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Comparative Analysis of Transient Dynamic Responses on Timoshenko Beams

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I. INTRODUCTION

Beams are well-known and commonly used structural elements on buildings construction. In addition to this, these are analysed and designed using commercial softwares. However, there are diverse engineering studies as helicopter propellers, aircraft wings, where a beam computational model is needed for a major accuracy on results [1].

In literature, there are two beam theories usually used on similar work research. The traditional Euler-Bernoulli theory, which is applied on buckling beams researches, where a non-linearity is considered on the geometry element [2]. Although the mentioned affirmation, shear effect is not analyzed on this theory, hence obtained results may not be correct and accurate [3]. For that reason, Timoshenko theory is introduced due to including shear effect on beams, permitting new ways of research using consistent methods, even to studying non-conventional materials [4]. Diverse authors have approached innovative materials for beams, as well known as functionally graded materials, which use Timoshenko theory for their development [5], [6]. Additionally, these have been compared with obtained results using Euler-Bernoulli theory under a same condition of materials composition [7]. Chen et al. [8] combined the shearing effect with the micro-beams effects applying Timoshenko theory, as Karami and Janghorban [9] with porous nanotubes under a free vibration, and the Saint-Venant-Kirchkoff law related to Timoshenko theory [10]. Mentioned research were modeled on a steady-state, therefore a dynamic analysis is required for evaluating Timoshenko beam behavior in a period of time. Moving loads on a moving beam [11] and elastic supports fixed to a mobile spring-mass system [12] were approached through dynamic analysis. A stable Alpha-Newmark method was applied during time integration for diverse dynamic cases due to its great accuracy and versatility [13].

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Despite all obtained information on past studies about beam behavior under different material, geometry conditions and theories, a lack of knowledges about beam behavior related to the time is observed. More research is needed to be developed considering this time factor in order to understand its influence in a transient dynamic state and improve the accuracy of the model.

The purpose of this paper is the development of a computational model for the analysis of beams based on the theorical formulation of Timoshenko arriving on Hamilton's Principle. Three independent variables for the approximation field are considered on this model. Alpha-Newmark method is applied for fully discretization. References [14] and [15] are compared to this research for validation of the model.

II. THEORETICAL FORMULATION

A. *Timoshenko beam theory*

The first-order theory or Timoshenko theory, includes the shear deformation, where plane sections remain plane but not perpendicular to the longitudinal axis after deformation. Field displacements are shown on (1),

$$u_1(x_1,x_3) = u(x_1) - x_3\phi_1(x_1)$$

$$u_2(x_1,x_3) = 0 \tag{1}$$

$$u_3(x_1,x_3) = w(x_1)$$

where x_1, x_3, ϕ_1 are the independent variables.

A linear geometrical behavior of a beam is considered, thus the strain is defined as

$$\varepsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \tag{2}$$

Equation (2) is included and replaced on field displacements in (1), obtaining

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = \frac{du}{dx_1} + \left(\frac{d\emptyset_1}{dx_1}\right)x_3 \tag{3}$$

$$\varepsilon_{13} = \varepsilon_{31} = \frac{1}{2}\left[\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right] = \frac{1}{2}\left[\emptyset_1 + \frac{dw}{dx_1}\right] \tag{4}$$

$$\varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = \varepsilon_{23} = \varepsilon_{32} = \varepsilon_{33} = 0 \tag{5}$$

Components on above contain mentioned three independent variables. The total strain is calculated through Green-Lagrange strain tensor.

$$\varepsilon = \varepsilon_{11}(e_1 \otimes e_1) + 2\varepsilon_{13}(e_1 \otimes e_3) \tag{6}$$

B. Principle of Virtual Displacements

The body will be in equilibrium only if virtual work of internal and external forces applied on the body are zero against a virtual displacement. Mathematically, it was defined as

$$\delta W_i - \delta W_e = 0 \tag{7}$$

Displacement along the beam is the main component for calculating external work. It is shown on (8).

$$\delta W_e = \int_{x_1} q\delta w \, dx_1 + \int_{x_1} f\delta u \, dx_1 \tag{8}$$

