On the optimal sizing and energy for a battery bank storage in renewable energy systems

Andrés García[1], Eduardo Guillermo[1,2]
[1]Departamento de Ingeniería Eléctrica, Universidad Tecnológica Nacional, Facultad Regional Bahía Blanca. 11 de abril 461, Buenos Aires. Argentina, andresgarcia@frbb.utn.edu.ar
[2]Grupo de Estudios sobre Energías (GESE), Universidad Tecnológica Nacional, Facultad Regional Bahía Blanca. 11 de abril 461, Buenos Aires. Argentina, eguiller@frbb.utn.edu.ar

***Abstract*– This paper presents a complete deep-cycle battery charging model with an emphasis on optimizing the number of batteries in the bank subject to charging energy constraints. The algorithm is able to determine the minimum number of batteries needed for a given application given the output power as well as the distribution of them. A comparison with a real case is presented as long as future work-**

***Keywords*—Deep-cycle battery, optimal energy, charging algorithm.**

I. INTRODUCTION

Renewable energy storage is a very significant problem today, as shown in [1] and [2]. Moreover, the survey in [1] covers all the possibilities available to store energy emphasizing that batteries need to be optimally charged to extend their lifespan. In addition, the battery package size must be minimized for applications like electric vehicles, mobility or even electronic equipment.

Undoubtedly, battery energy storage is by far the preferred method in many applications, and as such, a proper charging algorithm that takes care of the energy from the incoming renewable source and a sizing method have to be applied in order to take the most profit from the battery bank (see for instance [3] and [4]).

In this way, optimal sizing with optimal renewable charging energy is mandatory. However, the existent literature does not take into account upper bounds on the incoming energy (renewable) or even including the datasheet curves provided by the battery manufacturer (see [5], [6] and [7]).

In this paper, an upper and lower bound on the number of batteries able to supply a continuous load during a prescribed design time will be obtained. Moreover, the battery charging dynamics is taken into account using available data provided by manufacturers: deep of discharge (DOD), time to charge at 100%, minimum and maximum voltages/currents during charge algorithms, as well as constraints in the availability of charging energy coming from renewable energy sources: input energy constraints.

This paper is organized as follows: Section II describes the available charging methods to optimally charge a deep-cycle battery, Section III presents the battery model for charging with a focus on energy balance, Section IV develops an optimization formulation with a ready to use upper/lower bound on the number of batteries to use, Section V studies a real case with a comparison with an existent algorithm and real measurements, Section VI presents some discussions, whereas Section VII presents some conclusions and future work.

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II. DEEP CYCLE BATTERY CHARGING METHODS

As reported in [8], a Deep-cycle battery charge can be accomplished using three different methods:

- Constant voltage
- Constant current
- Two/three stage charging algorithm

A. Constant voltage

In this method, a constant voltage is applied all along the charging time (see Figure 1). Then two instants of time are clearly visible:

- T_1: The current remains constant
- T_2: defined for every battery type to complete a charge

Then, the energy consumption per charge-cycle can be approximated by:

$$E_{in} = V_{\max} \cdot I_{\max} \cdot T_2 + \left(\frac{I_{\max} - I_{\min}}{2} \right) \cdot (T_2 - T_1) + I_{\min} \cdot (T_2 - T_1)$$

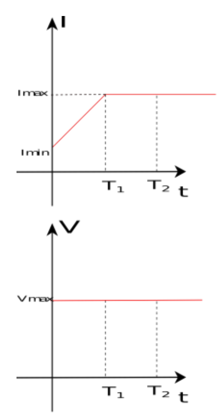


Fig. 1 Charging voltage and current vs time for a constant voltage method.

B. Constant current

In this method, a constant current is applied all along the charging time (see Figure 2). Then two instants of time are detectable and complementary to the constant voltage method:

- T_1: the voltage remains constant
- T_2: defined for every battery type to complete a charge

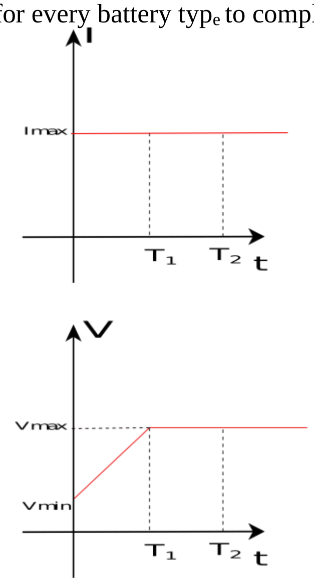


Fig. 2 Charging voltage and current vs time for a constant current method.

