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Nonlinear Transient Dynamic Behaviour of Functionally Graded Beams

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Abstract*– The objective of the following investigation deals with the geometrically nonlinear transient dynamic analysis of functionally graded beams. The effect of thickness stretching, and shear strain is considered within the formulation of the improved beam theory, which requires five independent variables and fully utilizes the constitutive equations. The graded material properties are realized under two material phases and are distributed by power law. The dynamic formulation is based on the Hamilton principle, which comes from the principle of minimum energy and being able to use the Lagrange´s function. The model is implemented by means of the Finite Element method for its numerical resolution, for this the modified Newmark method is used, in which the Newton Raphson method is applied to solve the system of nonlinear equations. High order interpolation functions are used to reduce the Poisson locking effect. Finally, the results are compared with benchmark problems and proposing new case studies.

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I. INTRODUCTION

In Japan by I. Shiota and Y. Miyamoto, in the 1980s, materials began to appear with new trends for aeronautics, and civil structures, among other engineering topics [1-3]. This material is based on taking specific advantage of the different materials and with this combining them smoothly and continuously without causing sudden changes or singularities in the stresses due to the interfaces [4]. These are called Functionally Graded Materials (FGM), for more details (see II.C). It is widely used with two materials that provide ductility or high strength properties and high thermal resistance, which are metal and ceramic respectively [5]. These two materials complement each other since steel does not have high thermal resistance and ceramic does not have high resistance to deformation.

The high order theories are based on adding the shear strain and different strain fields. In our case we will evaluate two types of them, the Timoshenko theory (FSDT) and the improved first order theory (IFSDT). The first one is widely known; however, the improved theory has not yet been evaluated in different applications. This improvement comes from the works of (M. Bischoff and E Ramm [6], Sansour [7]) where the use of a rotation tensor is not considered. In addition, the consideration of the stretching thickness requires the use of the totality of constitutive equations.

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These concepts were extensively used by Arciniega [8-9] for shell-type elements, considering a material configuration update process. Furthermore, it is possible to use this theory for different geometries (plates, beams, curved shells).

On the other hand, K. J. Bathe et al [10] evaluated shells and beams with a total Lagragian formulation and an updated Lagragian formulation for dynamic cases, considering non-conservative loads. He concludes that when using different time increments the solutions vary, and it is preferable to use small variations. This will be seen in the present investigation.

J.N. Reddy [11] analyzed the linear behavior of isotropic, orthotropic, and anisotropic composite plates, using a dynamic formulation (transient). He also considers the effects of shear strain and rotational inertia. Also, other authors such as H. Reismann and Y. Lee studied plates in dynamic motions. In the same way, he concludes the importance of the time scale in convergence. Likewise, K. Chandrashekhara and J. N. Reddy [12] studied doubly curved shells under a geometrically nonlinear transient analysis, using shear strains and Von Karman theory. The authors present an element not recommended for severe geometry changes. G. N. Praveen and J. N Reddy [13], were the first to present a nonlinear transient thermoelastic analysis for functionally graded plates, with two characteristic materials, they found that the gradient of the material properties plays a determining role in the response. FGM materials have influence on the inertia of the element.

More recently M. Gutierrez and J. N. Reddy [14] studied shell elements with the improved theory under a nonlinear transient analysis. Among other authors, M. Yao, and W. Zhang [15] under the same conditions studied cylindrical panels with (FSDT). V. Svalbonas [16] also considers the plasticity of the shells for his transitory analysis. C. Aksoylar, A. Ömercikoglu, Z. Mecitoglu and M. H. Omurtag [17] studied FGM plates with a blast charge showing that rectangular plates show different dynamic behavior (strain amplitudes) in the direction parallel to the long and short boundary.

For the most part, it is evident that the majority of analyzes are of plates and shells. There are few investigations that study a geometrically non-linear (transient) dynamic analysis of functionally graded beams. S. O. Waheed, M. A. Al-Shujairi and M. J. Aubad [18] studied functionally graded beams with Timoshenko's theory (FSDT) but did not consider the non-linear geometric part.

