Persistence detection for the application of a complex algorithm in volatility prediction

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***Abstract*—The research studies the dynamics of the volatility of the nominal exchange rate of the Peruvian nuevo sol against the US dollar, with the aim of identifying signs of dependencies over long time distances. The existence of persistence in the increases in yields and the volatility process is explored by applying two tests based on periodograms, the Hurst exponent is estimated, its consistency over time and the graphical analysis of its distributions for the subsequent application of the algorithm of complex dynamics. For this objective, the time series of the nominal exchange rate of the nuevo sol against the dollar from January 3, 2013 to February 11, 2021 is used. A non-geometric decay was identified in the correlograms in its non-linear transformation and estimates of the fractional integration parameter $d > 0$ together with the empirical and rescaled Hurst estimates $H > 0.5$, therefore it is concluded that there is sufficient evidence of a non-random fractal pattern conducive to the use of the complex algorithm in process.**

***Keywords*—Exchange rate, Fractal, Hurst exponent, Persistence, Volatility.**

I. Introduction

The exchange rate is one of the most important macroeconomic variables, it is a measure relationship between two currencies, for this reason it is neuralgic in the establishment of relationships trade between countries with different monetary systems and takes a regulatory role in the economic cycles Therefore, the volatility and unpredictability of the exchange rate has a high potential to cause damage at the macroeconomic and microeconomic levels, deteriorating exports total and damaging external competitiveness, consequently volatility is considered as a means of contagion from external crisis to the internal economy. Currently hybrid models such as in the investigations of Xinyu Song (2021)[12], which expand the GARCH-Ito model of Kim and Wang (2016), applied in financial data based on dissemination processes which incorporates the GARCH structure. On the other hand, A. Ratnasari1, Sugiyanto and W Sulandari, (2021)[1], use the GARCH and Markov switching volatility models to explain and analyze volatilities and the changing conditions of the real exchange rate of Indonesia, Thailand, South Korea, with aim to detect signs of financial crisis, while investigations such as Xiaofei Wu, Shuzhen Zhu, and Junjie Zhou. (2020)[11], which show that the MSGARCH models they have a greater precision in the prognosis compared to the classic GARCH regimen Unique in terms of value at risk (VAR) in the case of Renminbi (RMB).

It is defined that a series has long-term dependence according to Mcleod and Hippel (1978) [10], when observing and analyzing the theoretical tendency of autocorrelation of the series under study. Currently, different tests are used to identify whether a time series has a long memory. In this study, the correlogram analysis, the rescaled range of Hurst (1951) [5], the tests of the local Whittle estimator proposed by Robinson (1995) [8] and Geweke and Porter-Hudak (1983) [3] to find evidence of black noise in its volatility and thus in future research to be able to capture the durability of the persistence of some shock through some extension of the GARCH model such as the FIGARCH model proposed by Baillie, Bollerslev and Mikkelsen (1996) [2].

The research is divided as follows: It begins with the introduction, a theoretical review is carried out to define series with long memory derived from persistent processes, its study through fractional calculation, its relationship with the fractal dimension and with the different types of noise , then the statistical-graphic analysis of the exchange rate dynamics is presented, the application of the different tests to detect the type of noise, such as those of the correlogram, rescaled, corrected and empirical Hurst rank, its stability and tests based on the periodgrams of the local Whittle estimator proposed by Robinson (1995) and that of Geweke and Porter-Hudak (1983).

II. Methodology

The first objective is to detect persistence in the latent process that generates the volatility of the nominal exchange rate series in its increments, thus evidencing a process with dependent increments. Therefore, a first nonlinear transformation is performed as a first approximation to the volatility process. Then it is decided to apply semiparametric methods based on periodograms such as the Whittle local estimator test(1995) and the Geweke - Porter and Hudak (1983) or GPH test to then estimate the Hurst coefficient and analyze its stability over time. In this way, with sufficient evidence, make the decision to implement in the second order moment of the Adam optimization algorithm a parameter that captures the complex noise (Koop) called an algorithm of a complex nature in process. As shown in figure 1.

