Dynamics of Covid-19 infection, using Delay Differential Equations

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Abstract

Due to the current Covid-19 pandemic, the scientific community has needed to generate increasingly fine and realistic models to predict the behavior of the virus over the weeks, so that local authorities can make decisions to slow the spread of the virus. The differential equations of delay will be used to capture the incubation cycles of the virus and susceptibility, adjusting the model to the reality of the city of Milagro in Ecuador . These equations are solved using the differential transformation method, a non-traditional numerical method, that takes advantage of its linear properties to find the solution through a Taylor series. Finally you will find the susceptible curve, infected and immunized over time will be found.

Keywords: Nonstandard numerical methods, covid, Delay Differential Equations.

I. Introduction

Delay Differential Equations

Models of dynamical systems are, for the most part, represented by one or more differential equations. Specifically, if we are talking about a biological system which changes with the time period t, we call the dependent variable x (depends on time t) state[18].

The theory of differential equations with challenges deals with models where the variation of the state variable x, over time depends on each instant t, not only on $x(t)$ but also on the values of x in previous instants. A Delay Differential Equation is one in which the expressure of the state, the state appears in one or several[3 instants. This is:

$$x'(t) = f(t, x(t), x(t-1), \dots, x(t-\tau_n)) \tag{1}$$

A general Delay Differential Equation has the form:

$$x'(t) = f(x(t), x(t-\tau)) \tag{2}$$

where the function f is continuous with parameter $r > 0$ called the delay of the equation.

An initial value problem containing retardes requires more information than its analogous problem without delay. The simplest case is one in which a single delay appears [1]

$$x'(t) = f(t, x(t), x(t-\tau)) \tag{3}$$

An example of a delay equation can be encontracting by studying the logistic equation which in its standard version describes the growth of a population by:

$$N'(t) = N(t)[b - aN(t)] \tag{4}$$

Assuming population density negatively affects the per capita growth rate of agreement with $\frac{dN}{Ndt} = b - aN(t)$ due to the environmental degradation. Hutchinson noted that the effects Negative What the High Densities of populated tion of the environment Rates of natality in Times Further due a delays in development and maturation [17]. This led him to propose the equation delay logistics

$$N'(t) = N(t)[b - aN(t-r)] \tag{5}$$

where $a, b, r > 0$ and r is called delay.

Differential Transformation Method

The differential transformation, like other transformed ones (Fourier, Laplace, Mellin, etc.), is an important recourse for solving problems with initial value and with value at the boundary for ordinary and partial nonlinear differential equacines, providing solutions in infinite series, many of which can be representation of functions known explicitas [25].

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The method of differential transformation has in recent years provided speed in the convergence, decreasing the time required for computing power series because they are required fewer terms to reach a margin of error acceptable. Given its rapid convergence, it is a method applicable to a wide variety of problems of physics and engineering, which require a method that allows to minimize calculation times and in some cases, find the exact solution to the problem itself from the power series that are thrown as solutions [25].

Definition

Let $y(x)$ be a function continuously differentiable around a value x_0. The transform is defined as differential tion of $y(x)$ written $Y(k)$ as: [19]

$$Y(k) = \frac{1}{k!}\left(\frac{d^k y(x)}{dx^k}\right)_{x=x_0}$$

The value of the function y(x) is obtained from the inverse differential transformation of $Y(k)$

$$y(x) = \sum_{k=0}^{\infty}(x-x_0)^k Y(k).$$

In particular, if $x_0 = 0$, you have to

$$Y(k) = \frac{1}{k!}\left(\frac{d^k y(x)}{dx^k}\right)_{x=x_0}.$$

Properties

Let be the functions $f(x)$, $g(x)$ and $h(x)$ (continuously differentiable) with difference transforms - them $F(x)$, $G(x)$ and $H(x)$ respectively. We state the following properties.

- If $f(x) = g(x) \pm h(x)$, then $F(k) = G(k) \pm H(k)$.

