

Analysis of Fatigue Crack Growth Resistance of Aluminum Matrix Composites

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Abstract– This work analyzes fatigue resistance of aluminum matrix composites with the support of robust statistics. The experimental samples studied were aluminum-based composites with a matrix containing Al, Cu and Mg and reinforced with diboride particles fabricated via both gravity and centrifugal casting. The variation in fatigue strength due to boron addition (forming diboride particles) was assessed via a comparison of the Paris equation exponent calculated for each composition as the slope of the da/dN vs ΔK curve plotted on a log-log scale. This approach, combined with the implementation of Kujawsky's crack driving force parameter and the application of robust statistics, allowed identifying variations in the crack growth behavior due to heterogeneities in the microstructure. The results showed that samples obtained via centrifugal casting have a higher fatigue resistance than those obtained via gravity casting. In addition, the analysis demonstrated how crack growth resistance decreased by increasing amounts of reinforcing particles in the material. The use of a robust statistics analysis was fundamental to avoid rejecting data from samples exhibiting variations in the crack growth curve.

Keywords– Fatigue crack growth; robust statistics; aluminum matrix composites.

I. INTRODUCTION

In the analysis of fatigue data, the behavior of the da/dN vs ΔK curve is usually assumed as linear in the Paris regime. This corresponds to stable crack growth stage or stage II, which Paris associated with the linear behavior exhibited by most materials when the crack growth curve is plotted in a log-log scale [1]. However, in highly heterogeneous materials fatigue crack growth (FCG) data deviate from such linear behavior as influenced by a number of factors: residual stresses, fatigue crack closure, microstructure, geometry, environment, overloads, underloads, temperature, frequency, and the effects of mixed-mode loading [2]. When such deviations occur, events detected during fatigue testing affect the slope of conventional linear regressions in the Paris regime. Even more, in some cases, the linear tendency of the material response could be questionable. Nonetheless, if such events are identified to be random and are not actually attributed to the behavior of the material, the reduction of the weight of those events on the linear regression in the Paris regime would allow using such data in the FCG study of a specific material. In statistics, such events are identified as outliers. Since these outliers are defined as data values with unexpected deviations with respect to the majority of the data points, they can alter the sample mean \hat{y} .

Robust statistics can be used to obtain linear models even in the presence of outliers. This data analysis tool is based on parameters such as the data median and the range, which are barely affected by outliers [3-5]. Therefore, the implementation of a robust statistical analysis could be a solution to avoid discarding the data in cases where the number of samples is limited. This work addresses such approach to study the crack growth response in the Paris regime of a series of Al-matrix composites reinforced with diboride particles. The intrinsic heterogeneity of these composites poses a unique challenge to crack growth analysis, affecting the experimental results, which are difficult to interpret without a methodology based on robust statistics.

II. EXPERIMENTAL PROCEDURE

All aluminum matrix composites studied in this work were fabricated via two routes: gravity and centrifugal casting. The target chemical compositions of the specimens are presented in Table 1. The composites are part of a series of studies aimed at producing lightweight, high strength alternatives for structural parts for the aerospace and transportation industries [6-10].

TABLE 1
COMPOSITE CHEMICAL COMPOSITIONS IN WEIGHT PERCENT

B	Cu	Mg	Al
0			
1			
2	2.5	1	Balance
3			
4			

Fatigue testing was carried out using a three-point bending apparatus in a 100-kN capacity MTS® uniaxial testing unit retrofitted with Instron® electronics. The dimensions and geometry of the sample, shown in Fig. 1, were selected according to the ASTM E1820 (2008) recommendation for single edge bend SE (B) specimen with width-to-thickness ratio $W/B = 2$ [11].

