Branch and Bound mixed nonlinear optimization for the GRP structure of Galápagos interisland small craft

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Abstract– The interisland transportation of passengers in Galápagos Archipelago is mainly carried out using high speed boats manufactured using Fiberglass Reinforced Plastic (GRP). The design of these small boats does not commonly follow advised construction standards and therefore it is possible that their structures are oversized. In addition, these boats navigate at high speeds and therefore it is advisable to investigate ways to reduce their weight. In this work, the structure of the central module of a 12-meter-long small interisland GRP craft traveling at 25 knots is optimized.

Mixed Integer Nonlinear Programming (MINLP) is used for the optimization process, since the structural design contains both integer and real variables. These mixed problems can be solved applying the Branching and Bounding algorithm (BB). Due to the combination of variables, the Gekko optimization library, capable of solving MINLP problems through Python programming language, is used. The process is divided into two phases: the first phase optimizes the number of stiffeners and the laminate of the plates, and the second optimizes the dimensions and laminate of the stiffeners.

The constraints of this structural optimization correspond to the requirements of the ISO 12215 standard for small craft: flexural strength of plates and stiffeners, inter-laminar resistance of plates, stiffness of stiffeners, and resistance to buckling of the web and flange of reinforcements. As a result of this structural optimization, the structural weight of the module is reduced by 20%.

Finally, the results of the optimization process are evaluated using the finite element method, to compare the current and the optimized hull module. The structure is modelled with plane elements of composite laminated type material; end cross sections are considered as clamped. The load is applied with a half-wave distribution in the transversal direction, centered on the quarter of the beam, and at centreline of the bottom. It is verified that in the current design modules are oversized and that, in the case of the optimized section, stress levels do not exceed the permissible values.

Keywords—optimal weight, GRP structure, mixed integer, branch and bound.

I. INTRODUCTION

In every ship design, reduction of weight is frequently necessary in order to decrease the fuel consumption of the main engines. In particular, for planing boats, this reduction in displacement could mean the possibility of overpassing the hump in the resistance at the beginning of the planing regime. This was the case of a 11m long boat which at the first tests reached only 16 knots, but with reduction of 7% in its weight, was able to pass the design speed of 32 knots, [1]. So, looking for alternatives to reduce weight, for example of its structure, it is always a goal of boat designers.

The process to find the best combination of variables for the design of a boat may become a very long process, because of the large number of potential combinations. Selection of material is also an issue faced at the beginning of the design process, and as [2] concludes, for small craft, the use of composite materials can reduce structural weight. In this case, the use of GRP structures and the characteristics of the layers of lamination are introduced as new design variables. So when looking for the design with minimum weight, the application of an optimization algorithm is a good option, combined with a computer implementation to accelerate the calculations [3].

Commonly, in an optimization procedure, the design variables are constrained by different conditions. In the case of a structure, these restrictions may be related to the possibility of reaching one of the modes of failure. Direct evaluation to reach yield point or critical buckling load may be complex analytic procedures [4]. In some cases, standardization organizations have developed procedures to evaluate modes of failure, for example, rules from ship classification societies [5], [6], or standards from ISO with amendments in 2014 [7]. This rule-based evaluation of constraints brings the possibility of the design exceeding safety requirements, but simplifies the calculations that facilitate optimization procedures.

The description of the structure of a boat includes integer and real parameters, for example, the number of reinforcements and the height of a hat-type section of the reinforcements. In search of optimum, calculations of rates of change of the variables suffer jumps when dealing with parameters with integer values. One simplistic option is to consider all variables as real, and at the end round up the integer values, [8], but this may result in a non-optimum solution, or could provide a result outside of the feasible region, as explained by [3]. Therefore, for the case of a structure, it is preferable that the optimization procedure considers both integer and real variables.

Considering the requirements previously mentioned, one option that has been employed for the optimization of boat structures is a genetic algorithm (GA). For example, in [9], one of these GA algorithms, when combined with direct evaluation of failure modes, produced a module of minimum weight. They considered a simple flat panel, however the spacing between reinforcements does not produce an integer number when divided by the length of the panel. Also in [10], an optimization

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was applied for the structure of a planning boat. In this case, the application of the ISO standard was compared with direct evaluation of failure modes of an GRP bottom structure. Also, in [11] a scheme was applied to reduce the weight of a fishing vessel by trying to reduce the amount of fiber content. And in [12], the artificial bee colony algorithm was applied to reduce the weight of a fishing vessel GRP structure. The main disadvantage of applying GA is the slowness to reach an answer.

In this work, a Branch and Bound (BB) optimization algorithm [13] has been applied to minimize the structural weight of the module of a GRP 12-m hull; this small craft provides interisland passenger transportation in the Galápagos Archipelago, and because of the required high velocity, a reduction in displacement is desired. This BB process includes mixed variables (both integer and continuous), and the problem is nonlinear. Common building practices are considered for the definition and optimization of the lamination. Since the complexity of the system is not very high, the requirements of ISO12215 standard [7] [14] are implemented as constraints, but a finite element analysis of the composite structure is applied to the resulting structure.

II. DEFINITION OF THE STRUCTURE

A. *Description of the structure*

The structure of a small craft is composed of several blocks separated by strong transversal elements, similar to bulkheads. As in general ship structures, framing can be longitudinal or transversal. The keel is one the main contributors to longitudinal stiffness of the bottom part of the structure. Plating laminate has low thickness, and also very low rigidity, which is compensated by stiffeners.

