Modeling the Impact of Covid-19 on the Prices of Commodities with a Damped Harmonic Oscillator

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Abstract- Covid-19 pandemic has affected nearly all aspects of international exchanges and the magnitude of the loss is still being investigated. Using a damped harmonic oscillator under forced excitation model, this paper examines the impact of the Covid-19 on the price of commodities such as a cobalt, copper, lithium, and gold. The study shows higher losses with cobalt and higher gains with gold during the period considered and could help decision-makers in their predictions.

Keywords: Vibrations, Damping, Harmonic Oscillator, Commodities, Covid-19

I. Introduction

Engineering models have been used in the literature to study economic markets and social behavior. As an illustration, Lesser [1] studied the neo-consumer theory of economic demand using the second principle of Thermodynamics and concluded that any reasonable consumer left alone without any interference will make decision leading to more happiness. The second principle of Thermodynamics [2] teaches that any system left on its own, without interference, will evolve towards less chaos and more order, or in other words, less entropy.

Sandoval [3] used damped harmonic oscillator models to study shock in international financial markets and was able to compare the financial market crashes including subprime mortgages and others, and comparing the losses by using different world indexes, including the NYSE, S&P 500, NIKEI and the DAX to name a few.

This research applies the damped harmonic oscillator model to the study of four key commodities markets in the wake of the Covid-19 crisis. The commodities selected are gold, cobalt, lithium, and copper. The last three commodities being the leading mining products in the electric vehicles manufacturing. The Covid-19 is modeled as a shock to the market and the price evolution is examined, considering the model. The key parameters of these, their interpretation based on the analogy to the engineering model are analyzed and discussed.

The study sheds light on the total loss corresponding to each commodity and the overall response of the different markets to the Covid-19 shock. The equivalent mass of the market, the elasticity or the stiffness, and the damping coefficient are the engineering dynamic-equivalent that help explain the market behavior and guide decision makers in future crises.

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II. Modeling

A. Damped Harmonic oscillator

The model used in the study is the mass spring damped harmonic oscillator (Fig. 1) discussed in [4] and in Dynamics and Vibrations textbooks [5].

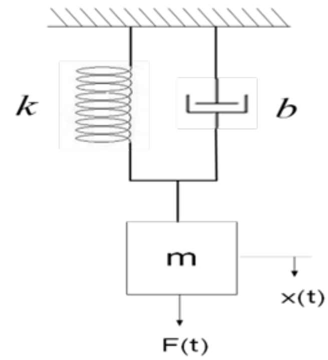

Fig.1 Loading of a sample

Under an external force F(t), the equation of motion from Newton's second law can be expressed as

$$m\ddot{x} + b\dot{x} + kx = F\left(t\right) \quad (1)$$

With $x(t)$ the vertical displacement from the equilibrium position of the system, m the system mass, b the damping coefficient, k the stiffness and $F(t)$ the applied force to the system.

If $F(t)$ is decomposed in a fixed part F_{eq} and a variable component $F_o e^{-at}$, equation (1) can be rewritten as

$$m\ddot{x} + b\dot{x} + kx = F_{eq} + F_0 e^{-at} \quad (2)$$

Substituting $x(t)$ with $P(t)$, the price of the commodity, we obtain

$$m\ddot{P} + b\dot{P} + kP = P_{eq} + P_0 e^{-at} \quad (3)$$

Equation (3) is the same as proposed by Leonidas (2011) [3] in his study of crashes of financial markets. $P_o e^{-at}$ is the external shock, assimilated to the Covid-19 impact in this paper. The decaying exponential assumes a stronger impact at the beginning and becoming weaker with time, as the world begins to understand the pandemic situation and attempts to bring an appropriate response.

The solution of equation (3) comes in two parts: a homogeneous solution assuming the right-hand side of (3) is zero and a particular solution considering the right end side, which accounts for the contribution of the non-zero forcing function. The total solution is of the form:

$$P(t) = P_{eq} + Be^{-at} + Ce^{-bt}\cos(\omega t - \phi) \quad (4)$$

With $P(t)$ the commodity price, P_{eq} the price at equilibrium after the shock Be^{-at}, the component of the price due to the shock and $Ce^{-bt}\cos(\omega t-\phi)$, the oscillation of the price around the price at equilibrium. The characteristics of the decaying oscillation are the amplitude C, the frequency ω and the phase ϕ, indicating the delay between the beginning of the shock and the effect seen in the brutal change in price.