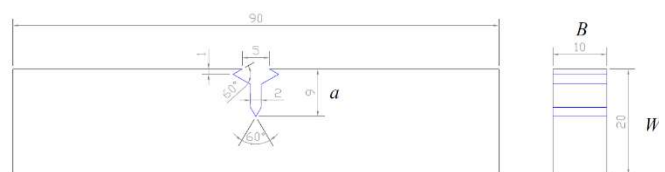


Fig. 1 Single edge bend SE (B) specimen diagram

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The load and the crack mouth opening displacement were the output data [11-12], which were employed to analyse the composites in the Paris regimen. The Paris Law (see Eq. (1)) yields to a quantitative prediction of residual life for a crack of a certain size and allows obtaining the fracture growth rate (FCGR) [1,13].

$$\frac{da}{dN} = C \Delta K^m. \quad (1)$$

where a is the crack length, N is the number of load cycles and C and m are material constants obtained from the fatigue crack growth curve. The concept of effective stress intensity factor ΔK_{eff} , from the modified Paris rule, as in (2), was used instead of ΔK to employ the minimum stress factor necessary to open the crack (K_{op}). The concept is utilized under the condition that K_{min} is lower than K_{op} [14-15].

$$\frac{da}{dN} = D \Delta K_{eff}^m. \quad (2)$$

where:

$$\Delta K_{eff} = \Delta K_{max} - \Delta K_{min} \text{ if } K_{min} > K_{op}. \quad (3)$$

$$\Delta K_{eff} = \Delta K_{max} - \Delta K_{op} \text{ if } K_{min} < K_{op}.$$

For a more accurate approach of crack growth and toughness, we used analytical tools of EPFM such as J-integral or crack tip opening displacement (CTOD). The relationship between CTOD and da/dN , represented in Eq. (4), takes into account the main factors that dominate crack growth [15].

$$\frac{da}{dN} = D' (CTOD)^{n'}. \quad (4)$$

As illustrated in Fig. 2, the applied load profile was triangular so as to simulate high loading rate conditions. The loading time was set to 0.1 second while the crack opening displacement was measured at the integral knives, as shown in Fig. 3.

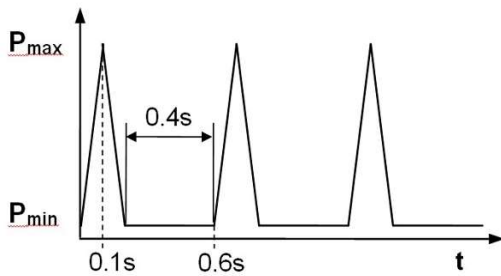


Fig. 2 Load profile

The calculation of K_I was carried out following the procedures stated by the ASTM E1820 standard for the measurement of the fracture toughness. Eq. (5) permits the calculation of K_I for bend specimens for a given force P_i .

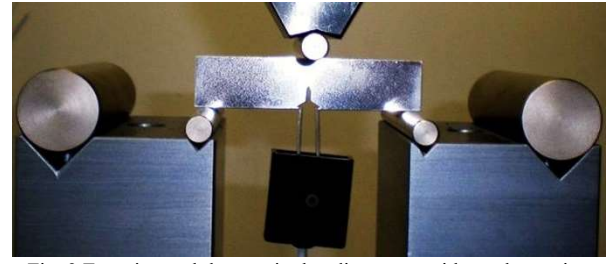


Fig. 3 Experimental three-point bending setup with crack opening displacement measuring device

$$K_{(i)} = \left[\frac{P_i \cdot S}{B \cdot W^{3/2}} \right] \cdot f(a_i/W) \quad (5)$$

where:

$$f(a_i/W) = \frac{3 \left(\frac{a_i}{W} \right)^{1/2} \cdot \left[1.99 - \left(\frac{a_i}{W} \right) \left(1 - \frac{a_i}{W} \right) \left(2.15 - 3.93 \left(\frac{a_i}{W} \right) + 2.7 \left(\frac{a_i}{W} \right)^2 \right) \right]}{2 \left(1 + 2 \frac{a_i}{W} \right) \left(1 - \frac{a_i}{W} \right)^{2/3}} \quad (6)$$

