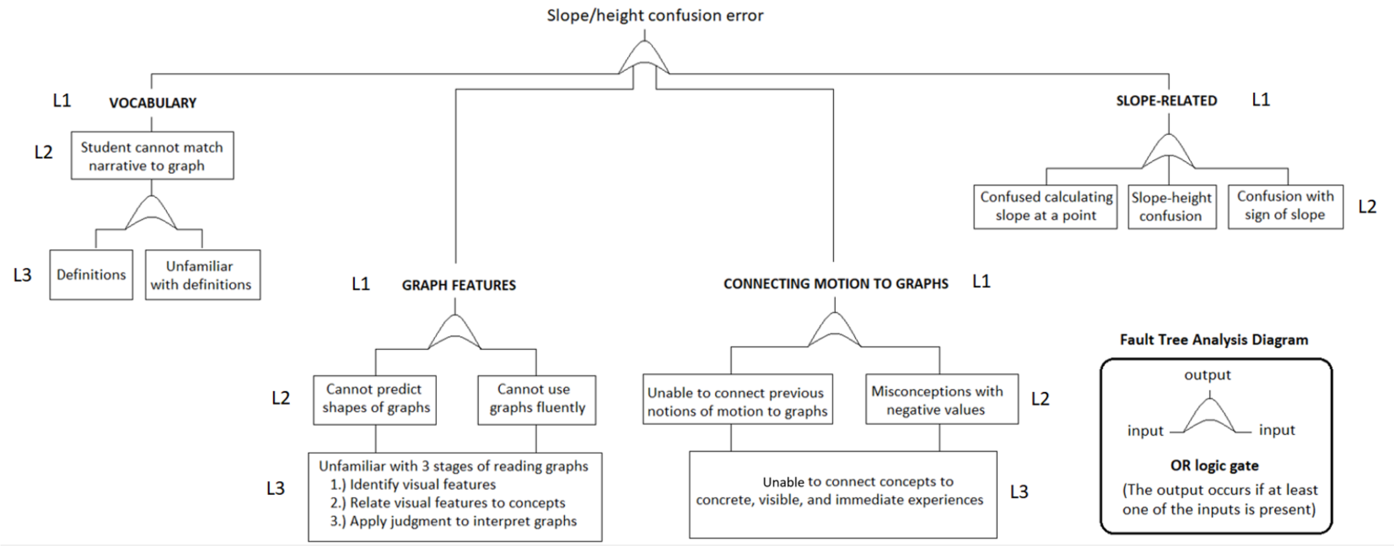

Fig. 4 Fault Tree Analysis diagram for the case of a "Slope/height confusion" error. The levels L1, L2, and L3 define the three priority levels of the failure chain. They were determined by using the "5 Whys" technique of RCA as explained in Table 2. The root causes are defined by the difficulties labeled "L3".

An examination of Fig. 4 shows four potential "L1" categories (slanted bones): "Vocabulary", "Graph Features", "Connecting Motion to Graphs", and "Slope Related". Two categories were not included - "Units" and "Area Related" - because they are irrelevant, i.e., there are no units in the problem and the problem is not related to area (it is related to slope).

Only the "Slope Related" category did not yield an "L3" root cause from within the "L2" potential difficulties which implies that difficulties related to slope, although important, are merely symptoms of a deeper problem. The remaining three categories yielded "L3" root causes.

As an example of how to disentangle the "L2" difficulties to reach the "L3" level, examine the "Connecting Motion to Graphs" category. For this category there are three "L2" difficulties. The resulting root cause, or "L3" difficulty ("Unable to connect concepts to concrete, visible, and immediate experiences) was judged to be at a more profound level than the other two ("Unable to connect previous notions of motion to graphs" and "Misconceptions with negative values"). The reason is that if a student can connect the kinematics concepts to concrete, visible, and immediate experiences, the student should be able to reconcile previous notions of motion to graphs, including the significance of negative values. The two "L2" misconceptions should disappear if the student is able to master the "L3" level (root cause).

Figure 5 summarizes the three root causes.

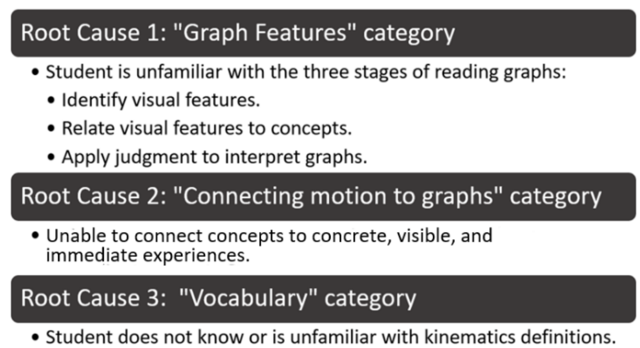

Fig. 5 Root causes of students' difficulties interpreting kinematics graphs generated by the FTA technique.