This research presents a computational model for nonlinear transient analysis for functionally graded beams. For the formulation of the model, the improved theory of beams mentioned before was used, which is detailed in the chapters below. In general, Hamilton's energy principle is presented using kinetic energy, strain energy and potential energy, this is developed with the Green Lagrange strain tensors (Von Karman or Sander Theory), the second Piola-Kirchhoff stress tensor and body-inertial forces. In addition, within the present framework for the dynamic solution (time integration) the Newmark method is presented mixed with the Newton Raphson method for the non-linear part, can be seen better [19]

II. Theorical Formulation

A. Beam Theory

The mathematical formulation of the beam theory is based on the expansion of the displacement field through a truncated Taylor series. It is also considered an expansion of the thickness coordinate. The quadratic term is included to avoid Poisson Locking [9]. We show in (1) the displacement field in which { x^i } is defined as the set of Cartesian coordinates with their orthonormal bases { $\mathbf{e_i}$ }.

$$\mathbf{v}\ (x^1, x^3, t) = \mathbf{u}(x^1, t) + x^3\boldsymbol{\phi}(x^1, t) + (x^3)^2\boldsymbol{\psi}(x^1, t) \qquad (1)$$

The coordinate of the neutral axis corresponds to x^1 and its respective displacement vector $\mathbf{u} = u_i(x^i, t)\mathbf{e_i}$. In addition, as already mentioned, x^3 it is the coordinate of the thickness. Also, the displacement vectors are divided into $\boldsymbol{\phi} = \phi_i(x^i, t)\mathbf{e_i}$ represent the bending rotation. Finally, corresponding to the last variable $\boldsymbol{\psi} = \psi_i(x^i, t)\mathbf{e_i}$ is the transversal quadratic deformation vector for stretching. It is important to note that 3D constitutive equations are required in accordance with the authors [20].

Giving the kinematic relations by means of the Green-Lagrange $\mathbf{E}$ strain tensor, the high order terms for the Von Karman nonlinearity are neglected.

$$\mathbf{E} = E_{ij}\ \mathbf{e_i} \otimes \mathbf{e_j} \qquad (2)$$

Separating the thickness to the neutral axis coordinate, we show the following equations (3), also neglecting the second coordinate x^2. Note that the terms that are multiplied by the coordinate of the direction x^3 (thickness coordinate) are separated and represented by the superscript 1, the rest by the superscript 0.

$$\begin{aligned} &\mathbf{E} = \mathbf{E}^{(0)} + x^3\mathbf{E}^{(1)} \\ &\mathbf{E}^{(i)} = E_{11}^{(i)}\ \mathbf{e}_1 \otimes \mathbf{e}_1 + E_{33}^{(i)}\ \mathbf{e}_3 \otimes \mathbf{e}_3 + E_{31}^{(i)}\ \mathbf{e}_3 \otimes \mathbf{e}_3 \end{aligned} \qquad (3)$$

Introducing the field displacement within the strain tensor and using indicial notation see the equations (4). These separated values help us for the mathematical formulation of the dynamic principle of virtual work. The terms are shown in detail below.

$$\begin{aligned} E_{11}^{(0)} &= u_{1,1} + \frac{1}{2}(u_{1,1}^2 + u_{3,1}^3) \\ E_{13}^{(0)} &= \frac{1}{2}(\phi_1 + u_{3,1} + u_{1,1}\phi_1 + u_{1,1}\phi_3) \\ E_{33}^{(0)} &= \phi_3 + \frac{1}{2}(\phi_1^2 + \phi_3^2) \\ E_{11}^{(1)} &= \phi_{1,1} + u_{1,1}\phi_{1,1} + u_{3,1}\phi_{3,1} \\ E_{13}^{(1)} &= \frac{1}{2}(\phi_{3,1} + \phi_{1,1}\phi_1 + 2u_{3,1}\psi_3 + \phi_{3,1}\phi_3) \\ E_{33}^{(1)} &= 2(\psi_3 + \phi_3\psi_3) \end{aligned} \qquad (4)$$

B. Hamilton's Principle

It is necessary to define the dynamic case for solid bodies. This principle comes from the minimization of the total potential energy. This system considers three energy functions: kinetic (*K*), strain (*U*) and potential (*V*). Furthermore, the energies can be expressed as functions of position and time see Reddy in [21].