In a classical way, the structure of a ship may be analyzed by superposing the three main effects, primary, secondary and tertiary, [15]. Also, in the case of small vessels, the internal forces from the primary model, that is, the bending moment and shear forces on the hull beam, are very small and their effects may be considered as part of the safety factor. The secondary structure is the lateral response of grillage composed by the stiffeners and girders, plus the effective breadth of the plating, which acts as the flange of both beams. Tertiary stresses correspond to bending of plate panels between stiffeners. Under linear assumption, these effects are superimposed.

Longitudinal and transversal reinforcements usually differ in inertia, assuming that one type supports the others. In the case of the transverse framing system, longitudinal girders support transverse frames, and in the limit, these are assumed as clamped in the longitudinal; the very strong longitudinal elements are assumed as clamped in their ends, which correspond to the end points of the module. This assumption simplifies the calculation of maximum bending moment for both, $pl^2/12$, where p is the force per unit length and l is the unsupported span length, see Fig. 1. Maximum shear force is $pl/2$. This force per unit length is calculated with the pressure and the spacing between stiffeners or girders.

For the tertiary analysis, which considers each plate panel between the stiffeners, the simple model is to consider it with all edges as clamped under uniform load, see Fig. 1. In this case, the bending moment per unit length may be estimated while keeping in mind the plate aspect ratio, uniform pressure and the shortest dimension, [16].

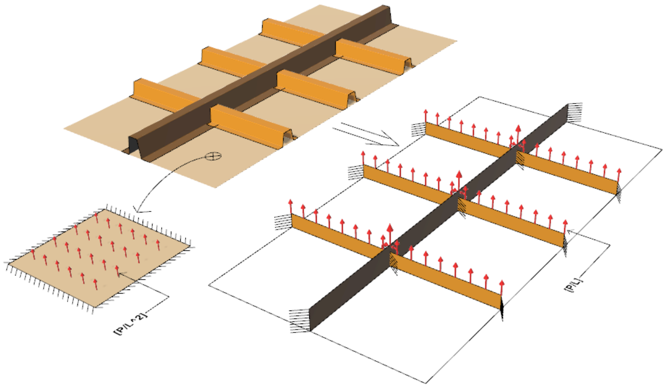

Fig. 1 Model of the bottom structure: grillage formed by longitudinal and transverse beams, and, clamped panel of plating between reinforcements.

B. *ISO standard*

The ISO12215 is a standard developed to design small craft of different materials. It includes simple formulations to estimate dynamic pressure on the hull due to hydrodynamic impact; these formulations depend on the velocity, main geometric characteristics, and, mass of the boat, [7]. Also, it includes formulations to estimate shear force and bending moment on the secondary grillage structure, based on pressure and spacing between stiffeners and girders, and bending moment per unit length to analyze plating, depending on pressure and main dimensions.

The ISO standard also covers structural use of GRP in the design of small craft. The standard explains how to estimate maximum allowable stresses, both normal and shear, based on the weight of the layer and mass of fiber.

The above-mentioned standard discusses different modes of failure: bending of beams and plating, shear of beam and plating, impact from floating objects, and, stiffness of the keel. For each one, values of safety factors are recommended with respect to ultimate stress of the laminates.

III. OPTIMIZATION PROCESS

A. *General process*

To apply an optimization procedure to the structure of a boat, [10] recommends to proceed in two steps. In the first step, the number of stiffeners and girders are selected together with the lamination scheme for the plating. Results of this process are the inertia and sectional modulus of reinforcements in both orthogonal directions and the lamination of the plating. In the second step, based on the bending requirement of the reinforcements in both directions, the dimensions and lamination are optimally selected, considering an effective breadth of plating attached to the stiffeners.

The above two-step process is applied to the structure of a small interisland craft, while taking into consideration the transversal framing system. First, the application of a BB to the bottom results in an optimum number of longitudinal girders and transverse frames, and bottom plating laminate. The same spacing between transverse frames is taken to design the side structure, but the process needs to determine the number of longitudinal reinforcements. In this way, there is continuity in transverse stiffness between bottom and sides. Longitudinal and transversal elements include a wooden core covered by GRP laminate. In the second step, optimization of the geometry of the reinforcements and its laminate is reached.

Because of results in this project, the bending stiffness of the keel was also optimized for minimum weight. This was because the optimization procedure determined that, because of the transverse dimension of the bottom of the boat, it was not necessary to include any longitudinal girder. As a result, the hull stiffness is reduced, and even though the requirements by ISO are satisfied, it is included an increment in stiffness for the keel.

B. *Optimal lamination*

For this process, a group of common practices of GRP construction are considered to complete the design of the lamination. These practices assume that the plate lamination is symmetric, since bending generates normal stresses in both upper and lower parts. The end layers are always of the Chopped Strand Mat type (CSM) with mass of 300 gr/m^2, since both surfaces later receive a coating paste for painting. Finally, since only CSM and Woven Roving (WR) fibers are considered, and the second type is very rough, it needs a smoother one, which is of the mat type. Therefore, lamination includes pairs of CSM and WR of different weight, and in the internal laminate, number of those layers must be equal.

For the definition of the lamination, layers of CSM of 300, 350, 450 and 600 gr/m^2 and layers of WR of 600 and 800 gr/m^2 are considered. Mechanical properties of fibers are presented in the appendix. For the case of the WR, as in the general construction process, its fibers are aligned in the longitudinal direction of the vessel. The number of each type of these 6 fibers are going to be design variables in the optimization. In Fig. 2 a summary of the symmetrical lamination process is presented, with *t* as the total thickness.