Proof.

$$\begin{aligned}
F(k) &= \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \\
&= \frac{1}{k!}\left(\frac{d^k (g(x)+h(x))}{dx^k}\right)_{x=x_0} \\
&= \frac{1}{k!}\left(\frac{d^k (g(x))}{dx^k}\right)_{x=x_0} + \frac{1}{k!}\left(\frac{d^k (g(x))}{dx^k}\right)_{x=x_0} \\
&= G(k) + H(k). \square
\end{aligned}$$

- if $f(x) = \alpha g(x)$ with $\alpha \in \mathbb{R}$, then

$$F(k) = \alpha G(k).$$

Proof.

$$\begin{aligned}
F(k) &= \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \\
&= \frac{1}{k!}\left(\frac{d^k (\alpha g(x))}{dx^k}\right)_{x=x_0} \\
&= \alpha\frac{1}{k!}\left(\frac{d^k (g(x))}{dx^k}\right)_{x=x_0} \\
&= \alpha G(k). \square
\end{aligned}$$

- If $f(x) = \dfrac{dy}{dx}$, then

$$F(k) = (k+1)Y(k+1).$$

Proof.

$$\begin{aligned}
F(k) &= \frac{1}{k!}\left(\frac{d^k\left(\frac{dy}{dx}\right)}{dx^k}\right)_{x=x_0} \\
&= \frac{1}{k!}\left(\frac{d^{k+1} y(x)}{dx^{k+1}}\right)_{x=x_0} \\
&= (k+1)\frac{1}{(k+1)!}\left(\frac{d^{k+1} y(x)}{dx^{k+1}}\right)_{x=x_0} \\
&= (k+1)Y(k+1). \square
\end{aligned}$$

Epidemiological Models

A particular case of host-parasite interaction is the study of the spread of a pathogen in a population, even if in its modeling there are some unique aspects. This case focuses mainly on the epidemic process within the host population. There is a great diversity of proposed models due to the immense variety of infectious agents and epidemiological patterns. Among which are the following: individuals who overcome an infectious disease, such as measles, mumps, chickenpox, are immunized. On the other hand, in diseases such as influenza the individual does not become immune. In different models it is often assumed that an infected individual is immediately contagious; however, there are diseases in which an incubation period is required before the individual can infect others. For example, in diseases such as measles, neonates are not vulnerable to infection as they are born with defences received from the mother; in contrast, in other cases such as AIDS, the baby may carry the virus acquired by the infected mother. There are other infections, such as tuberculosis in which an individual can be a carrier and transmit the virus despite having overcome this disease. [8]

In many of the diseases mentioned above, an analysis of the dynamic behavior of a system consisting of behaviors in which the host population is distributed in different categories of the infectious process can be performed. In this article two levels of complexity in order to identify fundamental features of most epidemiological processes.

The S-I-R model

This model is suitable for diseases that result in the ummunization of recovered individuals. Individuals who are born are incorporated into the category of susceptible (S); these can be infected as a result of contact with infected individuals (I), who eventually pass into the compartment of recovered (R) immune to the disease, being $S + I + R = N$ (mortality of any of the types is not considered). In the case of a closed population with no demographic changes, the equations that describe the model are:[24]

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = -\beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

The model assumes a well-mixed population, with a homogeneous and random distribution of its individuals. The transmission of the pathogen depends on the density of susceptible and infectious individuals, and on a parameter, β, which is an infection rate (number of susceptibles infected per infected individual and per unit of time). Infected individuals pass into the recovered compartment at a rate described by the parameter γ,the inverse of which, $1/\gamma$, is interpreted as the average duration of infection.[9]

$$\frac{dS}{dt} = -\beta SI.$$

It is easy to check that the condition for the expansion of the disease ($dI/dt > 0$) occurs when $\beta S/g > 1$. The growth of the infected compartment will stabilize when $dI/dt = 0$, which occurs when $\beta S/g = 1$ and finally the epidemic will be in retreat when $S/\beta g < 1$.

This allows this quotient to be considered as a reproduction rate of infection which, in analogy with the concept of a population's net reproduction rate, is referred to as R_0:

$$R_0 = \frac{\beta S}{\gamma}.$$

If the number of infected ceases to increase by $R_0 = 1$, it can be deduced that, under these circumstances, the critical population size or threshold necessary for an epidemic to occur is

$$S^* = \frac{\gamma}{\beta}$$

II. Case Study

San Francisco de Milagro, better known as Milagro, is an Ecuadorian city, located 40 km away from Guayaquil (most popular city in Ecuador). According to the latest census, Milagro has 133,508 inhabitants, being the 14th most populous city in the country.