Also, is known that the difference between potential and kinetic energy is called *Lagrangian* function. This is explained in depth in chap. XIX of the Lecture on Physics of Feynman. Defining it in a general way, *it is the movement of a body due to conservative forces in a time interval*, mathematically it is described by the line integral of the *Langragian*. For our formulation the virtual work δ stored a dynamic body $\mathscr{B}$

$$\int_0^t \delta L dt \equiv \int_0^t [\delta K - (\delta U + \delta V)]dt = 0 \qquad (5)$$

The virtual kinetic, strain and potential energies are defined by the following equations respectively. Note that ρ_0 is the density of the body, $\mathbf{b_0}$ is the body force vector and $\mathbf{t_0}$ is the traction vector, all expressed at the initial configuration.

$$\int_0^t \delta L = \int_{\mathcal{B}} (\delta\dot{\mathbf{v}} \bullet \rho_0\dot{\mathbf{v}} - \delta\mathbf{E} \bullet \mathbf{S} - \delta\mathbf{v} \bullet \rho_0\mathbf{b}_0)d\mathcal{B} - \int_{\Gamma_\sigma} \delta\mathbf{v} \bullet \mathbf{t}_0 ds \qquad (6)$$

Separate virtual energies are shown:

$$\begin{aligned} \delta K &= \int_{\mathcal{B}} \delta\dot{\mathbf{v}} \bullet \rho_0\dot{\mathbf{v}} d\mathcal{B} \\ \delta U &= \int_{\mathcal{B}} \delta\mathbf{E} \bullet \mathbf{S}\ d\mathcal{B} \\ \delta V &= -\int_{\mathcal{B}} \delta\mathbf{v} \bullet \rho_0\mathbf{b}_0 d\mathcal{B} - \int_{\Gamma_\sigma} \delta\mathbf{v} \bullet \mathbf{t}_0 ds \end{aligned} \qquad (7)$$

Also, the virtual strain energy the **S** is called second Piola-Kirchhoff Stress tensor. ($\mathbf{S} = S^{ij}\mathbf{e_i} \otimes \mathbf{e_j}$)

$$\mathbf{S}^{(i)} = \mathbb{B}^{(i)}\mathbf{E}^{(0)} + \mathbb{B}^{(i+1)}\mathbf{E}^{(1)} \tag{8}$$

$$\mathbb{B} = \int_{-h/2}^{h/2} (x^3)^k\, \mathbb{C}\, dx^3 \tag{9}$$

Introducing the displacement field into the energy equations we obtain the following. Due to not having more simplification, the virtual potential energy is not shown in detail.

$$\begin{aligned}\delta K &= \int_\Omega \int_{-1}^{1} \rho_0 \left(\delta\dot{u}_i + x^3\delta\dot{\phi}_i + (x^3)^2\delta\dot{\psi}_i\right) \\ &\left(\dot{u}_i + x^3\dot{\phi}_i + (x^3)^2\dot{\psi}_i\right) J dx^3 d\Omega \\ &= \int_\Omega [I_0(\dot{u}_i\delta\dot{u}_i) + I_1(\dot{u}_i\delta\dot{\phi}_i + \dot{\phi}_i\delta\dot{u}_i) \\ &+ I_2(\dot{\psi}_i\delta\dot{u}_i + \dot{u}_i\delta\dot{\psi}_i + \dot{\phi}_i\delta\dot{\phi}_i) \\ &+ I_3(\dot{\psi}_i\delta\dot{\phi}_i + \dot{\phi}_i\delta\dot{\psi}_i) + I_4(\dot{\psi}_i\delta\dot{\psi}_i)] d\Omega\end{aligned} \tag{10}$$

This expression comes from applying the variation of virtual work and represent the consistent mass matrix. Reddy [19], mention that the use of this expression is undesirable, due the fact the matrix can never be explicit and for less computational time is affordable apply the diagonalization of the terms. This can be solved by sum-row lumping technique, among others [22-23].