Internally, beam is governed by the strain and the virtual internal work is defined as

$$\delta W_I = \int_V \sigma_{ij}\delta\varepsilon_{ij}dV \tag{9}$$

The beam theory used for this approach only considers two degrees of freedom located on the longitudinal and transversal axes (x_1 and x_3).

$$\delta W_I = \int_{x_1}\int_A (\sigma_{11}\delta\varepsilon_{11} + \sigma_{13}(2\delta\varepsilon_{13}))dA \, dx_1 \tag{10}$$

Strains are expanded according to components defined in (3), (4) and (5). Hooke's law is also applied on (10) to finally obtain

$$\delta W_I = \int_{x_1} \Big[A_{11}{\varepsilon_{11}}^{(0)} \, \delta{\varepsilon_{11}}^{(0)} + D_{11}{\varepsilon_{11}}^{(1)}\delta{\varepsilon_{11}}^{(1)} + 4K_sA_{13}{\varepsilon_{13}}^{(0)}\delta{\varepsilon_{13}}^{(0)}\Big]dx_1 \tag{11}$$

in terms of displacements, it is expressed in (12).

$$\delta W_I = \int_{x_1} \Big[A_{11}\frac{du}{dx_1}\frac{d\delta u}{dx_1} + D_{11}\frac{d\phi_1}{dx_1}\frac{d\delta\phi_1}{dx_1} + K_sA_{13}\phi_1\delta\phi_1 + K_sA_{13}\frac{dw}{dx_1}\frac{d\delta w}{dx_1} + K_sA_{13}\frac{dw}{dx_1}\delta\phi_1 + K_sA_{13}\phi_1\frac{d\delta w}{dx_1}\Big]dx_1 \tag{12}$$

where:

$$A_{11} = EA \tag{13}$$

$$D_{11} = EI$$

$$A_{13} = GA$$

C. Hamilton's Principle

The mathematical formulation is constructed by the Hamilton's Principle. An extremum value, along the movement curve, is obtained by solving the integration below

$$\int_{t1}^{t2} Ldt \tag{14}$$

where L is defined as the Lagrangian function of the mechanical system.

$$L = K - \Pi \tag{15}$$

The kinetic energy is symbolized by K and the potential energy by Π.

Hamilton explains the development of the system through the time as a stationary action integral. It is expressed in (16)

$$\delta J = \delta\int_{t1}^{t2} Ldt = 0 \tag{16}$$

Every conservative mechanical system presents a potential energy Π, which includes a deformation internal energy W_I and the potential of external forces W_e.

$$\delta J = \int_{t1}^{t2} [\delta K - (\delta W_I - \delta W_e)]dt = 0 \tag{17}$$

Kinetic energy can be analyzed using its traditional definition that involves velocity $\dot{\mathbf{u}}$, density ρ and volume V during a period of time.

$$\int_{t_1}^{t_2}[\delta K]dt = \int_{t_1}^{t_2} \delta\left[\frac{1}{2}\int_V \rho.\dot{\mathbf{u}}\cdot\dot{\mathbf{u}}dV\right]dt = \int_{t_1}^{t_2} \delta\left[\frac{1}{2}\int_{V_0} \rho_0.\dot{\mathbf{u}}\cdot\dot{\mathbf{u}}dV_0\right]dt \tag{18}$$

Equation (18) is expanded in order to disappear all vector terms appropriately and to define the volume integral in function of x_1.

$$\int_{t_1}^{t_2}[\delta k]\,dt = -\int_{t_1}^{t_2}\left[\int_{x_1}\big[(\ddot{u}\delta u + \ddot{w}\delta w)I^{(0)} + (\ddot{u}\delta\phi_1 + \ddot{\phi}_1\delta u)I^{(1)} + (\ddot{\phi}_1\delta\phi_1)I^{(2)}\big]dx_1\right]dt \tag{19}$$

III. Finite Elements Model

A. Variational Equation

Finite elements method is used as part of the process of computational model elaboration. Thus, a variational equation is formed by the previous principles and theories defined in Chapter II.