Lamination of the reinforcements needs to ensure that layers do not separate from the wooden core. For this reason, the wooden cores are only covered by layers of CSM.

Finally, following average values taken from ISO standard, [14] the mass fiber content values as percentage of total mass are shown to be 0.30 for CSM and 0.48 for WR. To estimate the thickness of each ply, *t*, equation (1) relates thickness to mass per unit area, *w*, density of resin and fiber, ρ_R and ρ_F, and, fiber mass content, ψ, [7]:

$$t = \frac{w}{\rho_F\,\rho_R}\left(\frac{\rho_R}{\psi} - \left(\rho_R - \rho_F\right)\right) \qquad (1)$$

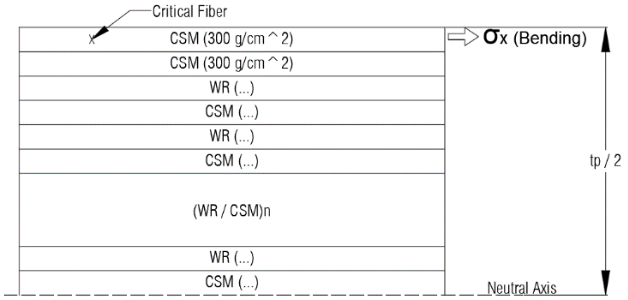

Fig. 2 Scheme for the lamination of the plating.

C. *Design variables*

For the first phase of the optimization process, the following design variables are considered:

N_T: number of transversal reinforcements,

N_{LB}: number of bottom longitudinal,

N_{LS}: number of side longitudinal,

N_{bi}: number of CSM layers of each one of the weights in the bottom plating, *i=1, 2, 3* and *4*,

N_{bi}: number of WR layers of each one of the weights in the bottom plating, *i=5* and 6,

N_{si}: number of CSM layers of each one of the weights in the side plating, *i=1, 2, 3* and *4*, and,

N_{si}: number of WR layers of each one of the weights in the side plating, *i=5* and *6*.

After completing the first phase, shear transverse area, section modulus and inertia for each one of the reinforcements are determined. Then in the second phase, the geometry of the reinforcements is optimized, see Fig. 3. In this work the reinforcements have wooden cores. The following variables are considered for the design:

h_l: height of longitudinal reinforcements of bottom,

b_l, width at base of longitudinal reinforcements of bottom,

h_{ls}: height of longitudinal reinforcements of side,

b_{ls}, width at base of longitudinal reinforcements of side,

h_T: height of transverse reinforcements of bottom,

b_T, width at base of transverse reinforcements of bottom,

h_{Ts}: height of transverse reinforcements of bottom,

b_{Ts}, width at base of transverse reinforcements of bottom,

N_{bli}: number of CSM layers of each one of the weights in the bottom longitudinal reinforcements, *i=1, 2, 3* and *4*,

N_{sli}: number of CSM layers of each one of the weights in the side longitudinal reinforcements, *i=1,2, 3* and *4*,

N_{sti}: number of CSM layers of each one of the weights in the side transverse reinforcements, *i=1, 2, 3* and *4*, and,

N_{bti}: number of CSM layers of each one of the weights in the side transverse reinforcements, *i=1, 2, 3* and *4*.

For the optimization of the keel the following design variables are considered:

h_k: height of keel,

b_k, width of keel, and,

N_{ki}: number of CSM layers of each one of the weights in the keel, $i=1, 2, 3$ and 4,

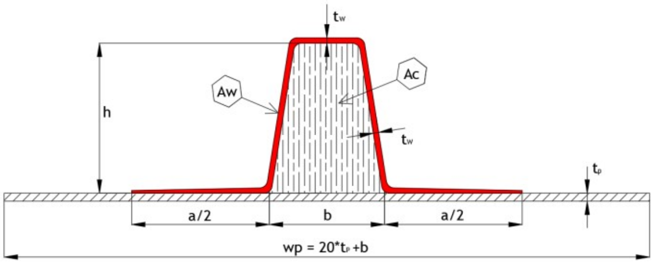

Fig. 3 Geometry of hat-type reinforcements, and effective breadth of plating.

Following common practice for this type of boat, reinforcements are hat-type, with a wooden core. In this project, the flange at top is equal to the width at base. Mechanical properties of the hard wood considered are presented in the appendix of this report.

Constraints: In the first phase of the optimization, bending and impact constraints are considered. Inter-laminar failure due to shear is not considered since, according to ISO [7], in cases of simple laminates, thicknesses are very small and shear flow is also small. To check the structure for impact, a minimum value of weight per unit area is specified, which depends on the velocity and the mass of the boat.

In the second phase, some constraints are specified differently. First the height of the stiffeners must be larger than its width in order to reduce spacing between them. Also, the strength of the structure is checked through bending stress in the flanges, shear stress in the vertical walls, and shear stress in the core. Also the bending stiffness of the reinforcements has a minimum value related to the maximum allowable deflection. Finally, to avoid local buckling of the flange, ratio of height to thickness and width to thickness must satisfy maximum values.

Maximum allowable stresses in the laminate: Following recommendations from ISO standard [7], allowable stresses can be a maximum of 40% of ultimate strength of the composite, which depends on mass content of fiber and mechanical properties of fiber. See Table I.