As in previous case, the energy consumption per charge-cycle can be computed approximately:

$$E_{in} = \left(\frac{V_{max} - V_{min}}{2} \right) \cdot I_{max} \cdot T_1 + V_{min} \cdot T_1 + V_{max} \cdot (T_2 - T_1)$$

C. Three-stage method

In this method, a constant voltage is applied all along the charging time (see Figure 3). Then two instants of time are clearly visible:

- T_1: The current starts to decrease and the voltage remains constant
- T_2: defined for every battery type to complete a charge

Finally, for this general method, the energy consumption yields:

$$E_{in} = \left(\frac{V_{max} - V_{min}}{2} \right) \cdot I_{max} \cdot T_1 + V_{min} \cdot T_1 + \\ + \left(\frac{I_{max} - I_{min}}{2} \right) \cdot (T_2 - T_1) + I_{min} \cdot (T_2 - T_1)$$

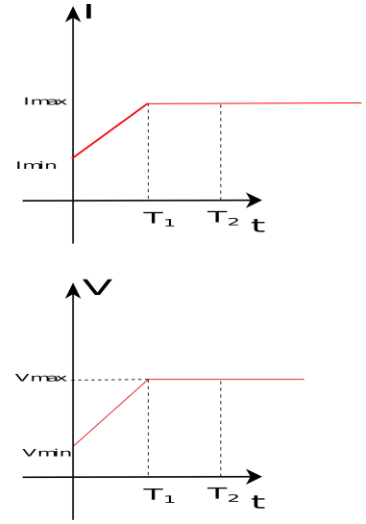


Fig. 3 Charging voltage and current vs time for a constant voltage method.

Next section develops the mathematical model for energy optimization in the case of constant voltage charging method.

III. Battery Model and Energy Balance

As every energy balance system, a renewable energy system, can be depicted as a block diagram including input (charging energy) and output power (load consumption), as described in Figure 4:

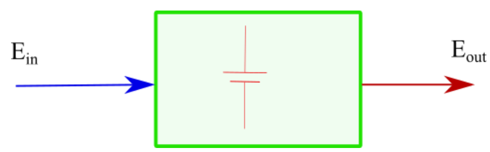


Fig. 4 Block diagram for input-output energy balance

Where E_{in} is the charging energy required to keep the deep-cycle battery bank charged and, E_{out} is the energy taken by the output load.

In addition to minimizing input energy (charging energy), the output energy needs to be taken into account, and this will be achieved by considering the current parameters for a constant voltage method.

Constant voltage optimization

To obtain closed-form formulas and an algorithm to properly sizing a del-cycle battery bank, according to the load power to feed, the constant voltage method will be assumed.

With a focus on energy optimization, it is necessary to calculate the input energy E_{in}:

$$E_{in} = \int_0^{T_2} V(t) \cdot I(t) \cdot dt \,.$$

Where V(t) is the voltage across the battery bank and I(t) the charging current. Taking into account the piecewise linear approximation in Figure 1:

$$\begin{aligned} E_{in} &= \int_0^{T_2} V(t) \cdot I(t) \cdot dt = \\ &= V_{\max} \cdot \left(I_{\max} \cdot T_1 + \left(\frac{I_{\max} - I_{\min}}{2} \right) \cdot (T_2 - T_1) + I_{\min} \cdot (T_2 - T_1) \right). \end{aligned} \quad (1)$$

According to [9, pp. 241-293], the total amount of charging time, can be approximated to be:

$$T_2 = \frac{C_{20}}{I_{\max}} \cdot DOD + 4h \,. \quad (2)$$

Where DOD means the Deep of Discharge state between 0 and 1 and C_{20} stands for the battery capacity at 20 hours of functioning expressed in *A.h*. In the same manner, we assume:

$$T_1 = 0.9 \cdot T_2 \,.$$

On the other hand, based on our own experience:

$$DOD = \frac{T_{use}}{T_{100\%}} \,.$$

Where T_{use} means the amount of time the battery bank is design to work and $T_{100\%}$ is the amount of time taken for the battery bank to be fully charged, in the previous notation: $T_{100\%} = T_2$.