$$\delta U = \int_\Omega \int_{-1}^{1} (\delta E_{ij}^{(0)} + x^3\delta E_{ij}^{(1)}) C_{ijkl} (E_{kl}^{(0)} + x^3 E_{kl}^{(1)}) J d\mathcal{B} \tag{11}$$

Due to the fact, that the material will be functionally graded along the thickness, it is commonly used by a simple distribution rule called power-law. For this reason, the expression of inertia as a function of said distribution is presented. The subscript *(i=0,1,2,3,4)* depends on inertia requires:

$$I_i = \int_{-h/2}^{h/2} \left(\left(\rho^c - \rho^m\right) \left(\frac{x^3}{h} + \frac{1}{2}\right)^n (x^3)^i + \rho^m (x^3)^i \right) dx^3 \tag{12}$$

In the same way, the simplified components of effective extensional, extensional-bending coupling and bending fourth - order stiffness tensor are presented.

$$\left\{A_{ijkl}, B_{ijkl}, D_{ijkl}\right\} = \int_{-1}^{1} \left\{1, x^3, (x^3)^2\right\} C_{ijkl} J dx^3 \tag{13}$$

C. *Functionally Graded Materials*

Functionally graded materials are a type of compound made up of a combination of different properties and characteristics of several materials, with certain changes that disregard the consideration of an interface between these materials. This is due to mitigating this abrupt transition between one and the other, and instead continuously and smoothly transforms [5].

As usual in micromechanics, it is possible to consider a homogenization of the material through several rules, for more details [24-25]. This can be given for several effective properties, such as thermal conductivity, Yong's modulus, among others. For the present investigation, a simple mixing rule of Voigt Kelvin will be used.

Usually, you can analyze the gradation of materials with two phases of materials, which are metal and ceramic. The first contributes the ductility against stress and the second the variation of high temperatures.

$$\breve{w}(x^3) = \breve{w}_c f_c + \breve{w}_c f_m \tag{14}$$

The effective properties are defined by $\breve{w}$. In addition, the subscripts *c* and *m* correspond to ceramic and metal. In the same way, f corresponds to the filling fraction of each phase.

$$f_c = \left(\frac{x^3}{h} + \frac{1}{2}\right)^n \qquad f_m = 1 - f_c \tag{15}$$

Note that n is the power law index, this represents the variation through the thickness x^3.

C. *Newmark Scheme*

In the temporary solution it is necessary to use the Newmark scheme. To know the deformed configuration, it is necessary to use this method since it is based on recursive equations. According to Bathe [26], it is the result of traditional methods plus the contribution of the mass. Therefore, the dynamic equation of a moving body is presented, and neglecting the damping.

$$[\mathbf{M}]\left\{\ddot{\boldsymbol{\Delta}}\right\} + [\mathbf{K}\left\{\boldsymbol{\Delta}\right\}]\left\{\boldsymbol{\Delta}\right\} = \left\{\mathbf{F}\right\} \tag{16}$$

Here the solution of the displacement vector is $\left\{\boldsymbol{\Delta}\right\}$, for non-linearity we know that the stiffness matrix $[\mathbf{K}\left\{\boldsymbol{\Delta}\right\}]$ depends on the deformed configuration. The mass matrix is represented by $[\mathbf{M}]$, which is multiplied by the acceleration vector $\left\{\ddot{\boldsymbol{\Delta}}\right\}$. The system is solved by the following equation ().

$$[\hat{\mathbf{K}}\left\{\boldsymbol{\Delta}\right\}_{s+1}]\left\{\boldsymbol{\Delta}\right\}_{s+1} = \left\{\hat{\mathbf{F}}\right\} \tag{17}$$