$$\begin{aligned}\delta J = \int_{t1}^{t2} \Bigg[\int_{x_1} & \big[(\ddot{u}\delta u + \ddot{w}\delta w)I^{(0)} + (\ddot{u}\delta\phi_1 \\ & + \ddot{\phi}_1\delta u)I^{(1)} + (\ddot{\phi}_1\delta\phi_1)I^{(2)}\big]dx_1 \\ & - \int_{x_1} \Big[A_{11}\frac{du}{dx_1}\frac{d\delta u}{dx_1} \\ & + D_{11}\frac{d\phi_1}{dx_1}\frac{d\delta\phi_1}{dx_1} + K_sA_{13}\phi_1\delta\phi_1 \\ & + K_sA_{13}\frac{dw}{dx_1}\frac{d\delta w}{dx_1} \\ & + K_sA_{13}\frac{dw}{dx_1}\delta\phi_1 \\ & + K_sA_{13}\phi_1\frac{d\delta w}{dx_1}\Big]dx_1 \\ & + \int_{x_1}(q\delta w + f\delta u)\,dx_1\Bigg]dt = 0\end{aligned} \tag{20}$$

B. Spatial Approximation (Semidiscretization)

Displacement terms are presented in (20), for that reason, an equation is required to generate an approximation of the original function (real solution). A displacements approximation is proposed applying a semi-discretization of the element.

$$u(x,t) \approx u_h^e(x,t) = \sum_{j=1}^{n} u_j^e(t)\psi_j^e(x) \tag{21}$$

The proposed approximation in (21) involves an analysis through "n" nodes number. This process is known as independent variable interpolation. For this approach, Gauss-Legendre and Lobatto-Gauss-Legendre approximation is applied.

Independent variables in the variational formulation are replaced by their approximated forms. Hence, mass and stiffness matrixes are obtained and shown on (22). This is the general equation of motion.

$$\int_{t_1}^{t_2} (\,[\mathrm{M}]\{\ddot{\mathrm{u}}\} + [\mathrm{K}]\{\mathrm{u}\} - \{\mathrm{f}\}\,)dt = 0 \tag{22}$$

C. Temporal Approximation (Full Discretization)

The equation of motion is in function of time yet. Due to this, the set of time-dependent equations need to be converted to a set of algebraic equations.

The approximation is done through the α- γ family, where Newmark-Alpha (also known as Newmark-Beta) method is chosen to determine stability and accuracy of the scheme.

First-and-second-order Taylor expansion is applied on the velocity during an instant s+1, obtaining

$$\begin{gathered}\mathrm{u}^{s+1} \approx \mathrm{u}^{s} + \Delta t\dot{\mathrm{u}}^{s} + \frac{1}{2}(\Delta t)^2[(1-\gamma)\ddot{\mathrm{u}}^{s} + \gamma\ddot{\mathrm{u}}^{s+1}] \\ \dot{\mathrm{u}}^{s+1} \approx \dot{\mathrm{u}}^{s} + a_2\ddot{\mathrm{u}}^{s} + a_1\ddot{\mathrm{u}}^{s+1}\end{gathered} \tag{23}$$

where:

$$\begin{gathered}a_1 = \alpha\Delta t \\ a_2 = (1-\alpha)\Delta t\end{gathered} \tag{24}$$

Equation (23) is replaced in the equation of motion to obtain the following fully discretized equations.

$$\hat{\mathrm{K}}\mathrm{u}^{s+1} = \hat{\mathrm{F}}^{s,s+1} \tag{25}$$

where:

$$\begin{gathered}\hat{\mathrm{K}} = \mathrm{K} + a_3\mathrm{M} \\ \hat{\mathrm{F}}^{s,s+1} = \mathrm{F}^{s+1} + \mathrm{M}\bar{\mathrm{u}}^{s} \\ \bar{\mathrm{u}}^{s} = a_3\mathrm{u}^{s} + a_4\dot{\mathrm{u}}^{s} + a_5\ddot{\mathrm{u}}^{s} \\ a_3 = \frac{2}{\gamma(\Delta t)^2} \\ a_4 = a_3\Delta t \\ a_5 = \frac{1}{\gamma} - 1\end{gathered} \tag{26}$$

Through the Newmark-Alpha method (average acceleration method), α and γ are defined with values in (27).

$$\alpha = \gamma = \frac{1}{2} \tag{27}$$

IV. Numerical Results

In this section, beams with different boundary conditions are evaluated using N-elements mesh with interpolation functions of P=4 in order to avoid shear locking. Obtained results will be compared with similar researches [14], [15] for validating the proposed model and observing the behavior of transient responses.