TABLE I

MAXIMUM ALLOWABLE STRESS OF CSM AND WR, N/MM^2

	Ultimate	Permissible
Normal stress, CSM	85	34
Interlaminar shear stress, CSM	17.25	6.9
Shear stress, CSM	62	24.8
Interlaminar shear stress, WR	14.1	5.64
Shear stress in wooden core	16.7	6.7

Objective function: to estimate the weight of the structure, it was categorized as four items: plating, transverse frames, longitudinal frames, and, keel. For all the reinforcements the area of overlap is included, as well as the weight of the cores. This calculation is developed for bottom and side of the hull.

D. *General procedure for optimization*

One option to solve minimization problems with discrete and continuous variables is the Branch and Bound method (BB), which begins by developing an optimization of continuous variables, [13]. This provides a starting point for the real problem, and a lower limit for the discrete solution. After this, one variable is increased to its next discrete value, and the optimization is developed with respect to the other variables. If the optimum value is greater than the previous, this variable is set to its next lower value and the process is repeated. If an improvement is found, the search continues in that direction until no new improvement is found. Then the variable is allowed to change value, subject to the actual limit, and the process is repeated for the next variables, until all discrete variables are examined.

This process is implemented with a special type of programming, Mixed Integer Nonlinear Programming, MINLP, which employs the Python language, [17]. In this project, the evaluation of the constraints and the objective function is implemented as functions which are linked with the Gekko library [18], which employs the BB method for the optimization. The functions employed in the optimization process are presented in Table II.

TABLE II

IMPLEMENTED FUNCTIONS FOR THE OPTIMIZATION PROCESS (PYTHON)

Area	Function (input variables)	Output
Bottom	span_space_stiff(nt,nl)	Spacings and unsupported length
	stiff_force(lul,sl,st)	Loadings on beams, shear forces and bending moments
	plate_force(bl,bt,lut,st)	Pressure, and, shear force, bending moment on reinforcements
	bottom_optimize(lp,bp,nt,nl, h,b,hT,bT,st,lul,sl,pbt,pbl,fdt, fdl,mdt,mdl,mdp)	Laminate, reinforcement dimensions and weight of bottom reinforcements and panel
Side	span_space_stiff_s(nls,st)	Spacing between longitudinals and unsupported length
	stiff_force_s(luls,sls,pb_l,d,t)	Loadings on beams, shear force and bending moment on longitudinals
	plate_force_s(bls,sls,b1ss, pb_p,d,t)	Pressure, shear force, bending moment
	side_optimize(nt,twl,twc,nls, sls,hs,bs,psl,fdsl,mdsl,mdps, luls,b_s,bside,psp,st):	Laminate and weight of side longitudinal reinforcements and panel
Keel	keel_optimize(KMat,Tmat, Wmat,b_t,Ep,Ec,dkeel,mdl, fdl,pbp,sl,lp)	Laminate and weight of keel

B. *Optimal lamination*

To keep the symmetry of the laminate, starting from the CSM 300 first layer, pairs of CSM+WR layers, of different weights are added. Then to complete the symmetry in the upper part of the thickness, the lowest layer is CSM type. In the lower half of the thickness an exact mirror of the upper completes the total thickness of the laminate. So, as design variables to optimize the laminate, number of CSM and WR layers of the different weights are considered, but as a constraint, the number of those layers must be the same, eqn. (2):

$$\sum_{i=1}^{4} N_{bi} - \sum_{i=5}^{6} N_{bi} = 0 , \tag{2}$$

where N_{bi} are the number of each one of the fibers available in the process. The first four are chopped mat and the last two are woven roving type.

IV. RESULTS OF THE OPTIMIZATION

A. *Description of the vessel*

Small craft which provide passenger interisland transportation in Galápagos are built with GRP, have open decks, and their lines include a hard chine to define the bottom area with a spray deflector (see Fig. 4). In Fig. 5, the general distribution of the vessel is presented, and main dimensions presented in Table III.

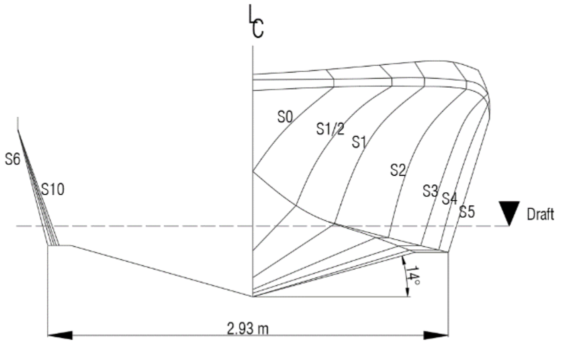

Fig. 4 Section plan.

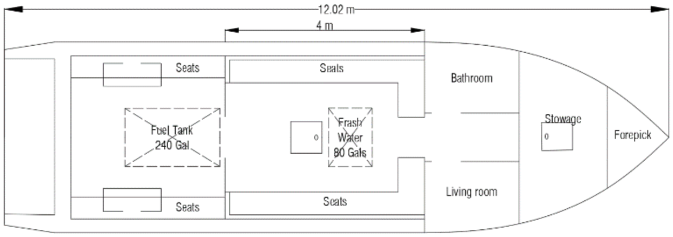

Fig. 5 General plan of the boat.