Where,

$$[\hat{\mathbf{K}}\left\{\boldsymbol{\Delta}\right\}_{s+1}] = [\hat{\mathbf{K}}\left\{\boldsymbol{\Delta}\right\}_{s+1}] + a3[\mathbf{M}]_{s+1} \tag{18}$$

$$\left\{\hat{\mathbf{F}}\right\}_{s+1} = \left\{\hat{\mathbf{F}}\right\}_{s+1} + \left[\mathbf{M}\right]_{s+1}(a3\left\{\boldsymbol{\Delta}\right\}_s + a4\left\{\dot{\boldsymbol{\Delta}}\right\}_s + a5\left\{\ddot{\boldsymbol{\Delta}}\right\}_s) \tag{19}$$

Note that it is necessary to use the stiffness matrix plus the contribution of the mass. Therefore, for these expressions (18-19) the stiffness matrix and force vector will be assigned the hat.

$$a_1 = \alpha \Delta t,\ \ a_2 = (1-\alpha)\Delta t,\ \ a_3 = \frac{2}{\gamma(\Delta t^2)}, \qquad (20)$$
$$a_4 = a_3 \Delta t, \quad a_5 = \frac{1}{\gamma} - 1,$$

This expression is known as effective stiffness, in the same way for the force vector, both masses are multiplied by coefficients (20) that generate stability and accuracy to the method. In addition, the velocity and acceleration expressions are shown, which are updated at each time step.

$$\left\{\ddot{\mathbf{\Delta}}\right\}_{s+1} = a_3(\left\{\mathbf{\Delta}\right\}_{s+1} - \left\{\mathbf{\Delta}\right\}_s) - a_4\left\{\dot{\mathbf{\Delta}}\right\}_s - a_5\left\{\ddot{\mathbf{\Delta}}\right\}_s \qquad (21)$$
$$\left\{\dot{\mathbf{\Delta}}\right\}_{s+1} = \left\{\dot{\mathbf{\Delta}}\right\}_s + a_2\left\{\ddot{\mathbf{\Delta}}\right\}_s + a_1\left\{\ddot{\mathbf{\Delta}}\right\}_s \qquad (22)$$

Finally, for non-linear cases, the method can be improved and combined with the Newton Raphson method since first the residue is solved together with the contribution of mass, and then the velocity and acceleration updates are made. The detailed scheme can be seen in [19].

$$\left\{\delta\mathbf{\Delta}\right\} = -[\hat{\mathbf{T}}(\left\{\dot{\mathbf{\Delta}}\right\}_{s+1}^{r})]^{-1}\left\{\mathbf{R}\right\}_{s+1}^{r} \qquad (23)$$

Where,

$$[\hat{\mathbf{T}}(\left\{\dot{\Delta}\right\}_{s+1}^{r})]^{-1} \equiv \left[\frac{\partial\left\{\mathbf{R}\right\}}{\partial\left\{\mathbf{\Delta}\right\}}\right]_{s+1}^{r} \qquad (24)$$

$$\left\{\mathbf{R}\right\}_{s+1}^{r} = [\hat{\mathbf{K}}\left\{\mathbf{\Delta}\right\}_{s+1}]\left\{\mathbf{\Delta}\right\}_{s+1} - \left\{\hat{\mathbf{F}}\right\}_{s,s+1} \qquad (25)$$

The total solution is obtained by the following combination (26).

$$\left\{\mathbf{\Delta}\right\}_{s+1}^{r+1} = \left\{\mathbf{\Delta}\right\}_{s+1}^{r+1} + \left\{\delta\mathbf{\Delta}\right\} \qquad (26)$$

II. Finite Element Formulation

For the formulation, the domain of the metric axes will be called Ω as in (27) we discretize the element in equal parts E.

$$\Omega = \bigcup_{e=1}^{E} \Omega^e \qquad (27)$$

Where Ω^e is the domain of the element, which also must be taken to an integration space for the integration by Gaussian quadrature and suitable to use the Lagrange interpolation functions with their continuous first and second derivatives.

$$\zeta(\ell) = \prod_{\substack{B=1 \\ B\neq A}}^{Ne} \frac{\ell - \ell^B}{\ell^A - \ell^B} \qquad (28)$$

The interpolation of the variables is given by the equations.