A. Transverse motion of a both-ends clamped beam

The proposed benchmark presents the transverse motion of clamped beam at both ends. A beam with square cross-section is considered for every cases. The vertical deflection was calculated at the center of the beam. Different P levels and N elements were used to analyze accuracy of results. Boundary conditions are defined below.

$$\begin{gathered}w(x_1,0) = \sin(\pi x_1) - \pi x_1(1-x_1) \\ \frac{\partial w}{\partial t}(x_1,0) = 0\end{gathered} \tag{28}$$

$$\theta(x_1,0) = -\pi\cos(\pi x_1) + \pi(1-2x_1)$$

$$\frac{\partial\theta}{\partial t}(x_1,0) = 0$$

Material parameters are given in Table I. A dimensionless geometry and constant material are considered in purpose of the analysis.

TABLE I

Parameter	Value
L	1
b	1
h	0.01
EI	1
GAK_s	$4/h^2$
ρA	1
ρI	$h^2/12$

Fig 1 shows the transient response of a clamped beam at both ends. Results illustrate similarity between the proposed model and the benchmark. The tendency is maintained, but obtained values vary insignificantly because of assigned P interpolation level and N elements. It is observed that N=2 and P=1 differs the most in comparing to other cases due to the analyzed number of nodes. There is a direct relationship between number of nodes with N and P values. When these increase, number of nodes are greater. Hence, the analyzed mesh is divided in many elements, giving to results more accuracy. In Fig 1, greater N and P values tend to be more similar between them until reaching a convergency in the real result. From this point, it is not enough to continue evaluating more results with greater N and P values.

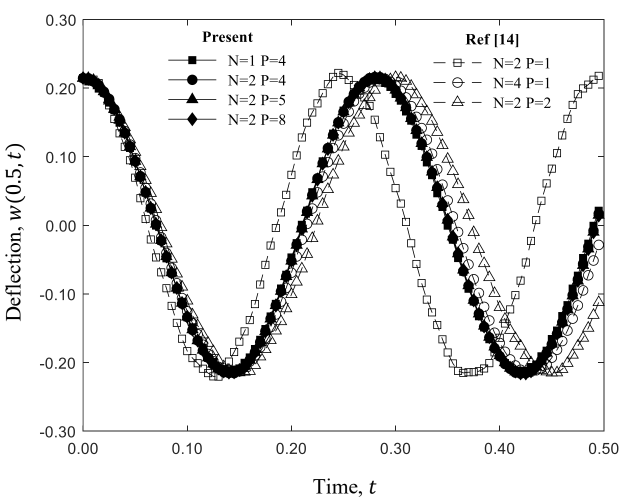

Fig. 1 Transient response of a clamped beam at both ends

B. *Transverse motion of a beam using a ratio of disturbance*

In [15], a free supported beam at both ends is analyzed using a ratio of disturbance r_d, which defines a central length l_d where initial displacement is applied.

$$r_d = \frac{l_d}{L} \tag{29}$$

Deflections and rotations were calculated along the beam considering different P levels and N elements. Boundary conditions are shown in (30), according to the estimated central length by the ratio of disturbance. Material parameters are given in Table II.

$$D(0) = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{2\pi x_1}{Lr_d}\right)\right], & \frac{-l_d}{2} < x_1 < \frac{l_d}{2} \\ 0 \qquad , & all\ other\ x_1 \end{cases}$$
$$\phi(0) = \begin{cases} \frac{-\pi}{Lr_d}\operatorname{sen}\left(\frac{2\pi x_1}{Lr_d}\right), & \frac{-l_d}{2} < x_1 < \frac{l_d}{2} \\ 0 \qquad , & all\ other\ x_1 \end{cases} \tag{30}$$

TABLE II

Parameter	Value
L	480 in
b	2 in
h	2 in
E	30000000 lb/in^2
G	12000000 lb/in^2
ρ	0.00073 $lb\cdot s^2/in^4$
K_s	0.322

Two scenarios were evaluated using a ratio of disturbance of 0.5, considering a P=8 interpolation function and N=8 elements. In Fig. 2, a transient response of a free beam at both ends using a Gauss-Legendre (GL) and a Lobatto-Gauss-Legendre (LGL) approximation is presented for comparing. A similar tendency is observed, however, results are delayed by 0.03 seconds approximately respect to [15] and it lags more by passing the time.

