Nonlinear Control Strategies for a Pendubot System

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***Abstract*– Nonlinear plants arise naturally in numerous engineering and natural systems, including robotics and biological systems, mechanical and automotive control, industrial process control, among others. The Pendubot is a nonlinear plant with two links used for research in nonlinear control and for education to teach some concepts as non-linear dynamics, robotics, and nonlinear control strategies. This paper presents simulation results for trajectory tracking using two different control strategies Takagi-Sugeno Controller and Sliding-mode Controller for a Pendubot System. The proposed controllers are used to track the response of angle q_2. To examine the robustness of the proposed controller, simulations using Matlab/Simulink prove the validity of this control method.**

***Keywords*- Pendubot, Takagi-Sugeno Controller, Sliding-mode Controller, Matlab, Engineering Design.**

I. INTRODUCTION

In real life, the physical dynamical systems cannot be represented by linear differential equations and have a nonlinear nature. Simultaneously, linear control methods rely on the critical assumption of a small range of operation for the linear model, acquired from linearizing the nonlinear system, to be valid. However, many systems highly nonlinear and discontinuous nature, for instance, mechanical and electrical systems, do not allow a linear approximation.

To solve the above problem, many control strategies are proposed [1-6]. In [1], the robust swing-up and balancing control method using a nonlinear disturbance observer for the pendubot system in the presence of dynamic friction is proposed. The performance and the validity of the proposed method are verified through both simulation and experimental results. In [2], the real-time optimal control of a Pendubot using nonlinear model predictive control (NMPC) combined with nonlinear moving horizon estimation (MHE) is presented. Experimental results illustrating the overall closed-loop control performance and the advantages of the nonlinear MHE-based NMPC scheme are shown. In [3], an advanced sliding-mode control (ASMC) with integral sliding function is proposed for the pendubot system, which is a planar two degree of freedom (2-DOF) robotic arm in the vertical plane with an actuator at the shoulder, but no actuator at the elbow.

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Simulation results demonstrate the performance and effectiveness of the proposed ASMC control compared to conventional sliding mode control applied in the swing-up and balancing scheme of the pendubot system. In [4], a novel block-backstepping based nonlinear stabilizing control law of a pendubot is presented. Lyapunov stability criteria have been used to analyze the stability of the overall system. Furthermore, zero dynamics stability is discussed to ensure the global asymptotic stability of the entire nonlinear system. In [5], a hierarchical sliding-mode controller with limited control torque to swing up a Pendubot is stated. The hierarchical sliding-mode control law is deduced in terms of the Lyapunov stability theorem. Finally, in [6], a new control strategy based on Fourier transformation and intelligent optimization for a planar pendubot with a passive second link is proposed; the planar pendubot can be treated as a second-order nonholonomic system whose control has been an open and challenging issue. A controller acting within a time corresponding to its fundamental harmonic term frequency is designed to realize the system control objective, which is to move the system from its initial position to the target position. A feedback control strategy based on a nonlinear disturbance observer is then applied to overcome the uncertainties/disturbances in the system.

In this paper, two nonlinear control strategies: Takagi-Sugeno Controller [7,8] and Sliding-mode Controller [9,10,11] for a Pendubot system, are proposed to achieve trajectory tracking. The fuzzy model proposed by Takagi and Sugeno is described by fuzzy IF-THEN rules, representing a nonlinear system's local input-output relations. A Takagi-Sugeno fuzzy model's main feature is to express the local dynamics of each fuzzy implication(rule) by a linear system model. On the other hand, to design a sliding-mode controller is more convenient to transform the system via diffeomorphism into the regular form, and the control law is obtained via the equivalent method control. The novelty consists in comparing nonlinear control strategies in the Pendubot system, which guarantees to track the response of angle q_2.

The rest of the paper is organized as follows: in section II Pendubot model is presented. In section III, controller synthesis and Membership functions for angle q_1 are reported. In section IV, simulation results to the proposed controller in the Pendubot system are presented. Finally, conclusions are drawn in section V.