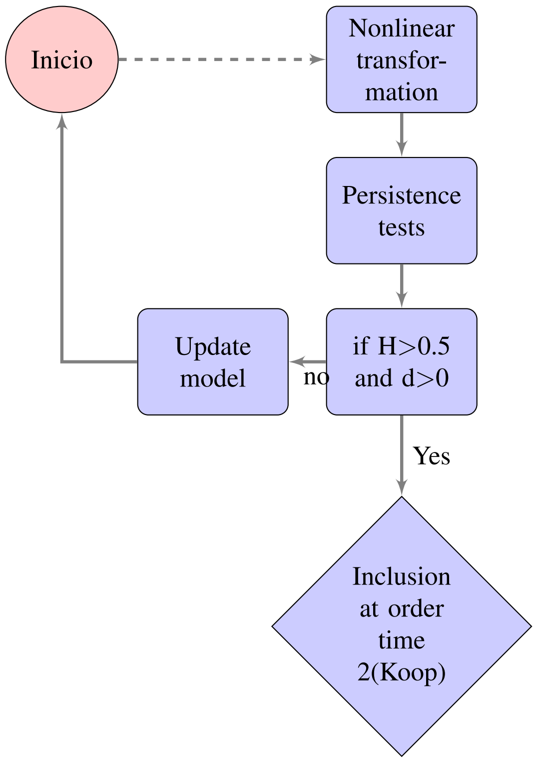

Fig. 1. Flowchart of the persistence detection process and inclusion of second order moments in the algorithm

A. Hurst exponent

For the exploration, the Hurst exponent or dependency index was used, initially investigated by the British hydrologist Harold Edwin Hurst (1880-1978) with the aim of identifying long-memory processes or long-distance dependencies. Then the Polish mathematician Benoit B. Mandelbrot (1924-2010) generalized the indicator and called it rescaled range (R/S) used to measure the relative mean reversion tendency of a time series to persist in one direction.

Be $Y_1, Y_2, Y_3, Y_4, ..., Y_k$ observations subjects of analysis ; where

- $X_t = \sum_{m=1}^{t} Y_m$
- $t + n \leq k$
- $R(t,n) = \underset{0\leq i\leq n}{Max}\{X_{t+i} - X_t - \frac{i}{n}(X_{t+i} - X_t)\} - \underset{0\leq i\leq n}{Min}\{X_{t+i} - X_t - \frac{i}{n}(X_{t+i} - X_t)\}$
- $S^2(t,n) = \frac{1}{n}\sum_{m=t+1}^{t+n}(Y_m - \hat{Y}_{t,n})^2$
- $\hat{Y}_{t,n} = \frac{1}{n}\sum_{m=t+1}^{t+n}(Y_m)$

The Hurst exponent is defined in terms of the rescaled range of the form $\lim_{n\to\infty}\mathbb{E}(\frac{R(t,n)}{S(t,n)}) = Cn^H$, where:

- $R(n)$ is the range of the first n deviations
- $S(n)$ is the sum of the first n deviations
- $\mathbb{E}(x)$ is the mathematical expectation
- n is the temporality
- C is the constant.

B. Persistence

According to Hosking (1981) [4] it proves that a process $\{y_t\}_{t\in\mathbb{N}}$ stationary long memory fulfills

$$\rho_k \approx C_\rho k^{2d-1}, k \longrightarrow \infty \tag{1}$$

Where ρ_k is the autocorrelation function of y_t. The sum of the autocorrelation tends to fall hyperbolically to zero, C_ρ is a positive constant and "d" is a fractional differentiation parameter, therefore if

$$d \begin{cases} \in (-0,5;0) & Process\ stationary\ antipersistente \\ = 0 & Movimiento\ Brownian\ standard \\ \in (0;0.5) & Process\ stationary\ persistent \\ \in (0.5;1) & Process\ not\ stationary\ persistent \end{cases}$$