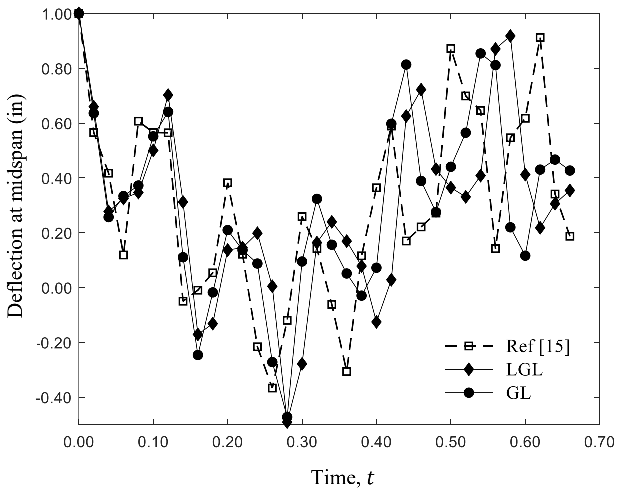

Fig. 2 Transient response of a free beam at both ends using GL and LGL approximation ($r_d = 0.5$) ($\Delta t = 0.02$)

Besides this, GL tends to be equal to [15]. Although this similarity, the condition of high-order interpolation function (P=8) meets better for LGL, giving more accurate results.

Initial and final times were taken in Fig. 3, Fig. 4, Fig. 5 and Fig. 6 for analyzing behavior of transient responses on a Timoshenko beam under a ratio of disturbance of 0.5 through displacements and rotations along the element.

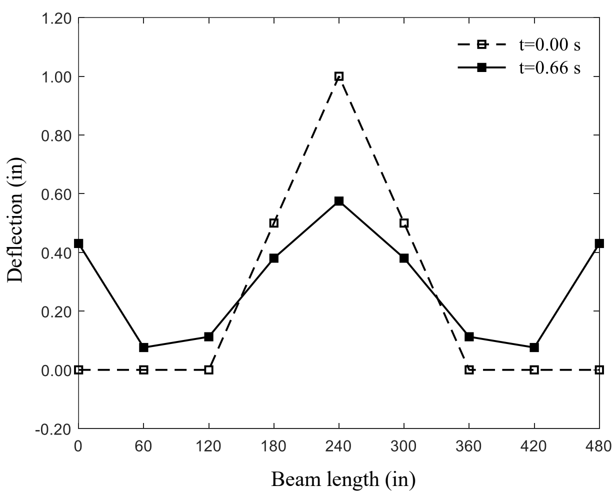

Fig. 3 Initial and final displacements of a free beam at both ends ($r_d = 0.5$) ($P = 4$) ($N = 2$) ($\Delta t = 0.02$)

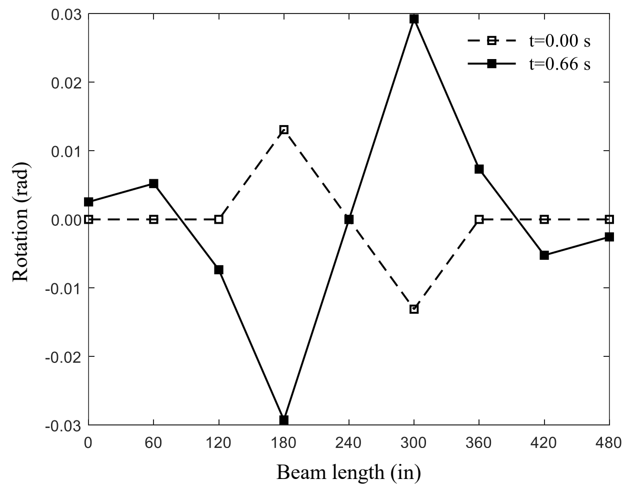

Fig. 4 Initial and final rotations of a free beam at both ends ($r_d = 0.5$) ($P = 4$) ($N = 2$) ($\Delta t = 0.02$)