II. PENDUBOT MODEL

The Pendubot is an electromechanical system consisting of two rigid links interconnected by revolute joints. A DC-motor actuates the first joint, and the second joint is unactuated. Thus, the second link is a simple pendulum whose motion can be controlled by actuation on the first link. The Pendubot is similar in spirit to the classical inverted pendulum on a cart or the more recent rotational inverted pendulum. The underactuated robot (Pendubot system) is illustrated as follows in Fig. 1.

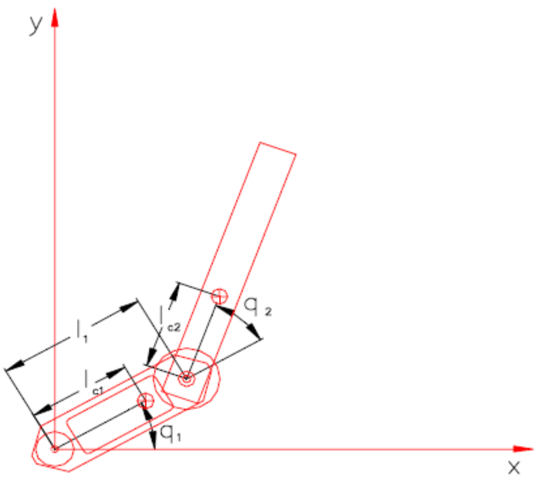

Fig. 1 Pendubot system.

The motor controls the angle q_1 and the angle q_2 is free. The pendubot model is like a two links robotic arm, with the difference that the torque only acts on the first link. The nonlinear mathematical model of Pendubot is given by

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q), \tag{1}$$

where, τ is the applied torque to the links and $q = [q_1, q_2]$ is the vector of the joint angle positions. $D(q)$ is the inertia matrix, $C(q,\dot{q})$ is the Coriolis matrix, and $G(q)$ is the gravity vector, details of above matrices and its derivation can be found in [12-14].

Defining the state space variable $x = [q_1, q_2, \dot{q_1}, \dot{q_2}]$, the subsequent system representation is obtained:

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ D^{-1}(B\tau - CX_2 - G) \end{bmatrix}, \tag{2}$$

where, $B = [1,0]^T$ and $X_2 = [x_3, x_4]^T$.

III. CONTROLLER SYNTHESIS

A. FUZZY CONTROL

Fuzzy logic controllers have succeeded in many control problems that the conventional control theories have difficulties to deal with. This control technique is critical for meeting the demands of complex nonlinear systems. A fuzzy logic controller chooses the switching states based on a set of fuzzy variables, which uses fuzzy rules, which are linguistic IF-THEN statements involving fuzzy sets, fuzzy logic, and fuzzy inference. Fuzzy rules play a crucial role in representing expert control/modeling knowledge and experience and linking the input variables of fuzzy controllers/models to the output variable for nonlinear systems. The two major types of fuzzy rules are Mamdani and Takagi-Sugeno (TS) [16,17].

Dynamic Takagi-Sugeno fuzzy models are not always easy to interpret, particularly when they are identified from experimental data. The Takagi-Sugeno inference method is an extension of the Mandani inference method, where the difference is that the TS output is a linear combination of the system states rather than a membership function.

For the control design, the following steps are followed:

1. The operation points of the system are defined.
2. The membership functions for the state to control are proposed.
3. The system (2) is linearized in the different operation points.
4. The state feedback gain is designed such that the closed-loop system is stable.