Then, replacing these formulas in (1):

$$\begin{aligned} E_{in} &= V_{\max} \cdot \left(\frac{C_{20}}{I_{\max}} \cdot \frac{T_{use}}{T_{100\%}} + 4h \right) \cdot \\ &\cdot \left(I_{\max} + \left(\frac{I_{\max} - I_{\min}}{2} \right) \cdot 0.1 + I_{\min} \cdot 0.1 \right). \end{aligned} \quad (3)$$

Moreover, $T_{100\%}$ is a function of the discharge current (output power current), as depicted in Figure 5 for several batteries:

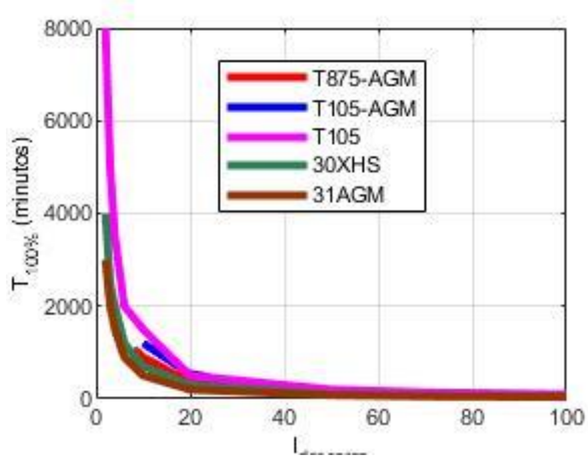


Fig. 5 Discharge current vs charging time

In this way, calculating the discharge current for a constant power load:

$$I_{\text{discharge}} = \frac{P_o}{M \cdot N \cdot V_{bat}} \quad (4)$$

Where P_o is the constant output power with a circulating current given by $I_{discharge}$, $\{M,N\}$ means a battery bank of N batteries connected in parallel with M branches (see Figure 6) and V_{bat} is the nominal battery voltage:

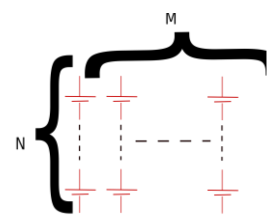


Fig. 6 Battery bank distribution

To simplify the algorithm, the non-linear curve in Figure 5, it is going to be linearized as follows:

$$T_{100\%} = a \cdot I_{discharge} + b, \qquad a < 0, b > 0 \quad (5)$$

Then, replacing (5) into (3) and adding a discharge constraint using (4):

$$\begin{aligned} E_{in} &= V_{\max} \cdot \left(\frac{C_{20}}{I_{\max}} \cdot \frac{T_{use}}{\left(a \cdot I_{\text{discharge}} + b\right)} + 4h \right) \cdot \\ &\cdot \left(I_{\max} + \left(\frac{I_{\max} - I_{\min}}{2} \right) \cdot 0.1 + I_{\min} \cdot 0.1 \right) \\ &\frac{P_o}{M \cdot N \cdot V_{bat}} \le \overline{I} \end{aligned} \quad (6)$$

Where $\overline{I}$ is an upper bound given by design. Then, the optimization problem is going to be defined and solved in next section.

IV. OPTIMIZATION ALGORITHM

Considering (6), the following nonlinear programming problem captures the optimal sizing battery bank formulation subject to a charging energy bound (see [10] for details on non-linear programming):

$$\min_{\{M,N\}\in\mathbb{N}} M\cdot N$$

such that:

$$\begin{cases} E_{in}=V_{\max}\cdot\left(\dfrac{C_{20}}{I_{\max}}\cdot\dfrac{T_{use}}{\left(a\cdot I_{\text{discharge}}+b\right)}+4h\right)\cdot \\ \cdot\left(I_{\max}+\left(\dfrac{I_{\max}-I_{\min}}{2}\right)\cdot 0.1+I_{\min}\cdot 0.1\right) \\ \dfrac{P_o}{M\cdot N\cdot V_{bat}}\le \bar{I} \\ E_{in}\le \bar{E} \\ V_{\max}=V \end{cases}$$