$$\mathbf{u}(x^i) = \left(\sum_{j=1}^{n} u_m^j \zeta^j(\xi)\right)\mathbf{e_m}$$
$$\boldsymbol{\phi}(x^i) = \left(\sum_{j=1}^{n} \phi_m^j \zeta^j(\xi)\right)\mathbf{e_m} \qquad (29)$$
$$\boldsymbol{\psi}(x^i) = \left(\sum_{j=1}^{n} \psi_m^j \zeta^j(\xi)\right)\mathbf{e_m}$$

III. Numerical Results

A. *Previous comments*

The results of the present investigation are shown in two general parts. The first consists of a transitory analysis with a single constant material throughout the beam, with the established geometric characteristics and properties. In addition, it will be compared with the first order theory of beams. The analysis shows the results for different time increments and boundary conditions.

In the second part the same results will be shown but including changes in the properties of the material turning it into a functionally graded material based on ceramic and metal. The establishment of the precision and stability by the Newmark method will be the same for all the cases presented, they are represented by two constants which are: $\gamma = 0.5$, $\alpha = 0.5$. High order interpolation polynomials (P=4) are used in the same way.

B. *Transient Analysis of Beam*

The beam has a length of 10 in, the lengths of its cross-sectional area are unitary, which are: width 1 (in) and height 1 (in). The modulus of elasticity is E=1.2 (psi), as well as a Poisson's ratio of $\nu = 0.2$. The density of the material is $\rho = 10^{-6}\ lb.s^2/in^4$. It will be subjected to a distributed transverse force of $q_0 = 2.85\ lb/in$.

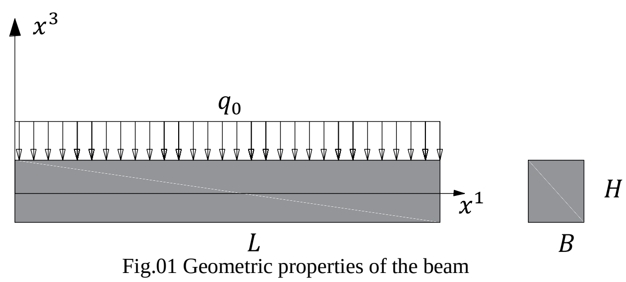

Fig.01 Geometric properties of the beam

The results shown in Fig.2 are compared with the first order shear theory (TBT-linear), which is detailed in ref. [19]. On the other hand, the agreement with the problem proposed by K. N. Bathe [10],[12] using the same properties and load cases was verified. Linear and non-linear result is shown.

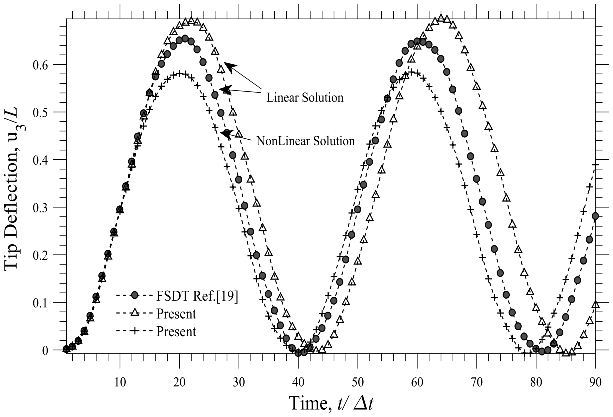

Fig. 2. Transient Analysis of an isotropic cantilever beam

In addition, the same case study is shown but with the variation of time increments. For all our cases we use the case: $\Delta_2: 1.35 * 10^{-4}$.

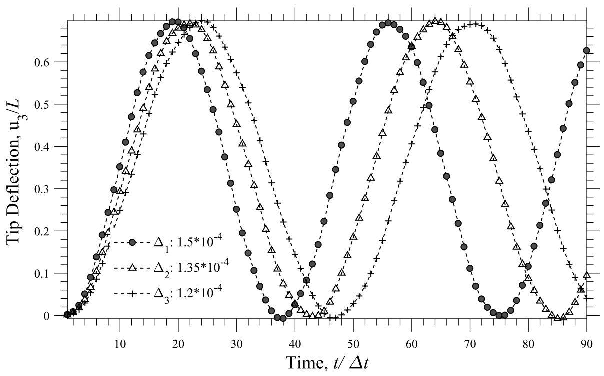

Fig. 3 Different Time Increments of C-F (Linear Response)

The next graphic shows the nondimensional rotations in time, between two theories represented in the legend for a FGM cantilever beam (n=2).