The S-I-R model will be applied to the population living in Milagro. This city is chosen as a study objective due to its medium population mass, it is not so affected by factors related to migration that usually accompany the largest cities. Still, Milagro has enough skills to draw meaningful conclusions based on statistics provided by the local government.

Based on the data obtained from the latest bulletin "National Situation due to Covid" to date of 20/12/2020, issued by the Ministry of Public Health of Ecuador. The city of Milagro has 1093 confirmed cases of Covid-19.

Methodology
Presentation of Dynamics Model for Covid

Below is the Covid-19 Infection Dynamic Model for the population of the Milagro City

$$\begin{cases} S'(t) = -\alpha S(t)I(t-\alpha_1) + I(t-\alpha_2) \\ I'(t) = \alpha S(t)I(t-\alpha_1) - I(t) \\ R'(t) = \beta I(t) - I(t-\alpha_2) \end{cases}$$

Presentation of useful delay values

We define the following constant values considered for the delays corresponding to the model, which are measured in weeks.

$\alpha 1 =$ Period in which people become susceptible to infection again $=1$ week.

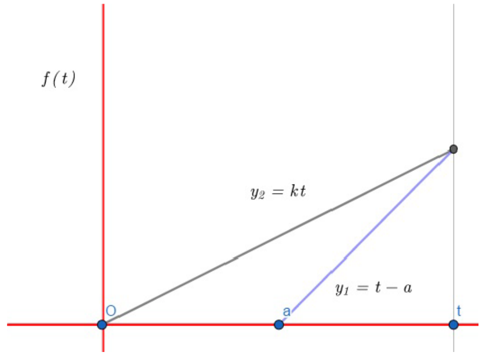
This value is set because this period includes cases of susceptibility due to a viral load much higher than that had in the initial infection, thus producing a relapse, so that they are included in the susceptibility group.

$\alpha 2 =$ Incubation period of the virus$=2.$ weeks. This value is considered by official announcements of the World Health Organization (WHO), which ensures that the incubation period of the virus goes from 1 to 14 days.

$\alpha =$ this is the percentage of infectious individuals among the number of exposed inhabitants $=0.\ 00019$

This value is obtained by applying a correction factor by the city of Milagro to the ratio between infected population and total population of Ecuador, based on data obtained from the latest bulletin "National Situation due to Covid" to date of 20/12/2020, issued by the Ministry of Public Health of Ecuador.

$\beta =$ is the percentage of recovered individuals among those who became infected at least once$=0.\ 88$ This last value was obtained from the latest bulletin "National Situation due to Covid" to date of 20/12/2020, issued by the Ministry of Public Health of Ecuador.



X-a shape model approximated to the kx shape

To solve the differential equations cited in the model Described formerly one replacement of the form x a that you have, by an approximation to a straight kx, it greatly facilitates the resolution of the same this approximation herself Has considerate linear how herself Appreciates in the Figure 1. The value of k is deducted:

$$k = \frac{t-a}{t}.$$

Apply the integral transformation

By applying the integral transformation we obtain recurring equations of one degree because they only dependen of the previous term. The values obtained are the coefficients of the solution power series of the dynamical system, thus, the precision is directly proportional to the number of recurring iterations to be performed.

The importance of the Integral Transformation method is based on the orthonormal bases when making an integral transformation of a model, this represents new projections in the path of the analyzed functions.

The result of an integral transformation is the expansion of the function from its standard representation to one of orthonormal functions that converge in spectral factorization.

Spectral Factorization helps the filtration of white noise with causal noise, that is, the integral transformation benefits the interpretation of trends in the study of projections in the growth of the virus by having a better appreciation of the functions that will give a better result.

The integral transformation is of utmost importance when its coordinates x, y, z has correlation and can be expressed and cannot be expressed in other orthogonal axes, but in other orthogonal systems.