When plasticity occurs, the calculation is performed in terms of the J-integral. For this purpose, we used a load-line displacement while the crack mouth displacement was utilized to calculate the crack size. Calculation of J-Integral includes an elastic component as well as a plastic component according to Eq. (7).

$$J_{(i)} = J_{el(i)} + J_{pl(i)} \quad (7)$$

where $J_{el(i)}$ is obtained according to Eq. (8).

$$J_{el(i)} = \frac{K_{(i)}^2 (1 - \nu^2)}{E} \quad (8)$$

$K_{(i)}$ is calculated using equation 1 with $a=a_0$.

Finally, $J_{pl(i)}$ is computed with Eq. (9).

$$J_{pl(i)} = \frac{\eta_{pl(i)} \cdot A_{pl(i)}}{B \cdot b_i} \quad (9)$$

where:

$A_{pl(i)}$ = area under force versus crack mouth opening displacement record to ν_i

$$\eta_{pl(i)} = 3.785 - 3.101 \left(\frac{a_i}{W} \right) + 2.018 \left(\frac{a_i}{W} \right)^{1/2}, \text{ and}$$

$$b_i = W - a_i$$

In a preliminary analysis, it was found a larger variation of the maximum load as a function of the percent of boron and the processing method. For instance, the maximum load corresponding to the base gravity cast material (0 wt.%B) was

around 2500 N. This value was used as a benchmark to define the fatigue load range for all composites. Thus, for 80, 70, 60 and 50% of maximum load, the corresponding fatigue load selected were 2000, 1750, 1500 and 1250 N, respectively. The large amount of data was reduced using the incremental polynomial method described in the ASTM-E647 standard [16] and processed to obtain the da/dN vs ΔK curve.

Then, fitting coefficients were computed by regression of the data from a log-log chart. Since it was observed that the regression models were affected by the presence of anomalous events (likely caused by the microstructure heterogeneities), we decided to employ a robust statistics strategy. MATLAB® provided the necessary computational tools: the *robustfit* function. This *robustfit* function uses an iteratively reweighted least square algorithm, where the program computes the weights for each iteration applying a bisquare function to the residuals from the previous iteration. The algorithm assigns lower weights to the points that do not fit well. As a consequence, the procedure decreases the sensitivity to outliers in the data as compared with ordinary least squares regression. Initially, a model was fitted by weighted least squares. In the weighted least squares regression, a scale factor w was included in the fitting process; hence, the error estimation is minimized according to Eq. (10) [17-18].

$$S = \sum_{i=1}^n w_i (y_i - \hat{y}_i)^2 \quad (10)$$

where w_i are the weights, which determine how much each response value influences the final parameter estimates. Then, the parameter estimates b is calculated as indicated in Eq. (11).

$$b = (X^T W X)^{-1} X^T W y \quad (11)$$

where W is composed by the diagonal elements of the weight matrix w . Then, Eq. (12) calculates the standardized adjusted residuals.

$$u_i = \frac{r_i}{K \cdot s \cdot \sqrt{1 - h_i}} \quad (12)$$

where r_i is the least square residual; h_i , the main diagonal of the hat matrix H (also known as leverages), which adjusts the residuals by down-weighting high-leverage data points; K is a tuning constant equal to 4.685, which gives 95% efficiency. $s \approx \text{MAD}/0.6745$ and MAD is the median absolute deviation of the residuals defined by Eq. (13).

$$\text{MAD} = \text{median}_i \{ |Y_i - \text{median}_j (Y_j)| \} \quad (13)$$

For a normal distribution, Eq. (12) becomes Eq. (13).

$$\text{MAD} = \text{median} \{ |Y - \mu| \} \approx 0.6745 \sigma \quad (14)$$

Finally, the robust weights are computed using the bisquare function given by Eq. (15). Subsequently, the model fitting is reached by convergence and the coefficients to fit a linear model are computed [17].