$\sum_{k=-\infty}^{+\infty}|\rho_k|$ does not converge. That is to say $\{y_t\}_{t\in\mathbb{N}}$, is long memory if:

$$\sum_{k=-\infty}^{+\infty}|\rho_k| = \infty \tag{2}$$

Where ρ_k is not absolutely summable. If fractional calculus is used for the analysis of this phenomenon, the stochastic process constructed for this theoretical body is defined as the fractional Brownian motion introduced by Andrei Kolmogorov[6].

It defines $B_H = (B_H(t), t \geq 0)$ a Gaussian centered stochastic process with continuous trajectories is a fractional Brownian motion of parameter $H \in (0,1)$ and with autocovariance function

$$C_{B_H}(s,t) = \frac{1}{2}(s^{2H} + t^{2H} - |t-s|^{2H}) \tag{3}$$

Where H it is called the Hurst parameter. So if $H \neq \frac{1}{2}$, it does not comply with the Markov property, signaling a dependency in the process of its dynamics.

In the case of $H > \frac{1}{2}$ points out that the increments of the fractional Brownian motion tend in the same direction, generating a persistent process. The mathematical definition of a stationary sequence exhibiting a dependence over long distances is presented below.

A stationary sequence of random variables $\{X_n\}_{n\in\mathbb{N}}$ has long-term dependence, if the sequence of covariances $\rho(n) = Cov(X_k, X_{k+n})$ con $n, k \in \mathbb{N}$ satisfies

$$\lim_{n\to\infty} \frac{\rho(n)}{cn^{-\alpha}} = 1 \tag{4}$$

For some constant c and $\alpha \in (0,1)$.

Then the non-geometric convergence of the function $\rho(n)$ towards zero is interpreted. There is a fractional Brownian motion whose increments show long-term dependence with Hurst parameter the satisfies the following conditions.

Si B_H a fractional Brownian motion and let the succession of increments of $\{X = B_k^H - B_{k-1}^H\}$ with $k \in \mathbb{N}$ has long-term dependency if $H > \frac{1}{2}$

Thus

$$\rho_H(n) = Cov(B_k^H - B_{k-1}^H, B_{k+n}^H - B_{k+n-1}^H)$$

$$= \frac{1}{2}((n+1)^{2H} + (n-1)^{2H} - 2n^{2H})$$

$$= \frac{1}{2}(n^{2H}(1+\frac{1}{n})^{2H} + n^{2H}(1-\frac{1}{n})^{2H} - 2n^{2H})$$

$$= \frac{n^{2H-2}}{2}(n^2(1+\frac{1}{n})^{2H} + n^{2H}(1-\frac{1}{n})^{2H} - 2n^2)$$

$$= \frac{n^{2H-2}}{2}(\frac{(1+\frac{1}{n})^{2H} + (1-\frac{1}{n})^{2H} - 2}{\frac{1}{n^2}})$$

L'Hôpital rule applies and $n \longrightarrow \infty$ so

$$\frac{(1+\frac{1}{n})^{2H} + (1-\frac{1}{n})^{2H} - 2}{\frac{1}{n^2}} \longrightarrow 2H(2H-1)$$

It concludes

$$\rho_H(n) \approx n^{2H-2}H(2H-1) \longrightarrow 0$$

$\forall H \in (0,1)$

Therefore the fractional Brownian motion is a long memory process and

$$\lim_{n\to\infty} \frac{\rho_H(n)}{n^{2H-2}H(2H-1)} = 1 \tag{5}$$

Be $c = (2H-1)H$ and $-\alpha = 2H-2$ in equation 4, so when $H \in (\frac{1}{2}, 1)$ fractional Brownian motion has long-term dependence so it is a persistent process. As shown in Table I. Analyzing their covariances, it is indicated that $n^{2H-2}H(2H-1)$ it is positive, therefore the trajectories of the process tend to the same direction.