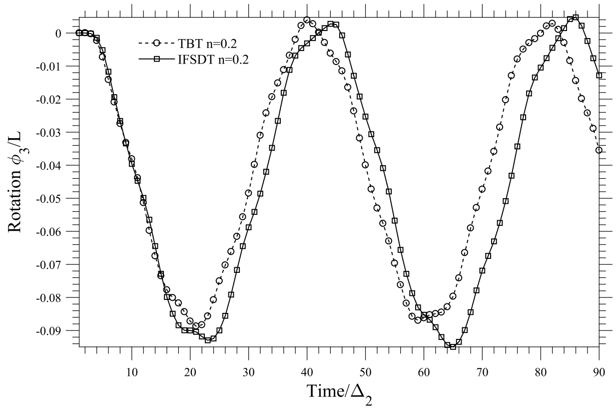

Fig. 4 Rotation $\varphi_3$of FGM Beam (Linear Response)

Next, Fig 5 represents the deformed configuration for each load step, in which it is noted that for step 5 the configuration stops deforming and returns to its original position due to the vibration response phenomenon.

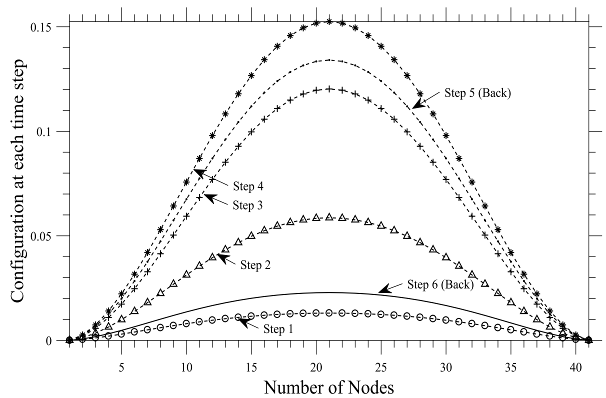

Fig. 5 Configuration at time step of C-C Beam (Linear Response)

C. *Transient Analysis of Funcionally Graded Beams*

The geometric properties of the beam are marked below (see Fig.6). Instead, the material properties are varied along the thickness by two materials, these are ceramic and metal. At the top it becomes totally ceramic and at the bottom metal equally. $H = 0.1$, $B = 1$.

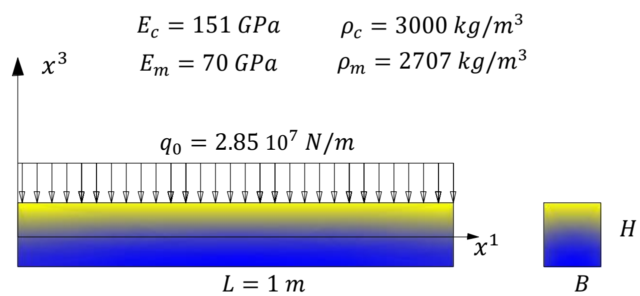

Fig. 6 Geometric properties of the FGM beam

The following image presents the results of the FGM beam for various power law indices. These were formulated with IFSDT.

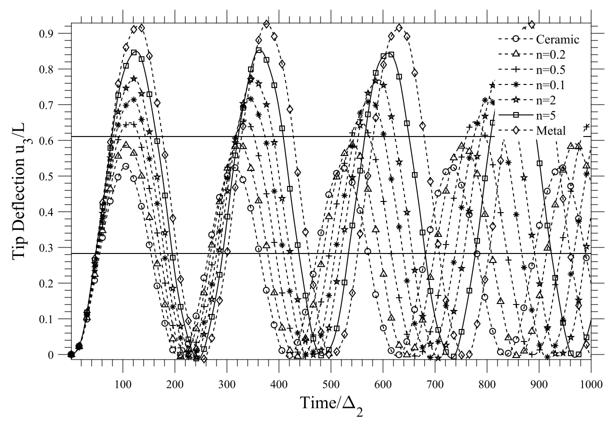

Fig.7 Linear Tip Deflection u_3 of Transient Analysis FGM Beam

In addition, Fig.7 shows two completely straight lines, which correspond to the linear solutions for the deflection of a completely ceramic beam (bottom line) and for a completely metal beam (top line). Which is consistent with the transient results.