One of the utilities of the Integral Transformations is that you can consider the use of the Laplace Transform mapping differential equations in the time domain in polynomial equations with this we will be able to have a better interpretation of the mathematical models considering the coefficient of determination that will make the model better validated than before the integral transformation.

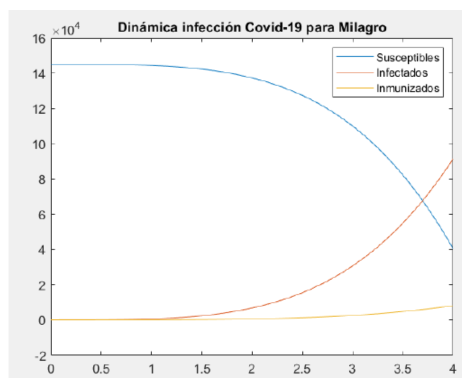
The Integral Transformation is very useful for finite intervals in this case it is advisable to use it since the life cycle of the virus is finite if the cycle grows it could already be used Fourier transformations.

Today the modeling in MATLAB is very appropriate to be able to corroborate the information obtained in the different mathematical models of the phenomena interpreted. The use of the dde23 function gives us the possibility to study differential equations with delay and interpret the data with those obtained from the solutions of the equations with standard and non-standard numerical methods.

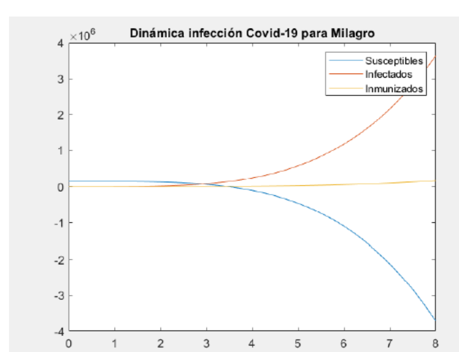
By comparing non-standard methods such as the Integral Transformation method with dde23, the relationship can be obtained, and it can be verified that the appreciation of viral growth and the maximum values of viruses are those appreciated by the non-standard method.

Results
Modeling at Matlab

We proceeded to make the model in a programming language obtaining the dynamics of Figure 1 and Figure 2



Dynamics of Covid-19 infection at an interval of 4 weeks



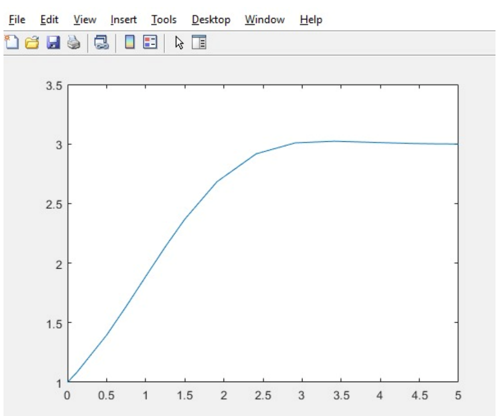
Dynamics of Covid-19 infection at an interval of 8 weeks

The results shown for 8 weeks cannot be interpreted in a real context because it shows a negative amount of susceptibles. This is because proportional delay has more distortion at the start of the interval due to the explained methodology.

Because Taylor series converge rapidly on exponential behaviors, an estimate was made for an order error $O(x^4)$.

The delay provided to the population dynamics model shows us a loving behavior of the spread of the disease since it considers the incubation period of the virus and the interval where it becomes susceptible again. In our model is not considered behaviors of contraction in the rate of contagion as four- rentenas or partial mobility restrictions therefore in 4 weeks two incubation periods have been fulfilled, producing an exponential increase pronounced from the third week.

Using MATLAB's dde23 function that solves differential equations with delay, we have plotted the behavior of viral growth by observing that the time of 5 weeks can be seen that growth in period 2, 5 has a linear trend and its monotony is not increasing, having greater growth in the later stages as shown in the graph below.