For the linear regulator problem, the following TS fuzzy laws are employed:

System rule i,

$$\textbf{IF the angle } \boldsymbol{q_1} \textbf{ is in } \boldsymbol{M_i}\textbf{, THEN } \begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A_i x} + \boldsymbol{B_i u} + \boldsymbol{P_i w} \\ \dot{\boldsymbol{w}} = \boldsymbol{Sw} \\ \boldsymbol{e} = \boldsymbol{C_i x} + \boldsymbol{Q_i w} \end{cases}.$$

Control rule i,

$$\textbf{IF the angle } \boldsymbol{q_1} \textbf{ is in } \boldsymbol{M_i}\textbf{, THEN } \boldsymbol{u_i} = -\boldsymbol{K_i x} + \boldsymbol{L_i w}.$$

By applying the center-average defuzzifier, product interference, and singleton fuzzier, the overall fuzzy controller can be represented by

$$u = \frac{\sum_{i=1}^{m} M_i\,(q_1) u_i}{\sum_{i=1}^{m} M_i\,(q_1)}. \tag{3}$$

Recall the necessary conditions for the regulator problem [15],

A1. The pair (A_i, B_i) is stabilizable.

A2. None of the eigenvalues of the matrix S coincide with any transmission zeros of the system (A_i, B_i, C_i).

If conditions A1 and A2 are fulfilled, then, for every matrix P_i and Q_i, there exist matrices Π_i and Γ_i than solve the next Francis equations,

$$\begin{aligned} \Pi_i S &= A_i \Pi_i + B_i \Gamma_i + P_i \\ 0 &= C_i \Pi_i + Q_i \end{aligned}, \tag{4}$$

where the control gain is $L_i := \Gamma_i - K_i \Pi_i$.

The reference signal is obtained with the next exosystem model:

$$\begin{aligned} \dot{w} &= Sw \\ y_{ref} &= Qw, \end{aligned}$$

where the matrix S and the output matrix Q defined as follows:

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \alpha \\ 0 & \alpha & 0 \end{bmatrix},$$

$$Q = [0, k, 0],$$

and α is the angular frequency and k is the magnitude of the reference signal y_{ref}.

The TS controller is design as follows: In the first step, the system operation points are defined, the operation points will depend on the reference signal to track, here a sinusoidal signal with angular frequency of $\alpha = \frac{2\pi}{5}$ and amplitude of 60° is employed as reference signal for angle q_2.

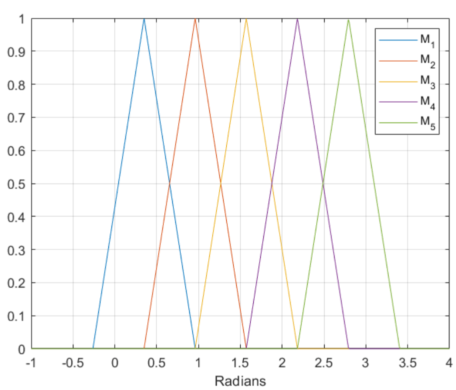

Fig. 2 Membership functions for angle q_1.

The operation points are chosen with the following restriction, $q_1 + q_2 = \frac{\pi}{2}$, as follows:

$$q_1^0 = [20,55,90,125,160]$$

$$q_2^0 = [70,35,0,-35,-70].$$

Next, in the second step, for each operation point of angle q_1 as shown in Fig. 2, five membership function are defined as

$$M_i(x_1) = \max\left(\min\left(\frac{(x_1 - a_i)}{(a_{i+1} - a_i)}, \frac{(a_{i+2} - x_1)}{(a_{i+2} - a_{i+1})}\right), 0\right), \quad \mathrm{i} = 1, \dots, 5$$

where,

$$a = [-15, 20, 55, 90, 125, 160, 195].$$

Next, in the third step, the linearization of system (2) at each operation point is carried out employing the Jacobian matrix. Then, in the fourth step, state feedback gains K_i are computed for each linearized system, such that each closed-loop system is stable.

Finally, in the fifth step, the regulator gains are obtained solving the Francis equations (4). Now the defuzzied method (3) is employed with m=5, with the control u_i defined as:

$$u_i = -K_i \begin{bmatrix} x_1 - q_1^0(i) \\ x_2 - q_2^0(i) \\ x_3 \\ x_4 \end{bmatrix} + L_i w + G_1, \tag{5}$$

with

$$\begin{aligned} G_1(q_1, q_2) = \; & m_1 g l_{c1} \cos(q_1) + m_2 g l_1 \cos(q_1) \\ & + m_2 g l_{c2} \cos(q_1 + q_2). \end{aligned}$$

B. SLIDING-MODE CONTROL

The sliding-mode control is a well-known discontinuous feedback control technique, which has been reviewed in several books and many journal articles. Relative simplicity for design, control of independent motion (maintaining sliding conditions), invariance to process dynamics, and external disturbances robustness are the main characteristics of sliding-mode control.