$$w_i = \begin{cases} (1 - (u_i)^2)^2 & |u_i| < 1 \\ 0 & |u_i| \geq 1 \end{cases} \quad (15)$$

III. EXPERIMENTAL RESULTS

To further understand why robust statistics are adequate in this work, we deemed important to review the heterogeneous nature of the composite microstructure. In both cases, i.e., gravity and centrifugally cast samples, the material presented heterogeneities characteristic of a particle-reinforced aluminum matrix composite. For instance, the micrograph in Fig. 4 illustrates the presence and distribution of the AlB_2 reinforcements, the intermetallic compound Al_2Cu or θ phase and some pores. These last (latter) defects are inherent from the cast Al-B master alloys and are the result of gas entrapment during the master alloy fabrication [19].

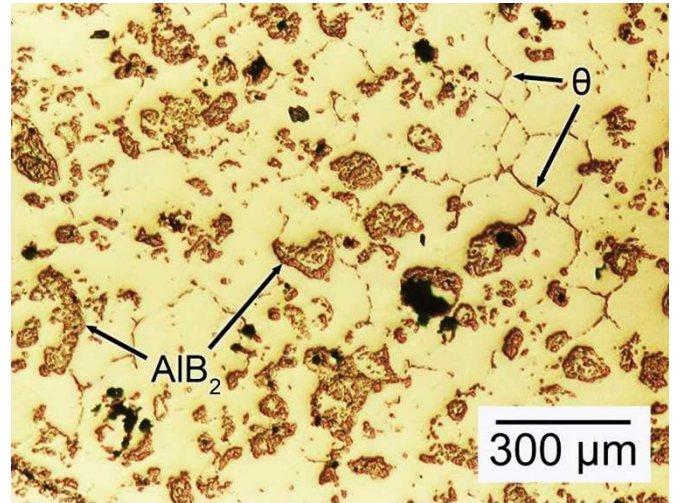


Fig. 4 Micrograph of a gravity cast composite containing 4 wt.% boron

The analysis of the fatigue data was performed by linear regression. The data fitting allowed obtaining the model parameters in the Paris regime. To do this, the asymptotic initial (dotted line) and final parts (dashed line) of the data were avoided, as illustrated in Fig. 5. The continuous line describes the data adjusted by least squares using a second order

polynomial to sets of $(2n+1)$ successive data points, with $n=3$. The data in Fig. 5 was used to obtain the da/dN vs ΔK curve plotted in the log-log graphic shown in Fig. 6 where significant changes in the crack growth (da/dN) are apparent, thus resulting in a disturbance on the Paris linear behavior. We found that such behavior repeated itself in all samples; hence, we discarded any artifacts caused by the instrumentation.