V. Generalization of Root Causes

The case study showed the FTA process of how to disentangle the priority levels between the difficulties to determine the root causes of a particular fault condition ("slope/height confusion").

The process was an inductive exercise, that is, it went from the specific to the general. The three root causes that were generated and shown in Fig. 5 can now be considered as a hypothesis that can be confirmed or invalidated for the rest of the data. This part of the exercise goes in reverse, that is, from the general (the hypothesis) to the specific. As such it is a deductive process.

An analysis of each of the remaining difficulties in the Fishbone diagram (Fig. 3) shows that all the difficulties conform to the hypothesis (Fig. 5).

For example, an important case to analyze is the "graphs-as-pictures" difficulty that resides in the category of

"Connecting Motion to Graphs" (see Fig. 3, third horizontal bone from top to bottom).

The "graphs-as-pictures" error was qualified by Beichner as "the most critical to address" of the six difficulties he mentions [1]. Beichner defined it as "The graph is considered to be like a photograph of the situation. It is not seen to be an abstract mathematical representation, but rather a concrete duplication of the motion event".

A Fault Tree Analysis diagram for the "graphs-as-pictures" fault condition would be similar to Fig. 4. The principal root cause is the students' inability to "connect concepts to concrete, visible, and immediate experiences" (Root Cause 2); however, the absence of approaching the problem within the framework of the three stages of reading graphs (Root Cause 1), and unfamiliarity with kinematics definitions (Root Cause 3), also underlie the problem. Therefore, the "graphs-as-picture" difficulty also shares the same three root causes listed in Fig. 5.

The same type of analysis was conducted on all the other difficulties in the fishbone diagram (Fig. 3) and the conclusion was that there were no additional root causes. Also, that the three root causes listed in Fig. 5 underlie all the other difficulties; therefore, these three root causes may be generalized to the entire data set.

Furthermore, it was also inferred that three of the categories in the Fishbone diagram (Fig. 3) do not lead to root causes: "Slope-related", "Area-related", and "Units". These are all important difficulties; however, they are merely symptoms of the deeper problems identified by the root causes.

The three root causes summarized in Fig. 5 are consistent with the findings of other researchers. Although not in the same words, Beichner [1] identifies all the issues contained in the three root causes in section VI of his article. McDermott, et al. (1987) [9] also mention these same issues throughout their entire article. Arons (1997) [17] was particularly emphatic in the "connecting motion to graphs" category. In fact, the phrase "connect motion to concrete, visible and immediate experiences" that was incorporated into Root Cause 2 was taken directly from Arons' teaching guide [17].

Skrabankova, et al. [19] also added additional and precise insight by focusing on the three stages of reading graphs stated in Root Cause 1. They used eye tracking equipment to determine where in the graph students fixated their gaze while they solved a problem.

Trowbridge and McDermott (1980) [7] provided a profound observation regarding the process of becoming familiar with kinematics definitions and concepts contained in Root Cause 3 of the "Vocabulary" category: "We have found that the inability to discriminate between related concepts often accompanies the indiscriminate use of technical vocabulary. We have also observed that as students begin to disentangle one concept from another, the process is reflected in more precise use of appropriate terms."

The ease with which the insights of these researchers was discussed and pieced together in the preceding paragraphs attests the value of the RCA method. The process of collecting the students' difficulties, and then classifying and analyzing them systematically with RCA techniques, provided a superior degree of clarity that had not been achieved previously in the literature.

VI. Implications for Instruction

The following list of implications for instruction may be inferred from the results of this investigation.

1. (Root Cause 1) Spend time discussing the three stages of reading graphs. Use them consistently as the framework to solve kinematics graphs problems. Provide practice and feedback so that students become familiar with the process. If the process is applied consistently, students should be able to avoid many of the "L2" level difficulties and will be able to refine their knowledge gradually.