On the other hand, the non-linear results for the same load case are represented in the following graph.

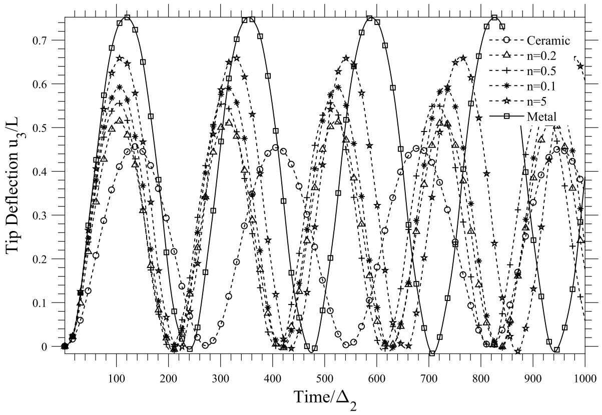

Fig.8 Nonlinear Tip Deflection u_3 of Transient Analysis FGM Beam

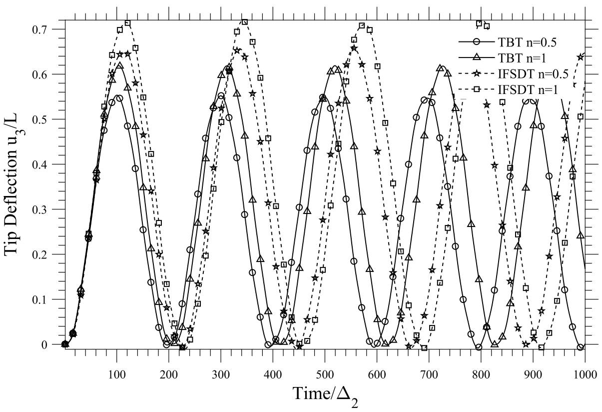

Fig.9 Nonlinear Tip Deflection u_3 of Transient Analysis FGM Beam (TBT vs IFSDT)

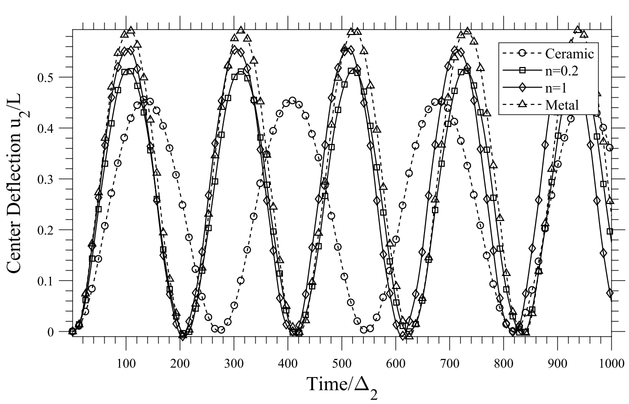

Fig.10 Nonlinear Center Deflection u_3 of Transient Analysis FGM Beam (IFSDT)

The figures 7, 8, 9 and 10 show in detail the variation between the FGM beams due to the behavior of the constitutive properties.

IV. CONCLUSIONS

In the present investigation, the geometric nonlinear behavior of functionally graded beams is examined. It is possible to carry out the computational implementation with a high order of convergence. The formulation consists of 5 fundamental and independent variables. In addition, the dynamic formulation of Hamilton's principle is carried out, obtaining the mass matrix. Using the power law for the graded change of the material which also affects the inertia of the material and the mass matrix equally. The Newmark method is used and precisely achieved the convergence of the solution. The reader is encouraged to develop further research on this topic, implementing new theories such as nonlocality, damage mechanics and microbeams.

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