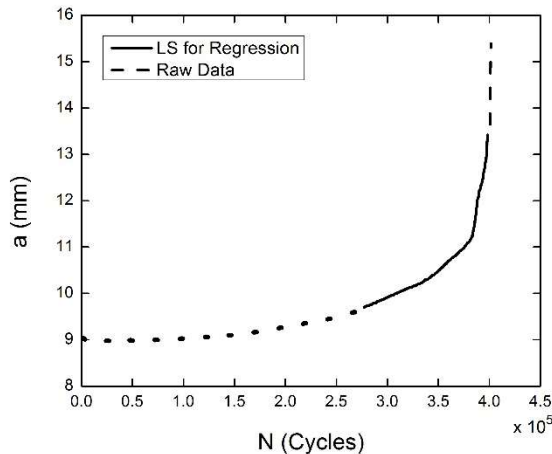


Fig. 5 Crack length as function of the number of cycles N

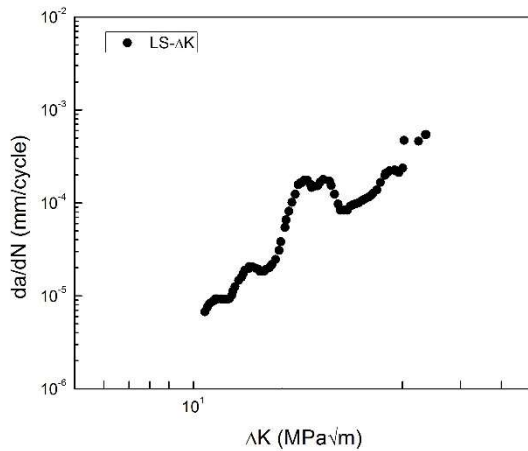
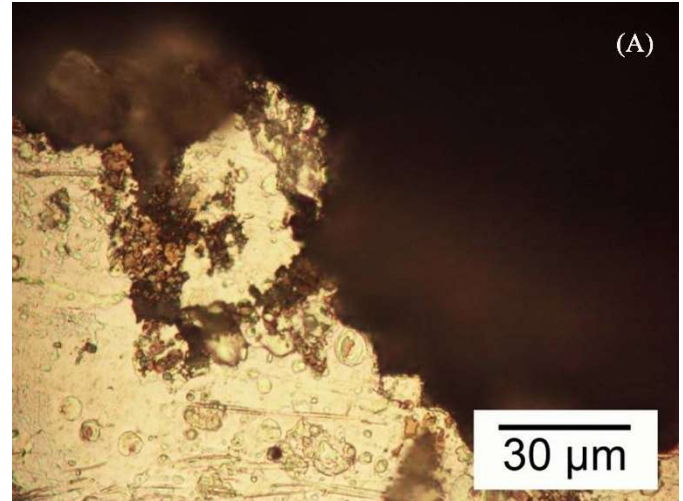


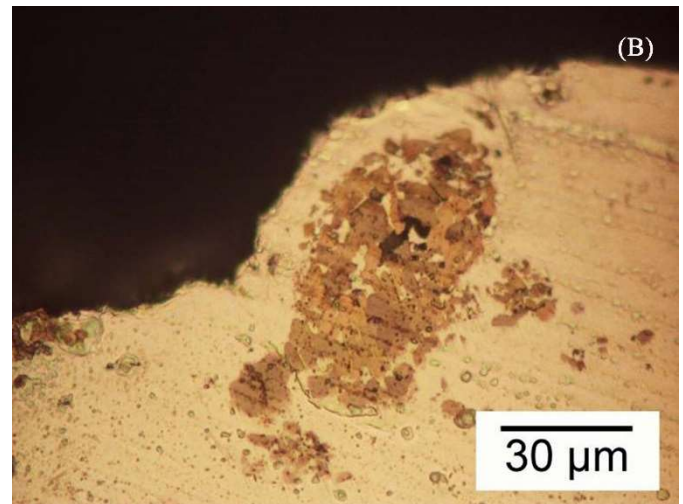
Fig. 6 Crack growth of centrifugally cast sample (2.5%Cu-1%Mg-1%B) showing anomalous behavior (discontinuities in the data)

As mentioned previously, this anomaly or data discontinuity affects the calculation of the slope via linear regression. However, through complementary scanning electron microscopy it was discovered that these events have occurred as consequence of the composite microstructure heterogeneity (non-uniform presence of reinforcements or casting defects), and do not correspond to the normal behavior of the composite. For instance, in Fig. 6 the presence of the mentioned heterogeneities affected the crack path and, therefore, the fatigue crack growth data.

Although a higher porosity results because of increasing amounts of AlB_2 particles, these defects were more evident in gravity samples and, in a lesser degree, in the centrifugally cast specimens (see Fig. 7(A)). This is expected as centrifuged specimens contained less porosity. On the other hand, when the particle clusters appeared free of casting defects, they do not separate from the matrix, letting the crack grow enveloping the cluster, as shown in Fig. 7(B). Thus, even though the clusters do not fail from defects, they could cause crack deflection, promoting more roughness in the fractured surface and the probability of increasing the number of crack growth rate acceleration/deceleration events.