Mandelbrot[9], defines a fractal as a set with the peculiarity that its Hausdorff dimension is greater than its topological dimension. In other words, he defined the fractal dimension as a non-integer number, thus facilitating the description of the fractal geometry. The relationship between the fractal dimension and the hurst exponent [5] is denoted through the following equation developed by Voss[7]:

$$2H + 1 = 5 - 2D \tag{6}$$

From equation 6 we obtain a relationship between the fractal dimension (D) and the Hurst exponent (H), then

$$D = 2 - H \tag{7}$$

Where D is the fractal dimension and H is the Hurst exponent. Table I presents a brief outline of the concepts discussed.

TABLE I
SURFACE ROUGHNESS

Parameter	Fract. Diff.	Characteristic	Noise
$H \in (\frac{1}{2}, 1)$	$d > 0$	Decreased fractal dimension	Black noise
$H \in (0, \frac{1}{2})$	$d < 0$	Increased fractal dimension	Pink noise
$H = \frac{1}{2}$	d=0	Randomness	White noise

III. RESULTS

The input database consists of 2116 observations on the nominal exchange rate nuevo sol with respect to the US dollar on a daily basis, ranging from January 3, 2013 to February 11, 2021 obtained from the central bank database. Reserve of Peru. The daily yields were expressed as follows:

$$r_t = Ln(B_t/B_{t-1}), \tag{8}$$

where B_t is the daily exchange rate in the period t.

Table II shows the main statistics of exchange rate returns (r) for the period under study. The average return is 0.00016826 %, a standard deviation of 0.0027. On the other hand, a negative leptocutic and asymmetric distribution of their yields is observed, which generates that the normality test of Jarque Bera (1987) rejects the null hypothesis of normality.

TABLE II
MAIN STATISTICS OF EXCHANGE RATE PERFORMANCE
3/01/2013-11/02/2021

Statistics	r
Mean	0.000168
Median	0.000293
Variance	0.000007
Deviation	0.002715
Asimetria	-0.610124
Ex.Curtosis	5.40072
Jarque Bera	2682.81
Prob.	(0.0000)

Figure 2, shows the width of the box compressed by the presence of a large volume of outliers with a slight bias to the left. Figure 3, confirms the absence of normality in its distribution, showing a lack of adjustment of the empirical observations in relation to the theoretical distribution.