TABLE III
CHARACTERISTICS OF THE INTERISLAND SMALL CRAFT

L_{OA} [m]	12.02
B_{max} [m]	3.45
B_{chine} [m]	2.93
D [m]	1.51
T_{Design} [m]	0.51
Velocity [knots]	25
Total mass of boat [kg]	8,864
Deadrise angle, β [°]	14

The module of the hull to be optimized corresponds to the first space for passengers, with a total length of 4.0 meters, see Fig. 5. This module is assumed to be prismatic.

Applying formulations from ISO [7], the basic design pressure for motor craft in planing mode, on the bottom is calculated, $P_{BMP\ BASE}$. This parameter depends on the mass of the boat, length on waterline, L_{WL}, the acceleration of CG, and, the design category of the boat. In this case, the design category applied is B ("offshore"), which means that "it is suitable to operate in seas with significant wave heights up to 4 m and winds of Beaufort Force 8 or less". The acceleration of CG depends on L_{WL}, width at chine, deadrise angle, total mass, and velocity of the vessel. In this case, the resulting acceleration of CG is 2.67 g, where g is the acceleration of gravity. The basic value of the pressure is $P_{BMP\ BASE}$ = 95.33 kN/m^2, and it is corrected when applied on plate or reinforcements, depending on the panel area ratio. A similar procedure is applied to estimate load on the sides. A summary of these values is presented in Table IV.

TABLE IV
LOADING ON THE STRUCTURE

Element	Side, kPa	Bottom, kPa
Plating	27.36	47.24
Longitudinal reinforcements	7.76	45.91
Transverse reinforcements	10.72	54.88

B. *Results of the optimization process*

Since this is a process which includes integer and real variables, the variation of the objective function may present discontinuities in tendency. In Figure 6, the final optimum number of layers of the different types of fibers is shown for different trials; in the figure, using the scale on the right axis, the value of the objective function is shown for the corresponding initial set of values. In Figure 7, the optimal number of transverse frames are shown for different trials; it is noticeable that in all of the reported trials, optimum bottom and side structure does not include longitudinal girders. As expected, the final optimal design is not always repeated.

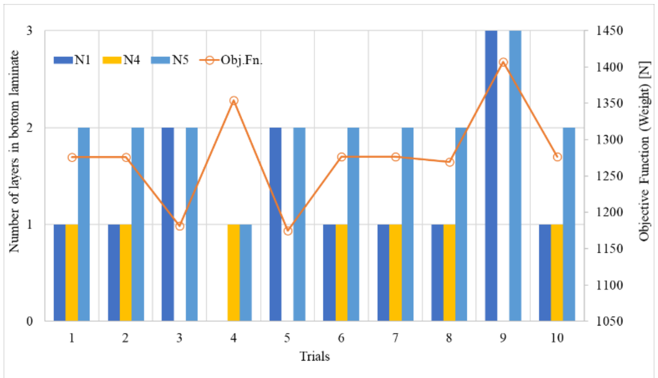

Fig. 6 Optimal number of layers in the bottom laminate.

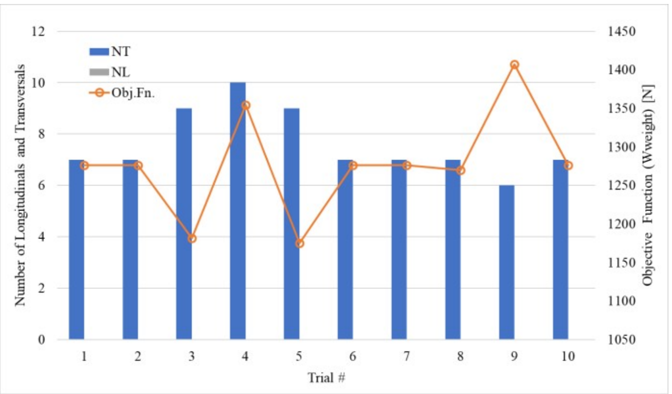

Fig. 7 Optimal number of transverse frames.

In Figs. 8 and 9, the final satisfaction factors of the different constraints are shown as functions of different initial design points. This factor is defined in two forms: the ratio of the actual parameter divided by the maximum allowable value, for example section modulus, inertia or vertical area, Fig. 8, and, for the case of buckling, the actual parameter divided by the value for instability, Fig. 9. From the results, it is determined that failure by approximating yielding from bending of the plate, shear of the wooden core, and the web, are the most common failure modes of the structure. From Fig. 9, in the case of buckling, none of the two constraints considered are close to failure.

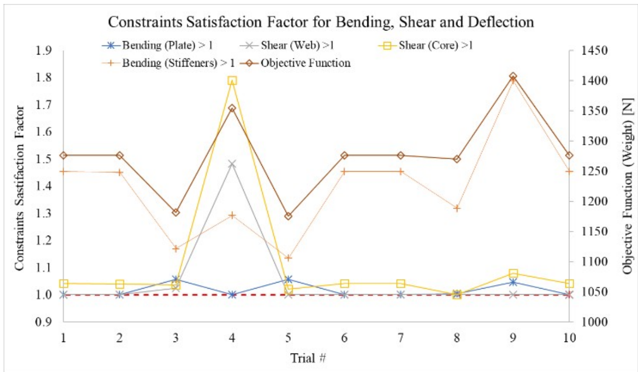

Fig. 8 Constraint satisfaction factor in bending for different trials.