2. (Root Cause 2) Always connect the motion implied in the kinematics graphs to a concrete, visible, and immediate experience. For example, the motion implied by the graph may be recreated by horizontally moving one hand over a real or imagined "Arons' Table" (Fig. 6). This kinesthetic activity (a physical activity conducted to assist in the processing of new information) was implemented successfully in the intervention designed by the author following this investigation with the RCA method (not yet published). The table was named in honor of Arnold Arons who devised it to assist in dissociating the shape of the graph from the path of motion [17]. This strategy is particularly useful for reconciling a horizontal motion plotted on a vertical axis, for identifying the sign of the kinematic variables, and for identifying direction reversals. It leads the learners "to confront and resolve the contradictions that result from his or her own misconceptions" [17]. In addition, there is no cost associated to its implementation.

3. (Root Cause 3) Continue teaching, as always, the definitions and concepts of kinematics as part of course coverage. Restate and review the definitions while solving kinematics graphs problems so that students can become familiar with the definitions as they gradually "disentangle one concept from another" [7].

4. For sure, issues will arise with the remaining specific difficulties at the "L2" level in Fig. 3 (these are all the horizontal bones except for those representing the three root causes); for example, calculating area units, or calculating the slope of a line that does not go through the origin. These specific difficulties may linger while the student works through several problems. However, these difficulties should turn into isolated issues that may be treated individually (with feedback by the instructor) because they are only symptoms of a deeper problem. Eradicating the root causes should be the priority.

Figure 7 provides a detailed solution of the case study problem based on these implications for instruction.

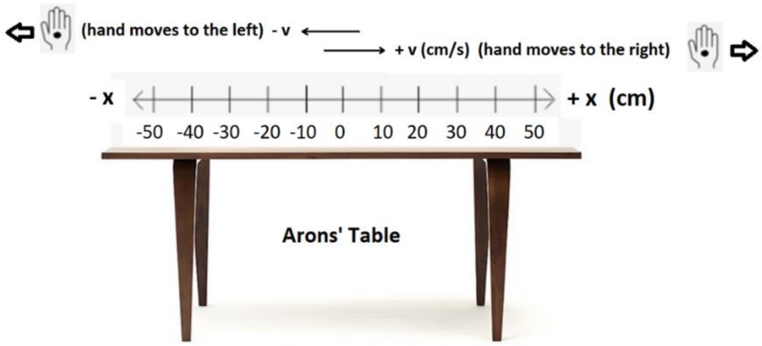

Fig. 6 "Arons' Table" for velocity used by the author in class and named in honor of Arnold Arons who devised the exercise [17]. Students move their hand horizontally over the real or imagined "Arons' table" to connect the motion implied in the kinematics graph to a concrete, visible, and immediate experience (Root Cause 2). There is no cost associated with this kinesthetic activity. It reconciles a horizontal motion plotted on a vertical axis, identifies the sign of the kinematic variables, and identifies direction reversals.