A) Sample exhibiting cluster separation



B) Crack bordering a cluster almost free of defects

Fig. 7 Optical micrographs of fracture zones with particle clusters in centrifugally cast specimen containing 4 wt.%B, tested at $R=0.02857$

With this in mind, robust statistics was used as a tool to compare the response of these composites with different levels of boron (i.e. different amounts of diboride particles). Robust statistics helped reduce the impact of the data anomalies on the linear fitting of the $\log(da/dN)$ vs $\log(\Delta K)$ curve, as illustrated

in Fig. 8. The following is an example for a centrifugally cast sample with 1 wt.%B (2.5%Cu-1%Mg-1%B).

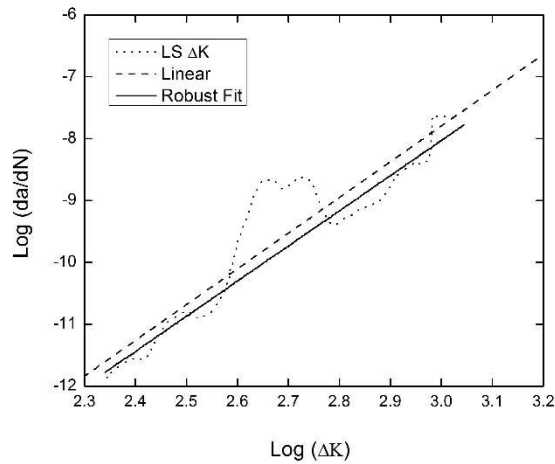


Fig. 8 Linear regressions of crack growth data shown in Fig. 6

In Fig. 8, the linear model is displaced up by the anomalous variations (dashed lines) in the da/dN while the *robustfit* function produces a better fit to the data (continuous line). Thus, robust statistics allowed effectively comparing the response of those specimens with anomalous behavior with specimens without such irregularities.

The study of fatigue resistance was based on the comparison of the Paris equation exponent calculated for each composition as the slope m of the da/dN vs ΔK curves corresponding to gravity cast samples (Figure 9) and centrifugal cast samples (Figure 10)).

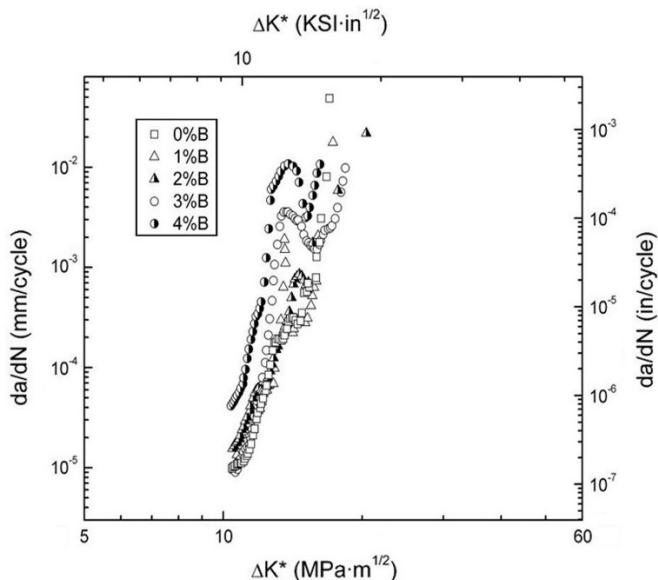


Fig. 9 Effect of boron weight percent on the da/dN vs ΔK curves of composites obtained via gravity casting (Max. load 1500 N)