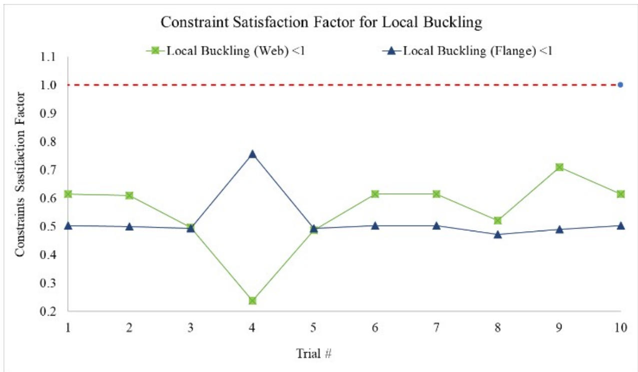

Fig. 9 Constraint satisfaction factor in buckling for different trials.

From the local optimal obtained starting with different sets of initial values for the design variables, an optimum is chosen. The optimum design characteristics can be seen in Table V, where they are compared with the current design.

TABLE V
THICKNESSES OF THE DIFFERENT LAYERS

Parameter	Current	Optimum
Bottom laminate (half thickness)	C4.5-C4.5-W8-C4.5-W8-C4.5	C3-C3-W6-C3-W6-C3
Side laminate (half thickness)	C4.5-C4.5-W8-C4.5-W8-C4.5	C3-C3-W6-C3-W6-C3
Number of transversals	6	9
# of longitudinals (one side)	1	-
Transversal laminate	C3.5-C3.5-C3.5	C6-C6-C4.5
Longitudinal laminate	C3.5-C3.5-C3.5	-
Longitudinal frame section	15x5 cm	-
Transversal frame section	5x5 cm	5.7x4 cm

Notation: C4.5: CSM of 450 gr/m^2, W8: Woven roving of 800 gr/m^2

Comparing results, it looks like installing more transversal frames with more stiffness allows a reduction in the thickness of the plating. This ends up in a reduction of the weight of the structure.

In Fig. 10, results of the weights are separated into groups, and plating, reinforcements and keel are shown. In the first two groups, weight is reduced compared with the original design. In the case of the keel, an increase in the stiffness of this element will compensate for there being no longitudinal girder in the optimized structure. As a final result, the weight of the module is reduced in about 20%.

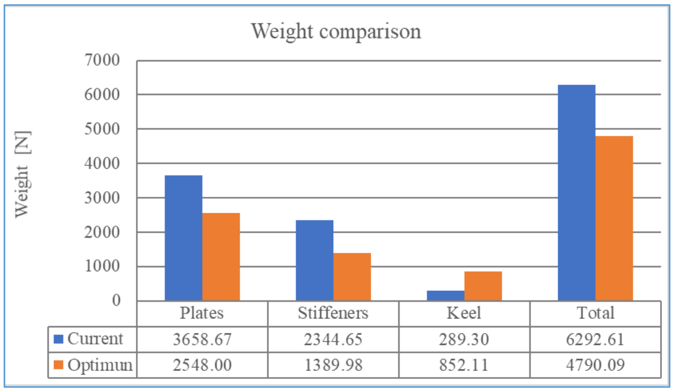

Fig. 10 Optimized weights compared with the current design.

Since the ISO calculations [7] included some simplifications for the secondary structure, it is necessary to complete the procedure with a finite element analysis, which follows. Also, results of an optimization for the boat operating at 30 knots are included in the appendix.

V. FINITE ELEMENT ANALYSIS OF THE STRUCTURE

It is always recommended to develop a full analysis of a structure. Because of the complexity of the geometry, the only option is the finite element method. In this case, the module of ANSYS software for composites is employed to prepare the model of the composite structure, and then the Static Structural module is employed to calculate the response to the equivalent static pressure.

The *Setup* of the finite element laminated structural model is completed in the following way:

Material Data: In the option *Fabrics*, the different option for the utilized fabrics are defined with its corresponding thicknesses (CSM300, CSM350, CSM450, CSM600, WR600 y WR800), see Table V. In the option *Stackups*, the sequence of the laminate is defined for bottom, side and reinforcements.

Rosettes: For the plates, *x* local axis is defined in the forward-aft direction of the hull, and, *y* is in the beam direction. For the web of the keel, *x*-axis points in the forward-aft direction, while *y* points upwards. For the flanges of the reinforcements, directions are similar to the plating.

Thicknesses for each one of the fibers are listed in Table VI. Equation (1) is applied together with the standard values of fiber mass content previously mentioned.

TABLE VI
THICKNESSES OF THE DIFFERENT LAYERS

Fabric and mass/area	Thickness [mm]
CSM300	0.7005
CSM350	0.8173
CSM450	1.0508
CSM600	1.4010
WR600	0.7760
WR800	1.0347

Some details of the structural model are shown in Fig. 11, where 25 elements are employed to represent the height of the keel. The discretization of the core of the reinforcements, keel and transversals, is developed with solid elements. The height of the transverse frames was discretized with 5 elements (see Fig. 12). In both figures, the overlap of the laminates of the reinforcements may be noticed, with lengths specified by ISO standard [7]. In Fig. 13, the full structural model of the 4-meter long module is presented.