Conclusions

- The differential transformation method is practical and accurate for Short Periods of time and with Proportional Delays.
- Unlike traditional methods, the all of transformation integral No Requires of dis- cretize expressions, working all the time with the Continuous function.
- Differential delay equations successfully model the actual delay cycles in the physical nature of the model to be solved.
- The spread of infection becomes uncontrollable without mitigation measures such as quarantine or social distancing.
- In this context the Delay Differential Equations are of utmost importance to be able to analyze the behavior of the virus in the populations, under a correct analysis it is possible to determine measures to be taken, including which are the best periods for the applicability of measures that reduce the risk of death in infected patients.
- The importance of this study is addressed in the observation of the delay and how this could be increased if strategies are found to be able to cushion the impact on this vulnerable sector of the Ecuadorian population.
- The study of an instant $(t\breve{\ }\tau)$ is necessary at this time since it will serve to observe the behavior of the graph of the mathematical model and to be able to observe the peaks that it could cause, these peaks that could be observed in the graphs must be well observed to be able to apply either appropriate drugs or also specific treatments to be able to reduce the effect of the disease with benefits for the infected.
- It is important to note that as the immunized grow according to the graph, the population susceptible to contagion tends to fall, thus producing a better effect of the measures taken both as vaccines and asepsis in the population.
- When studying this type of behavior, it has been denoted that the number of possible infected in the incubation period of two weeks that are represented in $\alpha 2$, must be observed stealthily and it is at those moments is that through the application of measures such as the confinement of people to avoid the contagion of more people in the population.
- It should be noted that at the beginning only in the city of Guayaquil there were more than 700 people killed in the streets of the city, seeing these results mandatory confinement measures are taken, to avoid excessive contagion of people. The hygiene conditions of the floors, the water, the use of masks, the sepsis of food must be controlled.
- It is concluded that the use of products for disinfection such as alcohol are very necessary also quaternary ammonium solutions, clear dioxide applied to areas, rooms, bathrooms are advisable to avoid the spread of the virus in the environment.
- The use of Vitamins E increases defenses according to research within the Department of Chemical Sciences of the Central University of Ecuador, increases the chances of having greater resistance and increased immune system.
- Proper balanced diet rich in antioxidants such as fruits, fishsuch as salmon, trout, clean water, will help strengthen the immune system of the population.

- It is recommended to purify the air through air purifying filters inside offices, enclosed spaces and hospitals which will help reduce the percentage of viruses; leaving the virus trapped in the filter and reduces the condition to people who work in these spaces.
- The use of Ozonates in environments confined to low concentrations is recommended since the use of high concentrations is toxic and dangerous for humans.
- The importance of the Integral Transformation method is based on the orthonormal bases when making an integral transformation of a model, this represents new projections in the path of the analyzed functions.
- The result of an integral transformation is the expansion of the function from its standard representation to one of orthonormal functions that converge in spectral factorization. Spectral Factorization helps the filtration of white noise with causal noise, that is, the integral transformation benefits the interpretation of trends in the study of projections in the growth of the virus by having a better appreciation of the functions that will give a better result.
- The integral transformation is of utmost importance when its coordinates x, y, z has correlation and can be expressed and cannot be expressed in other orthogonal axes, but in other orthogonal systems.
- One of the utilities of the Integral Transformations is that you can consider the use of the Laplace Transform mapping differential equations in the time domain in polynomial equations with this we will be able to have a better interpretation of the mathematical models considering the coefficient of determination that will make the model better validated than before the integral transformation.
- The Integral Transformation is very useful for finite intervals in this case it is advisable to use it since the life cycle of the virus is finite if the cycle grows it could already be used Fourier transformations.
- Today the modeling in MATLAB is very appropriate to be able to corroborate the information obtained in the different mathematical models of the phenomena interpreted. The use of the dde23 function gives us the possibility to study differential equations with delay and interpret the data with those obtained from the solutions of the equations with standard and non-standard numerical methods.
- By comparing non-standard methods such as the Integral Transformation method with dde23, the relationship can be obtained, and it can be verified that the appreciation of viral growth and the maximum values of viruses are those appreciated by the non-standard method.

Recommendations

- Establish limits based on the time interval and the delay period within which the solution does not exceed the initial population.
- Generate more examples of application of the delay Equations using the described methodological and continue to demostrate its usefulness in science.
- Create additional parameters to model mitigations such as quarantine or social distancing.

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