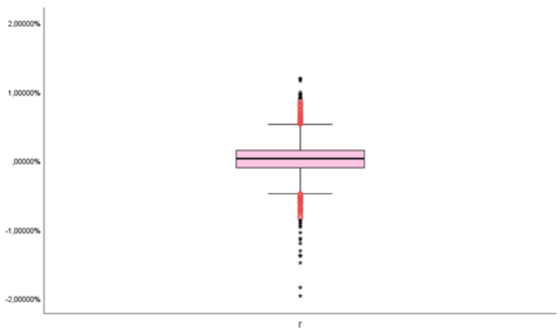

Fig. 2. Box plot of non- linear trasformation

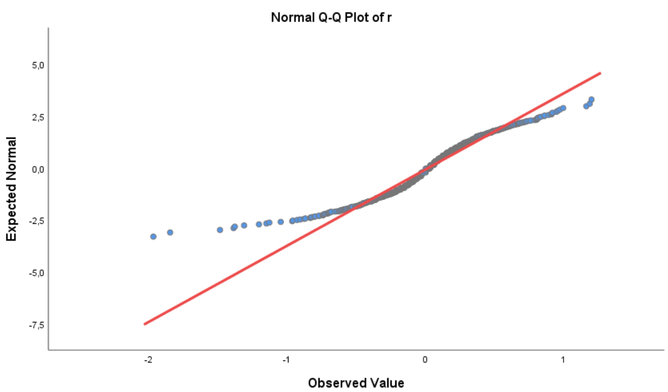

Fig. 3. QQplot. of nonlinear transformations

In Figure 4, the time series of the exchange rate (TC1), its returns (r) and its non-linear transformations (r2 and abr) are shown as a proxy for volatility, it is observed that the price series induces a non-linear behavior. linear. But it would appear that there is stationarity in the returns at least on average. On the other hand, to analyze the existence of non-linear time dependence in yields, transformations are observed, suggesting a sensitivity of volatility in time and market, volatility clusters or existence of volatility by groupings are also observed. If we observe the yields, its dynamics indicate an unconditional leptocurtic distribution, that is, an excessive concentration with respect to the mean and a large number of outliers, confirming that the yields do not follow a normal distribution.

To find evidence of black noise in the volatility process, sample correlograms are used for the first 100 lags of the series

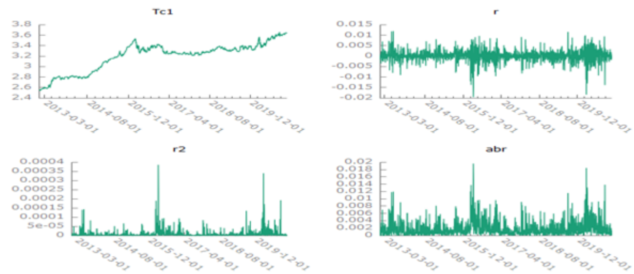

Fig. 4. Exchange rate daily series and its transformations from 3/01/2013 to 11/02/2021

of quadratic exchange rate returns (r2), as shown in Figure 5, it is evidenced a slow decay, consistent with a persistence behavior in the process of its transformation, which is why a decrease in the fractal dimension is suggested, that is, a smoother surface.

In Figure 6, the dynamics of the series of the absolute values of the yields (abr) is similar to the dynamics of the quadratic transformation (r2), a hyperbolic drop pattern is shown, which suggests evidence in favor of black noise .

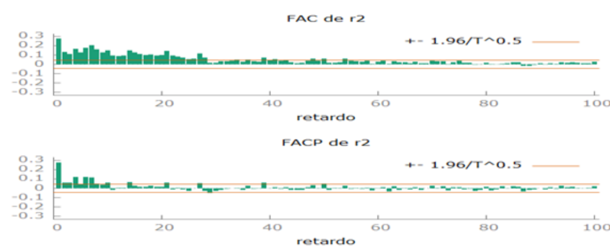

Fig. 5. Correlograms of quadratic Returns

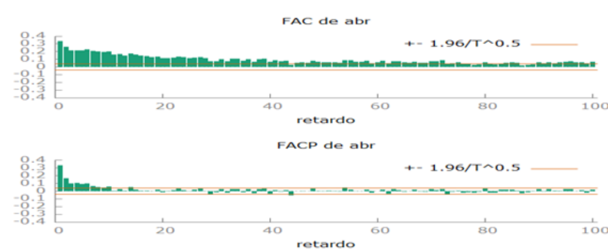

Fig. 6. Correlograms of absolute value of Returns

A. *Hurst exponent*

Table III presents the estimates of the Hurst exponent of the returns (r), the quadratic returns of the exchange rate (r2) and the absolute value of their returns (abr) using the following methods, rescaled range method, corrected rescaled range method, Hurst empirical exponent, corrected empirical exponent, and Hurst theoretical exponent. The estimates shown in Table III are reliable and consistent (Arango (2001)). It is observed that the quadratic returns have rescaled estimates corrected by Hurst higher than 0.5, with a value of 0.75 and greater intensity in their absolute values (Apr) with $0,80$. Estimates of the empirical Hurts exponent greater than 0.5 are observed in the nonlinear transformations, with 0.88 and 0.82 in the absolute value and quadratic returns, respectively. Therefore, there is evidence of black noise in the exchange rate volatility process.