Where V is the bank voltage given by the user (design). In order to minimize $M.N$, it is first considered the charging energy constraint:

$$V_{\max}\cdot\left(\frac{C_{20}}{I_{\max}}\cdot\frac{T_{use}}{\left(a\cdot I_{\text{discharge}}+b\right)}+4h\right)\cdot$$
$$\cdot\left(I_{\max}+\left(\frac{I_{\max}-I_{\min}}{2}\right)\cdot 0.1+I_{\min}\cdot 0.1\right)\le \bar{E}$$
$$I_{\text{discharge}}=\frac{P_o}{M\cdot N\cdot V_{bat}}$$

Then:

$$V_{\max}\cdot\left(\frac{C_{20}}{I_{\max}}\cdot\frac{T_{use}\cdot M\cdot N\cdot V_{bat}}{\left(a\cdot P_o+b\cdot M\cdot N\cdot V_{bat}\right)}+4h\right)\cdot$$
$$\cdot\left(I_{\max}+\left(\frac{I_{\max}-I_{\min}}{2}\right)\cdot 0.1+I_{\min}\cdot 0.1\right)\le \bar{E}$$

In other words:

$$M\cdot N\le\frac{a\cdot P_o\cdot\left(\dfrac{\bar{E}}{V_{\max}\cdot\alpha}-4h\right)\cdot\dfrac{I_{\max}}{C_{20}\cdot T_{use}\cdot V_{bat}}}{1-\left(\dfrac{\bar{E}}{V_{\max}\cdot\alpha}-4h\right)\cdot\dfrac{I_{\max}}{C_{20}\cdot T_{use}\cdot V_{bat}}\cdot b\cdot V_{bat}} \qquad (7)$$

Where $\alpha=\left(I_{\max}+\left(\frac{I_{\max}-I_{\min}}{2}\right)\cdot 0.1+I_{\min}\cdot 0.1\right)$. On other hand, the discharge constraint leads:

$$M\cdot N\ge\frac{P_o}{\bar{I}\cdot V_{bat}} \qquad (8)$$

Finally, from (7) and (8):

$$\begin{cases} \dfrac{P_o}{\bar{I}\cdot V_{bat}}\le M\cdot N\le\dfrac{a\cdot P_o\cdot\left(\dfrac{\bar{E}}{V_{\max}\cdot\alpha}-4h\right)\cdot\dfrac{I_{\max}}{C_{20}\cdot T_{use}\cdot V_{bat}}}{1-\left(\dfrac{\bar{E}}{V_{\max}\cdot\alpha}-4h\right)\cdot\dfrac{I_{\max}}{C_{20}\cdot T_{use}\cdot V_{bat}}\cdot b\cdot V_{bat}} \\ \alpha=\left(I_{\max}+\left(\dfrac{I_{\max}-I_{\min}}{2}\right)\cdot 0.1+I_{\min}\cdot 0.1\right) \end{cases} \qquad (9)$$

Where the upper bound is given by $\bar{E}=V_{\max}\cdot I_{\max}\cdot T_2$, providing an interval bounding for the battery number. It turns out that the number N is readily obtained once the battery voltage $V=V_{max}$ is defined:

$$V_{\max}=V=V_{bat}\cdot N$$

V. APPLICATION EXAMPLE

In this application example, a deep-cycle battery bank is designed to feed a 300W (average) load with a nominal voltage V_{max}=24V, for a continuous running period of 8 hours.