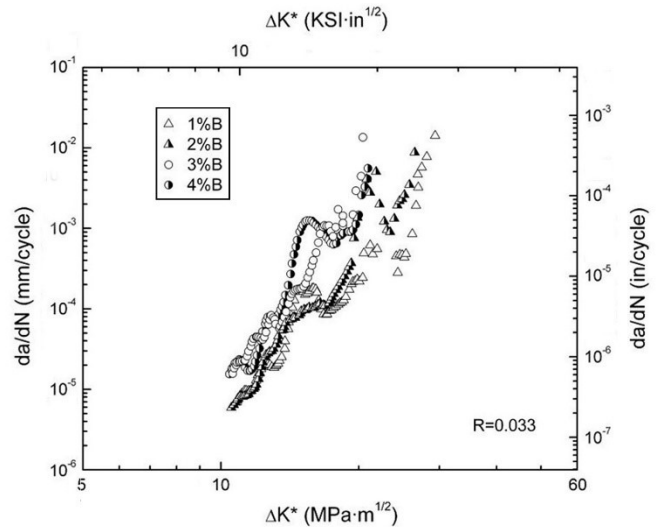


Fig. 10 Effect of boron weight percent on the da/dN vs ΔK curves of composites obtained via centrifugal casting (Max. load 1500 N)

Eq. (16) is the well-known linear model. This linear model was combined with the new crack driving force parameter (ΔK^*), proposed by Kujawsky [20] and the linear fit employing robust statistics [19]. The calculated slopes of each curve in Figs. 9 and 10, corresponding to gravity and centrifugally cast specimens, are presented in Fig. 11.

$$\log \left(\frac{da}{dN} \right) = m \log(\Delta K) + \log C \quad (16)$$

Fig. 11 reveals the tendency of the crack growth curve slope in the Paris regime to increase for levels of boron higher than 1 wt. %. Nevertheless, the computed slopes obtained for the matrix (0 wt.%B, i.e. unreinforced alloy) are higher than the slopes obtained in samples containing only 1 wt.%B.

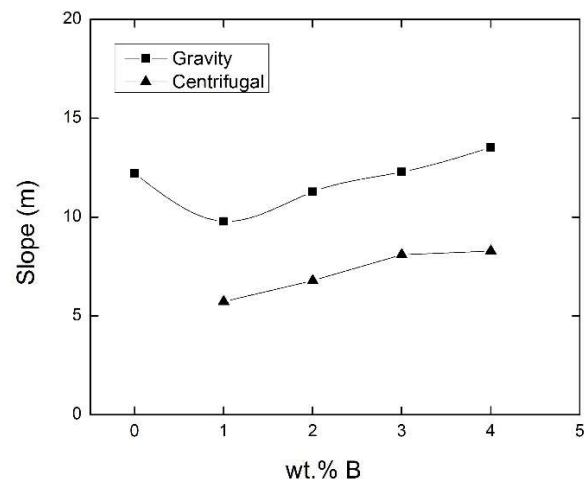


Fig. 11 Crack growth curve slopes of gravity cast and centrifugally cast samples, with crack growing in the centrifugal casting force (CCF) direction, as function of the weight percent of boron for 1500 N of maximum fatigue bending load.

Since the composites were particle-reinforced with different amount of boride, the authors expected a reduction in the slope (increment in the fatigue resistance). Toughness reduction can be related to the particle clusters that surround the pores of AlB_2 . This reinforced composite showed a predominant behavior of brittle fracture, which is observed in a mode of quasi-cleavage [19, 21]. Such behavior suggests a strong Al/AlB_2 interface, noting that the particle fracture promotes the crack growth where voids nucleate and coalesce with the crack tip [21]. On the other hand, the slope increment represents a shortening of the fatigue life since the crack growth rate increases.

IV. DISCUSSION

First and foremost, it is acknowledged that most particle reinforced metallic matrix composites have better ΔK threshold than the unreinforced matrix, due to shielding effects from the reinforcing particles. Also, at higher ΔK , within the Paris regime, particle cracking takes place, which causes the composite to possess lower fatigue resistance than the unreinforced alloy. However, the authors believe that it is important to underscore that the heterogeneities, as those present in this AlB_2 -reinforced composite, may mask a more detailed analysis. For that reason, this work presents an alternative understanding of the results, based on the assumption that the discontinuities in the da/dN vs. ΔK curve are indeed due to events related to the microstructure. This could have been further proved if an in situ, real time (upon each fatigue test) characterization equipment was available.