This control technique has been an efficient tool for complex high-order control in dynamic nonlinear plants operating under uncertain conditions, a typical problem for many real processes.

The sliding-mode control is designed in two stages. The first one is selecting the sliding manifold, and the second one is to find a discontinuous control law such that it drives the trajectories to the sliding manifold in finite time and then maintains them on this surface.

To design a sliding-mode controller for the nonlinear system (1), it is more convenient to transform the system (2) via a diffeomorphism $x' = \varphi(x)$, into the regular form [18]. Such transformation is proposed as follows:

$$\begin{aligned} x_1' &= x_1 \\ x_2' &= x_2 \\ x_3' &= x_3 - g_3 g_4^{-1} x_4 \\ x_4' &= x_4 \end{aligned}.$$

Thus, the Pendubot model in regular form is a follow

$$\dot{x}' = \begin{bmatrix} x_3' - D_{22} D_{12}^{-1} x_4' \\ x_4' \\ g_3 (D_{12} D_{22}^{-1} - D_{11} D_{12}^{-1})(C_2 + G_2) \\ g_4 p_4 + g_4 u \end{bmatrix},$$

where,

$$g_3 = \frac{D_{22}}{\Delta},\ g_4 = \frac{-D_{12}}{\Delta},$$
$$p_3 = \frac{D_{12}}{D_{22}}(C_2 + G_2) - C_1 - G_1,$$
$$p_4 = \frac{D_{11}}{D_{12}}(C_2 + G_2) - C_1 - G_1,$$
$$G_2(q_1, q_2) = m_2 g l_{c2} \cos(q_1 + q_2),$$

and,

$$\Delta = D_{11} D_{22} - D_{12}^2.$$

The sliding surface for trajectory tracking is defined as

$$s = ce + \dot{e},\quad c > 0,$$

where the tracking error is defined as

$$e = x_2' - Qw.$$

Taking the derivative of the error, the control law is obtained via the equivalent method control [19] as

$$u = g_4^{-1}(-c x_4' + c\alpha w_2 - g_4 p_4 + \alpha^2 w_3) - \rho sign(s). \quad (6)$$

IV. Simulation Results

Fig. 3 Pendubot Prototype.

To illustrate the applicability of the proposed nonlinear control strategies (TS fuzzy controller and SM controller), the following nominal parameters (Table 1) of the pendubot prototype (Fig. 3) are employed.

Table 1. Nominal Parameters for Pendubot System

Variable	**Value**
m_1	$0.8293\ Kg$
m_2	$0.3402\ Kg$
l_1	$0.2032\ m$
l_{c1}	$0.1551\ m$
l_{c2}	$0.1635\ m$
I_1	$0.0059\ Kg\ m^2$
I_2	$0.0043\ Kg\ m^2$
g	$9.81\ m/s^2$

Simulations are done using MATLAB Simulink with the Euler solver and a fixed step size of 0.5×10^{-4}. Simulations are performed as follows in Fig. 4. The reference signal for angle q_2 is given by