Fig. 3 and Fig. 4 present displacements and rotations along the beam using P=4 and N=2 values. It is clearly observed the evaluated low number of nodes. Despite the logical deflection shape, these values are not enough accuracy to conclude a statement. However, Fig. 5 and Fig. 6 show better results due to using N=16 elements. Hence, the number of nodes gets greater, obtaining more accurate results which demonstrate a more real behavior of the beam during from two moments at the time.

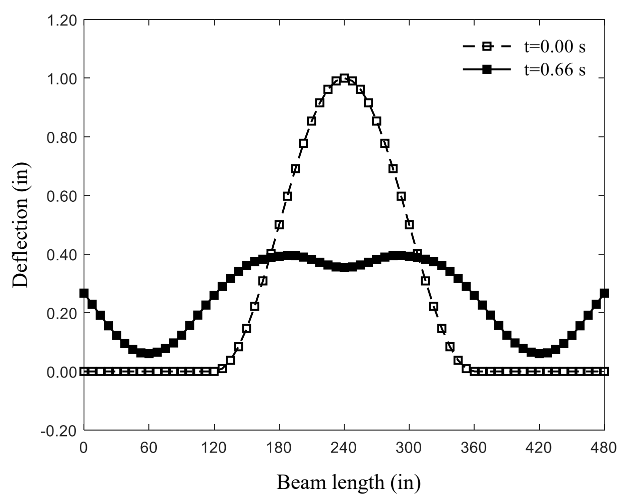

Fig. 5 Initial and final displacements of a free beam at both ends ($r_d = 0.5$) ($P = 4$) ($N = 16$) ($\Delta t = 0.02$)

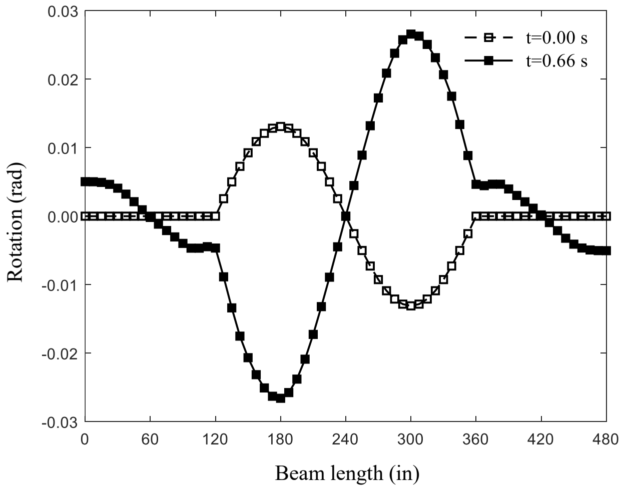

Fig. 6 Initial and final rotations of a free beam at both ends ($r_d = 0.5$) ($P = 4$) ($N = 16$) ($\Delta t = 0.02$)

V. CONCLUSIONS

The present research made a formulation to analyze the dynamic transient behavior of a beam under the Timoshenko theory, which includes the shear deformation. It was used for developing a finite element model based on Hamilton's Principle. The semidiscretization was done through the shape function interpolation. In order to avoid adverse phenomena as the shear locking, high-order interpolation functions were used. The full discretization of the independent variables was reached using the average acceleration method, as well-known as Newmark scheme.

According to the obtained results and the comparison with similar studies, the proposed model was adequately validated applying different boundary conditions and material parameters. Additionally, model results presented a considerable accuracy due to the greater number of elements discretized and the high-order interpolation functions applied through modern methods of approximation as GL and LGL, opening new ways to analyze others beams under innovative approaches.

For future publications, authors expect to improve the present investigation evaluating beams using high-order theories with more independent variables, including functionally graded materials (FGM) or adapting a non-linear geometry on its analysis.

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