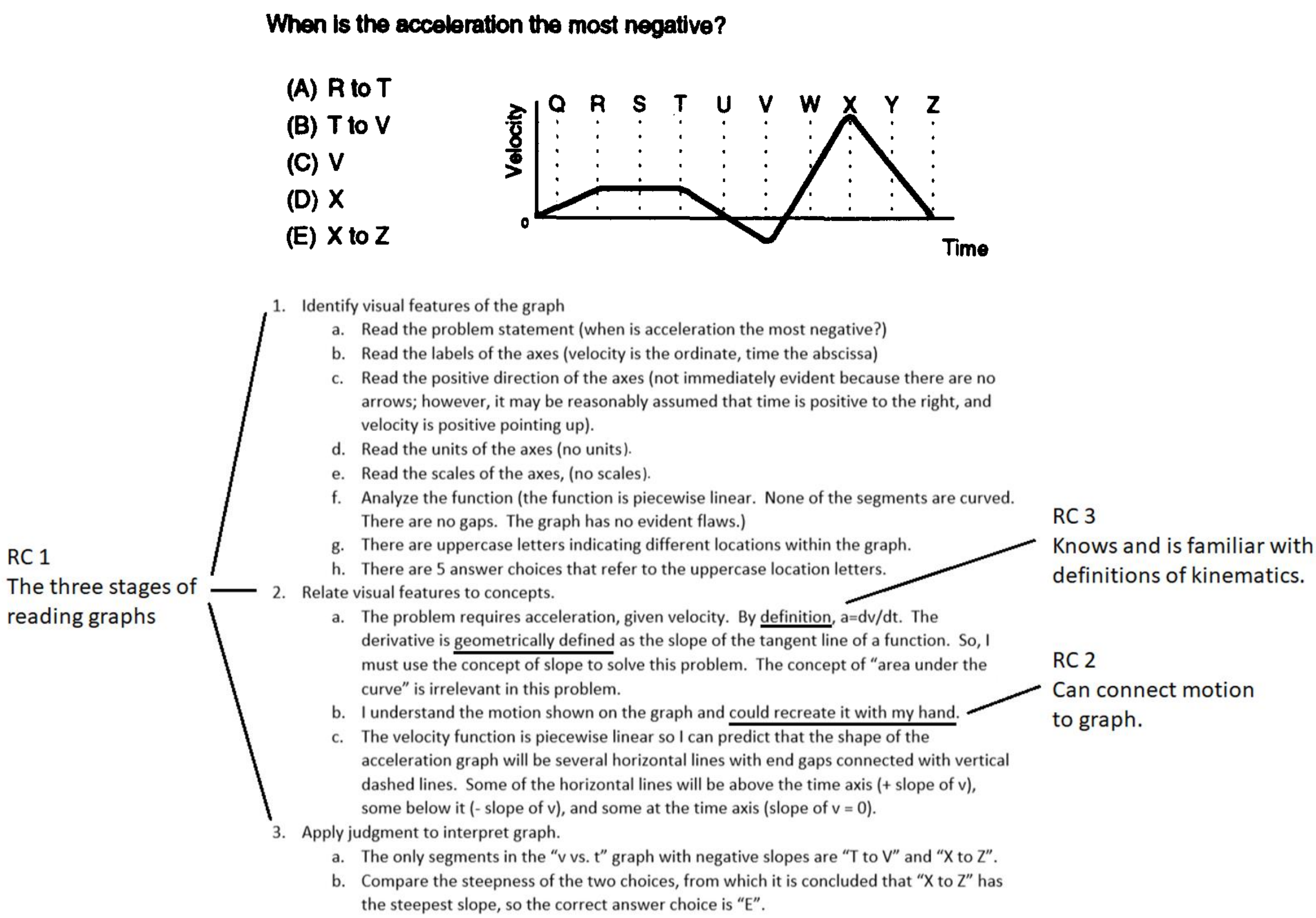

Fig. 7 "Correct solution of the case study problem with remarks related to the root causes. An expert may conduct the entire process in less than one minute. Novices should be instructed, allowed to practice, and feedback should be provided so that they can refine their knowledge gradually.

VII. LIMITATIONS

The literature review was based on the saturation criterion [2] and only analyzed 64 publications of the approximately 600 publications available in the literature. A more extensive review may perhaps uncover additional difficulties at the "L2" level, as defined in Fig. 4. However, it is believed that all the root causes of the problem ("L3" level) have been properly identified through the investigation presented in this article.

The six categories used in the slanted bones of the Fishbone diagram (Fig. 3) were based on the author's judgment of the data. Other researchers could perhaps decide on different categories, a larger (or smaller) number of categories, or different coding schemes.

VIII. CONCLUSIONS

The synthesis provided in this article fills a gap in the literature that may perhaps be as valuable to other researchers and instructors as it was for the author.

This article also provided an additional demonstration that the RCA method is readily adaptable to any problem from any discipline. RCA is useful when the problem has many potential causes, and particularly when the root causes remain hidden and tangled within the data. RCA forces the researcher to compile a comprehensive list of potential causes and classify them into categories, thus initiating a systematic process that provides a superior degree of clarity of the situation and leads to a potential resolution.

The Fishbone diagram proved to be a concise way of organizing, classifying, and visualizing all the difficulties. It synthesized all the difficulties into a coherent whole. It also served as the canvas upon which to discriminate between relevant and irrelevant difficulties during the case study analysis that determined the root causes. The Fault Tree Analysis diagram was outstanding in revealing the root causes when paired with the "5 Whys" technique.

Once the root causes were revealed through the RCA method, a precise list of implications for instruction emerged easily from the results. In summary, instructors should concentrate on eradicating the three root causes that lead to difficulties while interpreting kinematics graphs.

Finally, a word of caution: the RCA method in itself is not sufficient to disentangle a problem. A successful implementation of RCA requires good inputs (data), strong knowledge of the field to which it is applied, and good critical thinking skills. It may be appropriate to think of RCA just as an excellent framework and not as a "silver bullet" that can instantly resolve a complex problem.

ACKNOWLEDGMENTS

The author gratefully acknowledges the following: the US Air Force Summer Faculty Fellowship Program for providing the funding to conduct this research; Dr. Lachlan Belcher, Director of the Center for Physics Education Research at the US Air Force Academy, for providing guidance throughout the investigation; Dr. Wilson González-Espada for providing valuable suggestions during the preparation of this manuscript.

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