$$y_{ref} = sin\left(\frac{2\pi}{5} t\right).$$

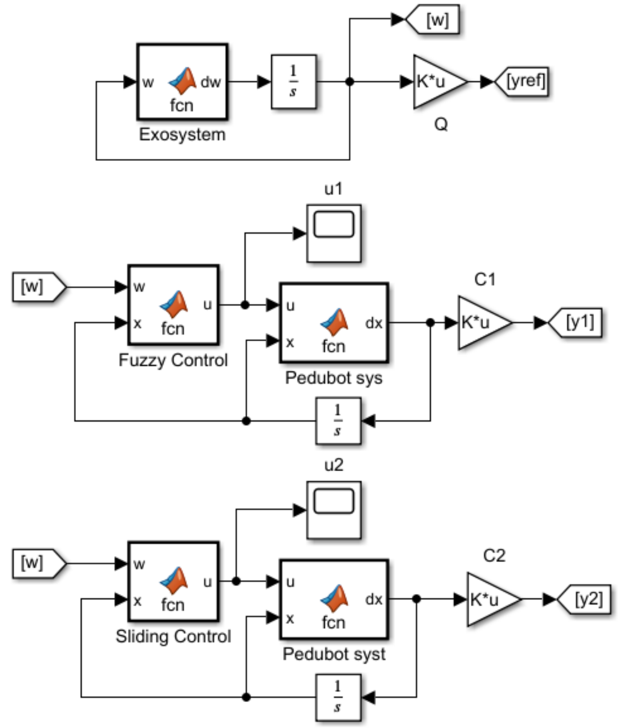

Fig. 4 Pendubot control simulation of the TS fuzzy controller and SM controller using an exosystem output as reference signal.

Employing TS control law (5) and SM control law (6) in the pendubot system (2), the tracking of the y_{ref} is achieved for the angle q_2 as can be seen in the Fig. 5.

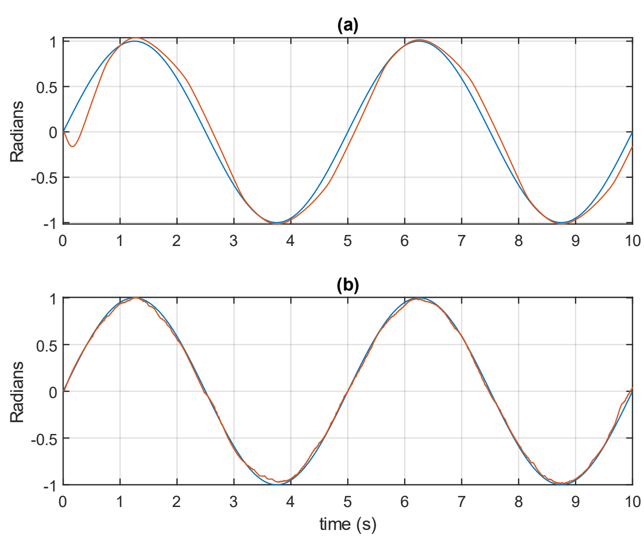

Fig. 5 Pendubot tracking response of angle q_2 with (a) TS fuzzy controller and (b) SM controller.

For the sliding-mode control design, the control law (6) in the pendubot system (2) with the following control parameters, $\rho = -2$, and $c = 10$ are employed, the tracking of a sinusoidal signal is achieved for the angle q_2 as is shown in Fig. 5b. Moreover, Fig. 6 displays simulation results for the average trajectory tracking error.

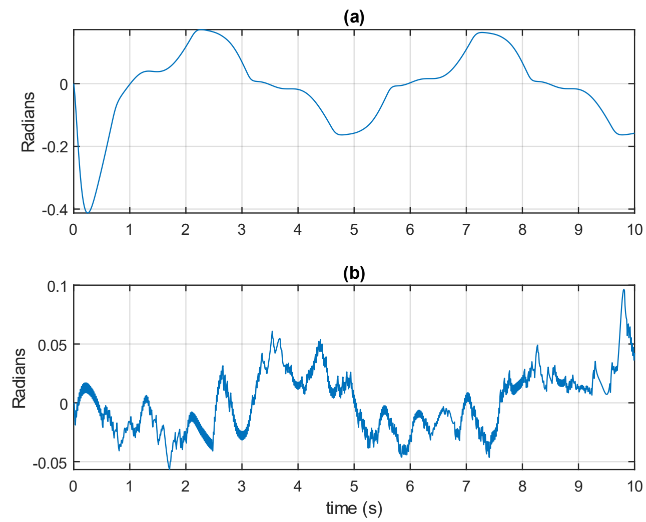

Fig. 6 Pendubot tracking error with (a) TS fuzzy controller and (b) SM controller.