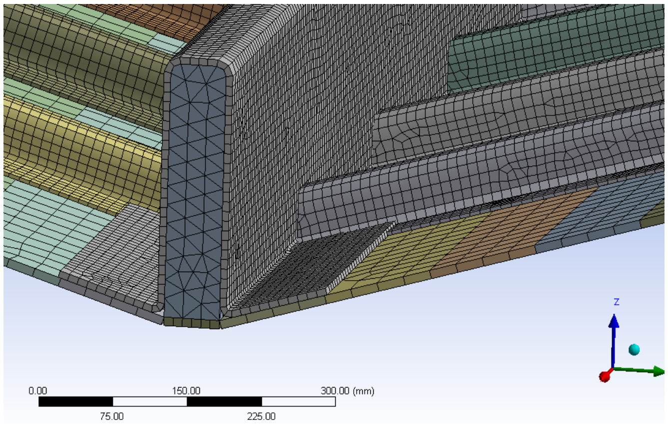

Fig. 11 Details of the model of the keel with its laminate overlap on each side.

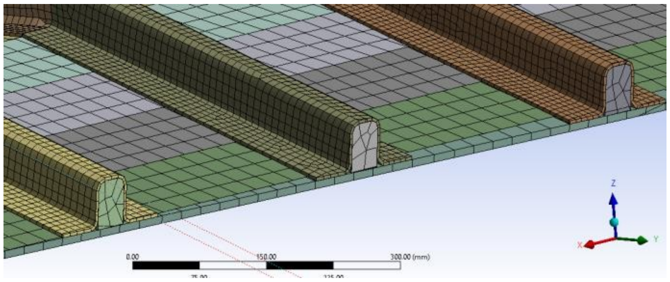

Fig. 12 Details of the model of the transversals showing overlap on each side.

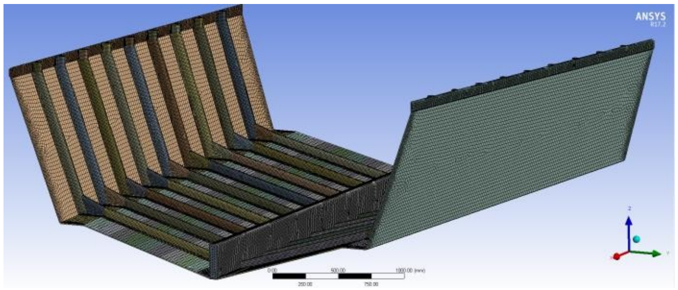

Fig. 13 Details of the finite element discretization of the structure.

Loading on the structure: Following the classical work by Heller and Jasper [19], pressure is considered with a linear variation in the longitudinal and a half-sine distribution in the transverse direction. From local experience, damages on the sides of small crafts are frequently reported, so this distribution of pressure is applied centered at *B/4*, Fig. 14, with acting load on the side of the vessel, named load condition 1. As a second load condition the distribution is applied centered at the keel, Fig. 15 of the vessel, load condition 2, which is transversely symmetric.

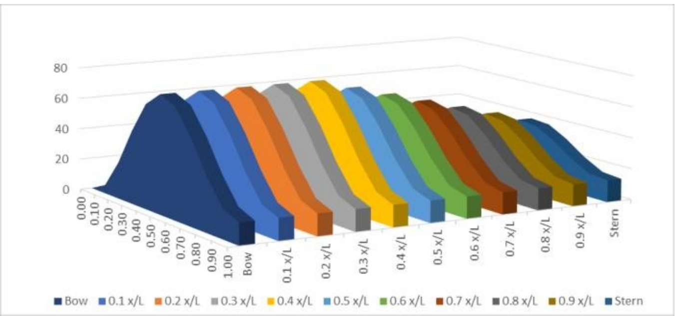

Fig. 14 Load condition 1, pressure centered at *B/4*, kN/m^2.

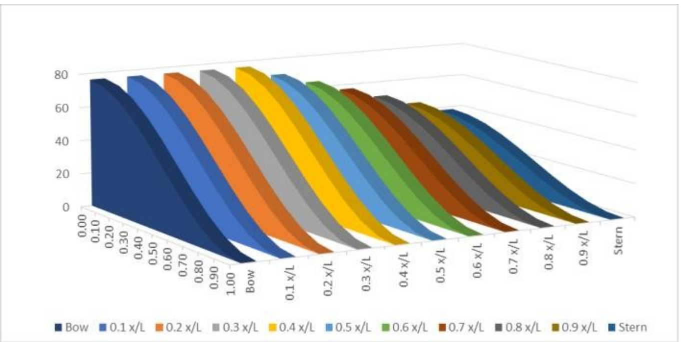

Fig. 15 Load condition 2, pressure centered at the keel (symmetric), kN/m^2.

To complete the model, joints in both end transverse sections, forward and aft, are considered as clamped. In the following figures, results from the finite element analysis are shown. In Fig. 16, equivalent von Mises stresses are very high at the points of support, and are thought to be due to the clamped assumption. Maximum deflection in load condition 2, that is symmetric impact on bottom, is about 12 mm, as shows Fig. 17. Normal stress levels at top and bottom faces of the bottom plating in Fig. 18 in the central part of the module present small values when compared with the allowable stresses. In this figure some very high values are also developed in the clamped ends. Normal stresses in the transversal stiffeners are shown in Fig. 19, with positive values in the flanges of the central region. This is expected if the frames are thought of as clamped beams with load pointing upwards.