To account for the inside interactions of elements of the systems, the last part of the solution (4) can be decomposed in more frequencies and phases, yielding a solution of the form:

$$\begin{aligned} P(t) &= P_{eq} + Be^{-at} + C_1 e^{-b_1 t}\cos(\omega_1 t - \phi_1) + \ldots \\ &C_2 e^{-b_2 t}\cos(\omega_2 t - \phi_2) \end{aligned} \quad (5)$$

The additional frequencies observed in equation (5) are akin to the phenomenon of aftershocks observed in earthquakes and studied in seismic vibrations. [6]

To solve equations (4) or (5), a best fit is found with the commodity prices recorded during the Covid-19 period as seen in Figure 2. [7]

Fig. 2 Trends of cobalt prices 2010-2020 **[7]**

The fitting with equation (4) will lead to a system of equations and several unknowns, depending on the number of data points used. The unknowns are P_{eq}, B, a, C, b, ω and ϕ. The best fit will account for minimum possible error between the provided data set and the price function obtained from equation (4) as shown in the equation below:

$$err = \sqrt{\sum_{i=1}^{n}\left[P(i) - P_i\right]^2} \quad (6)$$

With err, the error computed between the data P_i and the model response $P(i)$, n the number of data points.

B. Analogy of Damped Harmonic Oscillator with Commodity Market Model

To gain an understanding of the market's response using the oscillator model, an analogy is made between the model and the market. The mass m in dynamics represents the inertia or the ability of systems to stay at rest despite the external forces being applied to the system. In the market, the equivalent could be either the total production or the total amount of currency in the market.

The stiffness k in engineering represents the inability of an element of a structure to return to its initial configuration after the disturbance has been eliminated. An opposite to this concept is known as elasticity of the system. An elastic structure or an elastic element of a structure is one that returns to its initial posture after the removal of the acting outside forces. In comparison, the ability of the market to return to a new normal, not necessarily the old configuration, would be described by its elasticity.

The damping coefficient b, in engineering, measures the ability of systems to oppose motion. It is also a measure of the loss of energy of a system caused by relative motion of internal particles. A heavily damped system will not move while a lightly damped will oppose the motion and cause oscillations of the output being studied about an asymptotic or equilibrium position. The damping in the market situation will measure its response or opposition to the outside shock caused by a sudden force, in this case the Covid-19 pandemic. Hence, the emphasis on this parameter in the discussion at the end of this paper.

III. Results

The parameters of the model described above have been fit with the data from the evolution of the prices of commodities from 2018 and 2020 [7]. This bracket has been chosen as it coincides with the period of Covid-19 although the authors acknowledge the difficulty of pin-pointing the exact dates of the beginning and the end of the Covid-19 pandemic.

The octave software [8] has been used in the fitting process and the following correspondence of eleven parameters has been established in Table 1 for computation needs:

TABLE 1: CORRESPONDENCE BETWEEN THE MODEL AND THE FITTING PARAMETERS

p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11
P_{eq}	B	a	C_1	b_1	ω_1	ϕ_1	C_2	b_2	ω_2	ϕ_2

The results discussed include the fitting of the data of the price of commodities and the model equations (4) and (5). The maximum errors are also computed. Figures 3 to 6 show the evolution of prices of cobalt, copper, gold, and lithium, respectively.