Alternatively, the apparent contradiction observed in the behavior of the slopes with respect to the amount of boron can be explained by the combination of two phenomena. The first one is associated with direct and indirect strengthening mechanisms involved in the particle reinforcement of an aluminum matrix [22-23] and the second one relates to the material strength reduction promoted by a $\text{Mg}-\text{AlB}_2$ interaction; phenomenon studied before in these types of composites. This interaction is promoted by a substitutional diffusion process at the material processing (casting) temperature. In the process, aluminum atoms in the AlB_2 structure are substituted by magnesium ones, causing the depletion of Mg from the matrix and the reduction in the composite strength [6].

For lower particle loadings, for example 1 and 2 wt. %B, the crack growth curve slope would be influenced mainly by the strengthening mechanisms, promoting a reduction in the slope (higher fatigue resistance), which is the usual effect of particle reinforcing. The values of the slope increased with the maximum fatigue load for gravity casting samples because of monotonic damage. In centrifugal casting samples, a clear effect of increasing the maximum fatigue load was not observed. Hence, aluminum based composite fabricated via gravity casting showed lower strength than the functionally graded materials fabricated via centrifugal casting. On the other hand, the $\text{Mg}-\text{AlB}_2$ interaction exhibited higher effect on the slope values for 3 and 4 wt. %B specimens. With these levels of boron,

the higher quantity of particles resulted in a higher particle/matrix contact area, which increased the detrimental effect of the Mg diffusion into the aluminum diboride.

Moreover, the information for the boron content in the range from 1 to 4 wt.% in Fig. 11 was used to obtain regression model in Eq. (17). Table 2 summarizes the regression parameters. As aforementioned, the levels of maximum fatigue load had a large effect on gravity cast specimens while the effect of monotonic damage contribution was not quite apparent in the centrifugally cast specimens. These models allowed estimating the variation in the fatigue resistance of the composites as a function of the material's weight percent of boron. This information is relevant to understand the fatigue resistance of this series of materials and could be used to compare each composition with similar composites.

$$\text{Slope } (m) = X_1 \cdot (\text{wt. \%B}) + X_2 \quad (17)$$

TABLE 2
LINEAR REGRESSION PARAMETERS FOR EACH CASTING METHOD

Casting Method	Max Fatigue Load (N)	X_1	X_2	R^2
Gravity	1500	1.2114	8.637	0.994
Centrifugal	1500	0.8722	5.260	0.990

In addition, by comparing the results in Fig 11, it can be safely state that gravity cast $\text{Al}-\text{Cu}-\text{Mg}-\text{B}$ composites possess a lower resistance to fatigue crack growth because their computed slope m is steeper. This outcome is attributed to the processing characteristics, since in centrifugal casting the material is subject to acceleration of up to 48g, which helps to reduce defects due to shrinkage and gas porosity, common in gravity cast specimens.

V. CONCLUSIONS

The implementation of a robust statistics analysis was fundamental to avoid rejecting data from samples exhibiting perturbations in the linear behavior associated to the Paris regimen. This is important mainly in projects where the number of samples is limited.

The linear fit employing robust statistics also allowed the identification of variations in the crack growth behavior due to heterogeneities in the material microstructure and small variations in the testing conditions.

Centrifugally cast composites exhibited lower crack growth slopes m than gravity cast composites, which translates into higher crack growth resistance of the centrifugally cast composites.

In both gravity and centrifugally cast composites, the crack growth resistance decreases as the level of boron increases in the range from 1 to 4 wt.%, as indicated by unsophisticated linear regression models obtained for the crack growth curve slope as function of the amount of boron in the composites.

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