Figure 7 shows the changes in the Hurst value over time of the yields and their non-linear transformations of the exchange rate. Consistent estimates of $H > 0.5$ are observed in its transformations, generating evidence that supports the conclusions of the existence of the José effect in the performance and volatility processes.

TABLE III
CALCULATION OF THE HURTS COEFFICIENT OF THE TRANSFORMATIONS

Method	r	Abr	r2
Hurts R/S-Simple	0.5834	0.6705	0.6334
Corrected R over S Hurst exponent	0.5899	0.8004	0.7511
Empirical Hurst exponent	0.5158	0.8804	0.8257
Corrected empirical Hurst exponent	0.4846	0.8491	0.7919
Theoretical Hurst exponent	0.5345	0.5345	0.5345

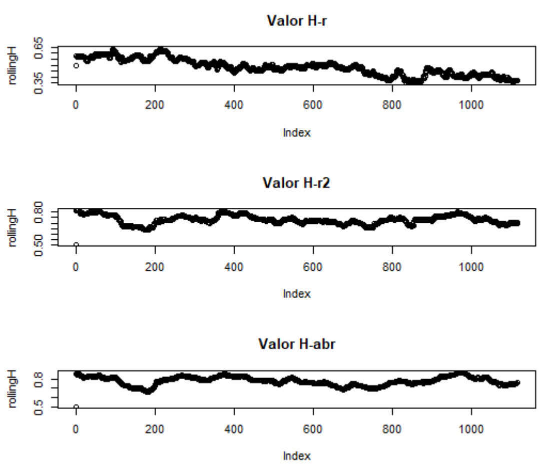

Fig. 7. Hurts stability in time of nonlinear transformations.

B. *Semi-parametric tests*

In Table IV, the Geweke and Porter-Hudak Estimator test [3] is shown through a scenario analysis for certain values of α, a bandwidth $m = T^{\alpha}$ was used , where $T = 2116$. The persistence presented by the quadratic performance is consistent for all values of α with a $d > 0$ and statistically significant $(p < 0.05)$, the presence of long memory is more intense in the absolute values of the returns (abr). The presence of sufficient evidence of a non-random fractal pattern in the dynamics of the volatility process is concluded.

Table V presents the Whittle Local Estimator test proposed by Robinson (1995) [8], following the same mechanism as the GPH test in Table IV, there is a bandwidth $m = T^{\alpha}$, where $T = 2116$. Black noise is observed in its non-linear transformations for the values of α, with $d > 0$, all estimates are statistically significant $(p < 0.05)$.

TABLE IV
GEWEKE AND PORTER-HUDAK ESTIMATOR TESTS

Variable	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.75$	$\alpha = 0.8$
r	0.15	0.16	-0.017	-0.002
r2	0.187	0.322	0.187	0.156
Abr	0.284	0.401	0.305	0.260

TABLE V
WHITTLE LOCAL ESTIMATOR TESTS

Variable	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.75$	$\alpha = 0.8$
r	0.043	-0.041	-0.022	-0.0093
r2	0.373	0.404	0.221	0.196
Abr	0.407	0.428	0.295	0.259

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CONCLUSIONS

A large number of outliers were observed confirming that exchange rate returns do not follow a Gaussian distribution and cluster volatility in their transformations. On the other hand, evidence of high sensitivity in volatility dynamics was found.

In the analysis of the correlograms, a hyperbolic decay was identified in their transformations, a black noise signal in the volatility process.

In the semiparametric tests, the null hypothesis of short memory was rejected in the volatility of the process and in the returns, with an estimate of the fractional differentiation parameter $d > 0$, all statistically significant $(p < 0.05)$

The estimates of the Hurst exponents in the quadratic returns and in absolute value generated $H > 0.5$ and its variation over time is consistent with the phenomenon of fractal dimension decrease, confirming black noise in the volatility of the process.

It is concluded that there is sufficient evidence of persistence in the volatility process of the exchange rate dynamics, therefore the objective of the application of the complex dynamics algorithm under development.

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