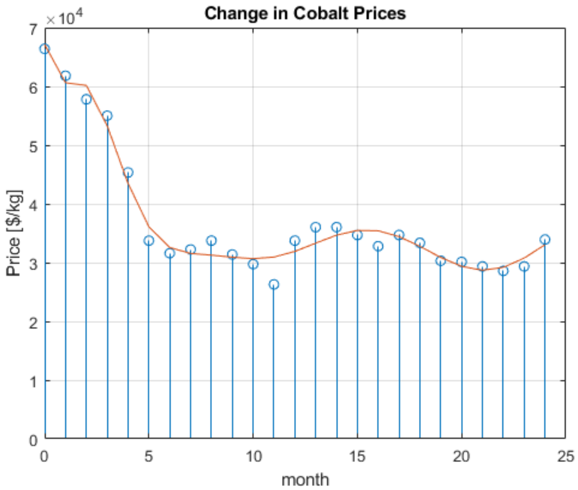

Fig. 3 Fitting of cobalt prices between 2018-2020

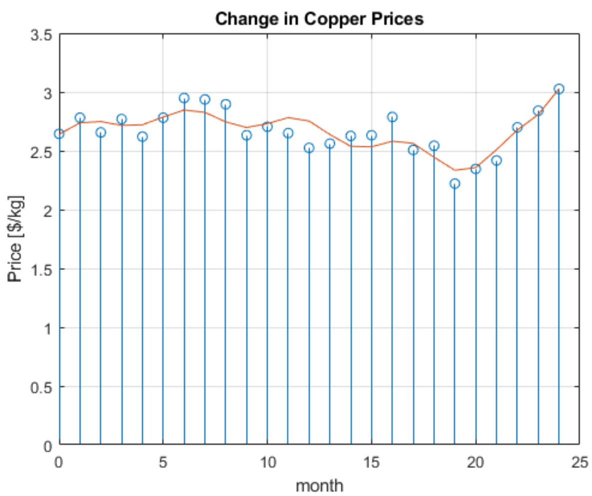

Fig. 4 Fitting of copper prices between 2018-2020

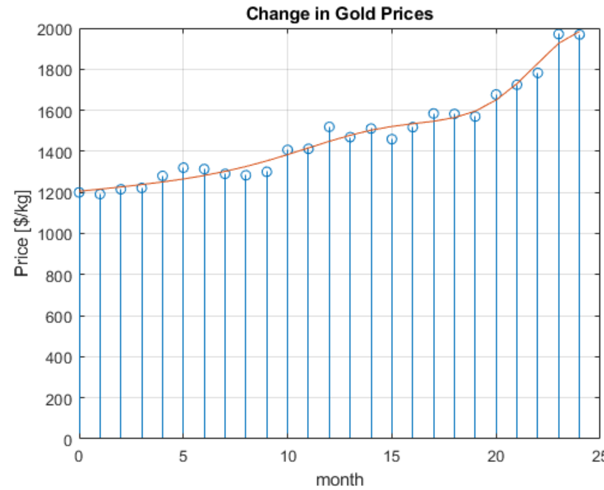

Fig. 5 Fitting of gold prices between 2018-2020

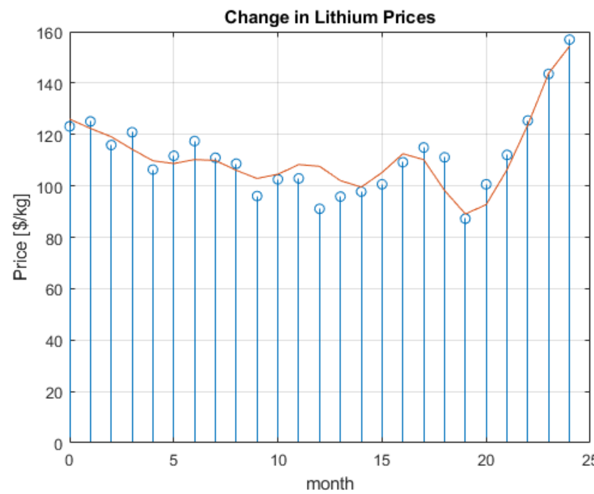

Fig. 6. Fitting of lithium prices between 2018-2020

Figures 7 and 8 indicate the errors in the models of copper and gold, respectively. Table 2 below is a summary of key fit parameters of the model. As an illustration, the parameter p5 indicates the value of the damping coefficient of the market. The parameter p2 or P_0, on the other hand, shows the intensity or the magnitude of the external force (Covid-19) P_0e^{-at} as felt by the commodity market. The lesser this force, the better the behavior of the studied commodity during the crisis.

Fig. 7 Errors in the copper model

Fig. 8 Errors in the gold model

TABLE 2
SUMMARY OF KEY PARAMETERS OF THE PRICE MODEL

Parameter	Cobalt	Copper	Gold	Lithium
p1	32138.7932	2.667500	1123.40700	104.7950
p2	11537.5407	-0.000392	82.75910	22.5178
p3	-30.0702	-3.232100	0.94215	-2.9640
p4	51941.5394	0.168270	1.71570	0.0252
p5	-2.6506	0.075032	2.15830	3.2661
p6	4.2496	2.884600	4.78900	8.3604
p7	-1.0168	4.498500	0.93858	-0.5744
p8	4218.3196	4218.31960 0	4.31300	2.5069
p9	-0.3963	-0.396250	2.17100	0.4136
p10	-2.6506	11.821500	1.07140	-13.1904
p11	4.5971	0.122410	0.87021	-2.2407

IV. Discussion

A review of the values of parameters in Table 2 and the trends of figures 3 to 6 allows the following observations. Cobalt had lost the most value by 55% from the beginning to the end of the pandemic or the two-year period span of the study. Copper is shown to have endure more stability with a fluctuation of 12%. Lithium shows a downward trend of 25% for the first 23 months and an increasing trend of 66% during the last months of the period in consideration. A rather interesting behavior is noted with the increasing trend of the price of gold during the entire period examined. God with a negative damping value of b=2.158 (parameter p5) has seen its value increase by more than 60%, confirming the refuge nature of this commodity. Errors were found be less than 10% for copper and less than 5% for gold.

V. Conclusion

A simple engineering model consisting of a damped harmonic oscillator, under forced vibration, has been used to study the response of commodities market due to an external force. The external force modeled the Covid-19 pandemic, and the response of the system was fitted to the data representing the price of these commodities during the two-year review period. The authors have acknowledged the difficulty to pinpoint the exact date of start and the end date of the Covid-19 period. The commodities examined included cobalt, copper, lithium, and gold. The latter has always been used as refuge in the financial transactions and as reserve currency by international financial institutions.

The other selected commodities besides gold were selected for their role in the industrial revolution of electrical vehicles in the fight against climate change. Highest losses have been recorded with cobalt and gold has seen its value increase during the observed period. The simple harmonic oscillator model has yielded interesting insights that could help decision-making authorities in their predictions of future similar crises.

VI. References

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