Also, Fig 7b. illustrated the control input signal u(t) applied, displaying the typical chattering characteristic for discontinuous control actions based on sliding modes. To eliminate or at least reduce chattering, boundary layer, observer-based, regular form, and disturbance rejection techniques can be used.

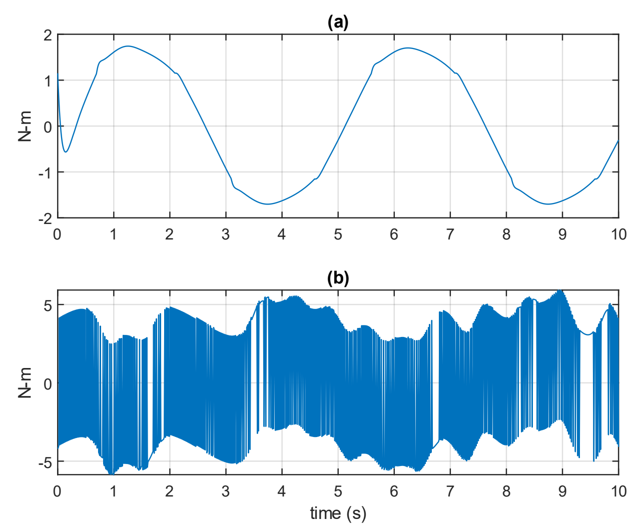

Fig. 7 Pendubot control signal of (a) TS fuzzy controller and (b) SM controller.

Finally, Fig. 8 displays pendubot tracking response of angle q_2 with disturbance to prove the robustness of the proposed controller. Before t = 4s, trajectory evolves freely without disturbance; at t = 4s and t = 8s, disturbances are incepted; however, trajectory tracking successfully even in the presence of external disturbances.

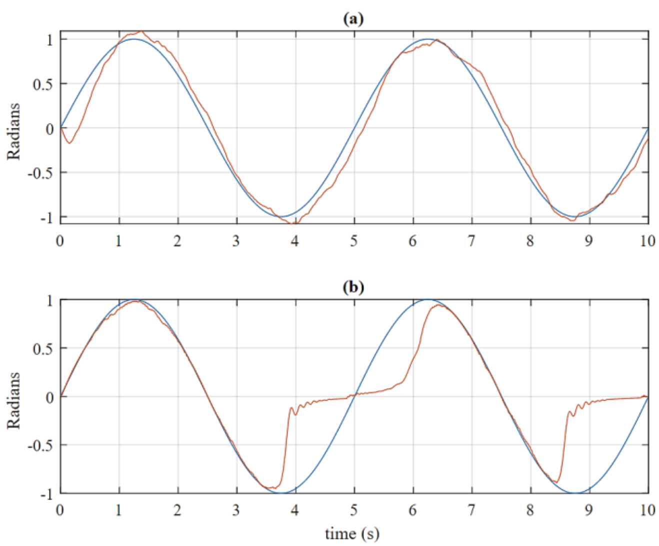

Fig. 8 Pendubot tracking response of angle q_2 with disturbance (a) TS fuzzy controller and (b) SM controller.

V. Conclusions

Nonlinear control strategies in the Pendubot system, which guarantees trajectory tracking for the proposed output y_{ref} in the angle q_2, were presented. Simulation results illustrate that trajectory tracking can be effectively achieved by using the proposed control scheme. Tracking response of angle q_2 with TS fuzzy controller and Sliding-mode controller is achieved. In addition, this setting presents a straightforward robust control algorithm.

Based on the tracking error, the sliding-mode controller has better performance in the tracking response of angle q_2. The average tracking error for TS fuzzy control is 0.13; on the other hand, SM control is 0.02.

Analyzing the performance of individual controllers in simulation, the system with the TS fuzzy controller maintains better trajectory tracking throughout the time, even in the presence of external disturbances.

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