Then according to (9), and considering Trojan T-105 batteries (6V nominal with 200Ah of capacity), the linear approximation expected by (5) is presented in Figure 7, along with:

$$T_{100\%}=\text{-}300[\text{h/A}]\cdot I_{discharge}+5000[\text{h}]$$

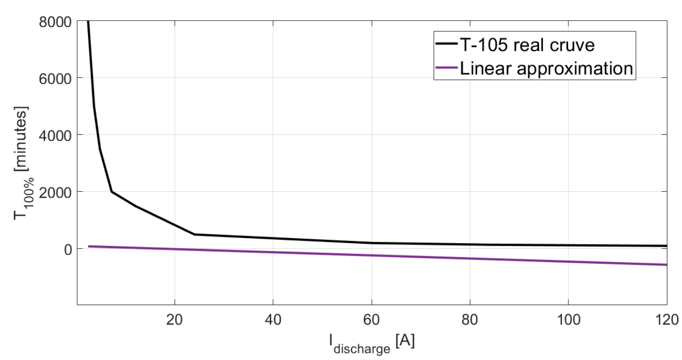


Fig. 7 Datasheet vs linear approximation for T-105 batteries

Considering $V_{max}=7.4V$ for a Trojan T-105 battery, the bounds given by (9) along with (2), provide the number of batteries to be used:

$$\begin{cases} \dfrac{300W}{17.5A\cdot 6V} = 2.8571 \le M\cdot N \le \dfrac{\text{-}300h/A\cdot 300W\cdot\left(\dfrac{\bar{E}}{7.4V\cdot 21.3A}-4h\right)\cdot\dfrac{20A}{200Ah\cdot 8h\cdot 6V}}{1-\left(\dfrac{\bar{E}}{7.4V\cdot 21.3A}-4h\right)\cdot\dfrac{20A}{200Ah\cdot 8h\cdot 6V}\cdot 5000h\cdot 6V} = 2.99 \\ a = \text{-}5.5\ h/A, \quad b = 96.3675h \\ \bar{I} = \dfrac{300W}{24V} + 5A \text{ (to add a conservative bound)} = 17.5A \\ \alpha = \left(I_{\max} + \left(\dfrac{I_{\max}-I_{\min}}{2}\right)\cdot 0.1 + I_{\min}\cdot 0.1\right) = 20A + 0.7A + 0.6A = 21.3A \\ \bar{E} = V_{\max}\cdot I_{\max}\cdot T_2 = 7.4V\cdot 20A\cdot\left(\dfrac{C_{20}}{I_{\max}}\cdot DOD + 4\right) = 9h \\ I_{\max} = \dfrac{C_{20}}{10} = 20A, \quad I_{\min} = 0.03\cdot C_{20} = 6A \end{cases}$$

Where an initial $DOD=0.5$ was considered (recall that in [7] a maximum of $DOD=0.7$ is pointed out). Clearly, the bounds show: $N.M=3$, however, with a nominal voltage of 24V for the load voltage and each battery with 6V, we choose: $N=4, M=1 \Rightarrow M\cdot N = 4$.

Then, the real charging profile on a bank of four T-105 batteries is depicted in Figure 8, confirming the charging time estimated by (2):

$$T_2 = \frac{C_{20}}{I_{\max}}\cdot DOD + 4 = \frac{225A.h}{10A}\cdot 0.1 + 4h = 6.25h$$

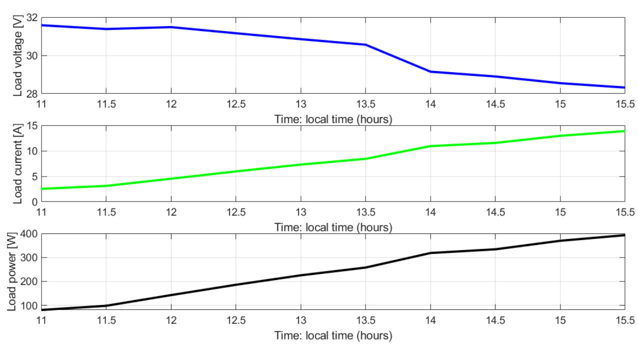


Fig. 8 Real measurement using Trojan's battery T-105 with a constant resistance load

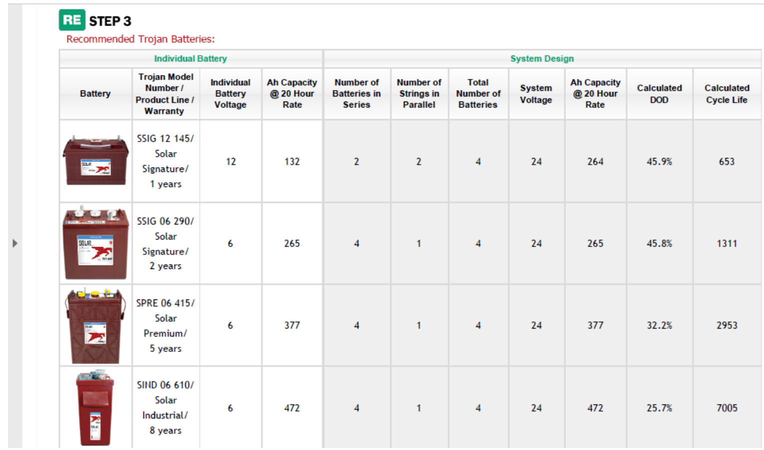
TABLE I

CHARGING MEASUREMENTS

Local time (hs)	Solar Rad. [W/m^2]	Voltage [V]	Current [A]
11	900	27.0	13
11:45	900	27.7	13
12	970	28.86	13.2
12:49	970	27.26	13.28
13	979	28.7	12.82
13:19	979	28.3	10
14:30	1002	29.4	9
15:18	900	29.5	8.73
15:53	900	28.49	8.79
16:10	469 (CLOUDY)	29.0	8.49
17:00	323 (CLOUDY)	30.3	8.5
18:10	211 (CLOUDY)	32.0	8.45

Whereas, real measurements for a four bank of batteries with $DOD=0.1$ and charging at $Imax=10A$, is depicted in Table I for a total charging time of 5 hours.

Comparing with Trojan's on-line algorithm (see Figure 9 and https://www.batterysizingcalculator.com/):



STEP 3

Recommended Trojan Batteries:

Individual Battery				System Design						
Battery	Trojan Model Number / Product Line / Warranty	Individual Battery Voltage	Ah Capacity @ 20 Hour Rate	Number of Batteries in Series	Number of Strings in Parallel	Total Number of Batteries	System Voltage	Ah Capacity @ 20 Hour Rate	Calculated DOD	Calculated Cycle Life
	SSIG 12 145/ Solar Signature/ 1 years	12	132	2	2	4	24	264	45.9%	653
	SSIG 06 290/ Solar Signature/ 2 years	6	265	4	1	4	24	265	45.8%	1311
	SPRE 06 415/ Solar Premium/ 5 years	6	377	4	1	4	24	377	32.2%	2953
	SIND 06 610/ Solar Industrial/ 8 years	6	472	4	1	4	24	472	25.7%	7005

Fig. 9 Trojan on-line calculator

It is clear that the algorithm in this paper performs a very tight calculation for the load power and time usage with a prescribed output voltage.

VI. Discussion

A ready-to-use algorithm is always helpful and welcome in engineering applications. However, the present analysis goes beyond a simple algorithm to simulate a battery bank capacity.

In addition to offering a bound for how many deep-cycle batteries to use in a given set of requirements (see (9)), this algorithm also takes into account the manufacturer's actual charge-discharge curves.

As it is well-known, deep-cycle batteries are expensive and, in most cases, heavy weight, so reducing to the minimum the number of units needed in each application with a guarantee of functioning in the worst-case scenario (namely continuous use), is very important and a must issue.

Notice that the algorithm requires, on the one hand, some data from the user-end: time of flight, application voltage, maximum load power but, on the other hand, the linear approximation to the real battery discharge curves.

In cases where few data are available, commercial deep-cycle battery curves approximate the application under study.

VII. Conclusions

In this paper an interval bound for the number of batteries to be used in a given renewable battery bank storage application was obtained.

As a matter of fact, the algorithm only requires rough bounds on the energy to be used for the charging method (constant voltage) and bounds on the load current, besides the battery data available from the manufacturer.

In order for the method to be effective, it requires the ability to include both the charging power (solar panels, wind energy, etc) and the dynamics of the battery.

As it can be noticed from the workout study case, the method is sensitive to the linear approximation of the discharge curve, opening the possibilities to more sophisticated approximations, for instance, using piecewise linear approximations but, at the same time, causing complications to produce a closed-form formula.

As a future work, the case in this paper and some other load powers will be studied with piecewise linear approximations.

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