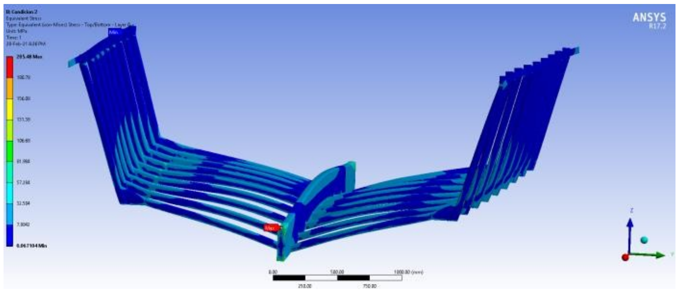

Fig. 16 Distribution of equivalent Von Mises stress of the elements of the reinforcement cores, load condition 1.

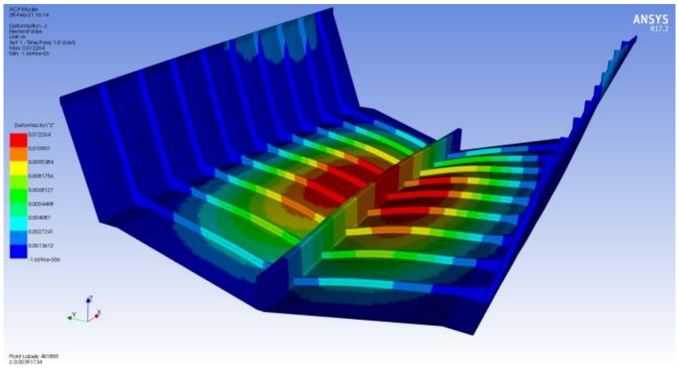

Fig. 17 Distribution of the vertical deflection, load condition 2.

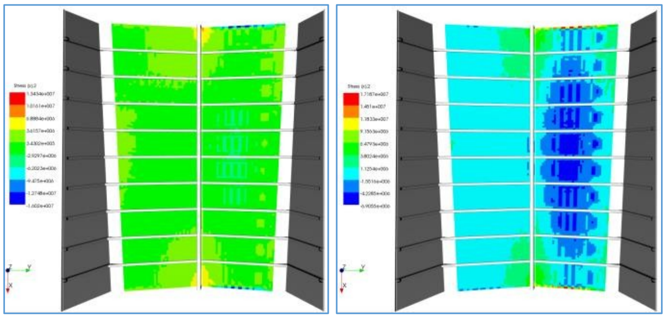

Fig. 18 Bottom panel normal x-direction stress [Pa], at the upper (left) and bottom faces (right), load condition 1.

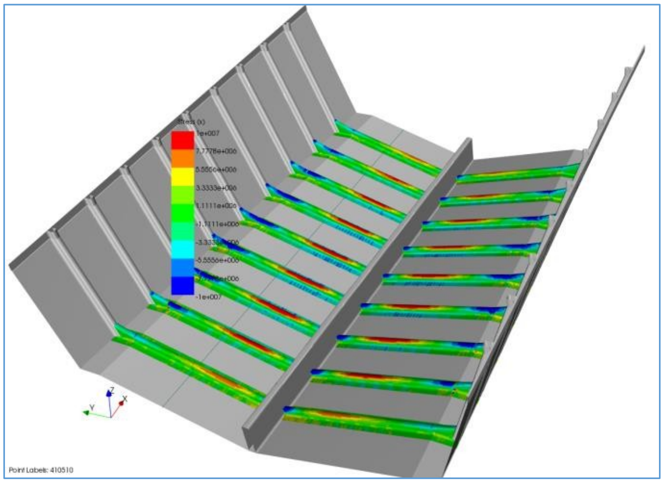

Fig. 19 Normal x-direction stress in the bottom transversal stiffeners, at the bottom layer, load condition 2.

In Fig. 20, normal stresses on the laminate of the keel are shown. In the flange of the central zone of this longitudinal element, stresses are positive, as it were a clamped beam under uniform load. Maximum values towards the clamped ends are about -10 N/mm^2.

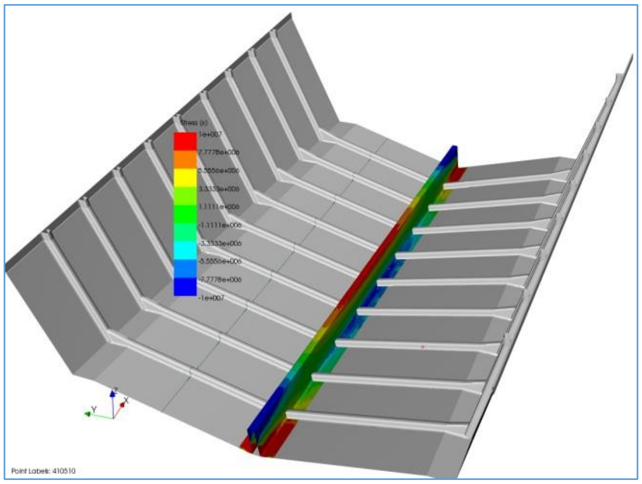

Fig. 20 Normal *x*-direction stress in the keel, N/m², at the bottom layer, load condition 2.

In Fig. 21 normal stresses in the longitudinal direction are shown as function of the vertical position in the bottom plating, between transverse frames at *y=B/4*. Normal stresses show jumps at the different layers of the laminate because of the different mechanical properties of the fabrics. Inter-laminar stresses may be estimated as the difference between normal stresses in the different layers, and as may be seen, are very small.

In Figs. 22, 23 and 24 of the appendix, results of the optimization process are presented for a velocity of 30 knots. It is found that no bottom longitudinal girders are needed, and even with the increase in keel size, the weight is still reduced.

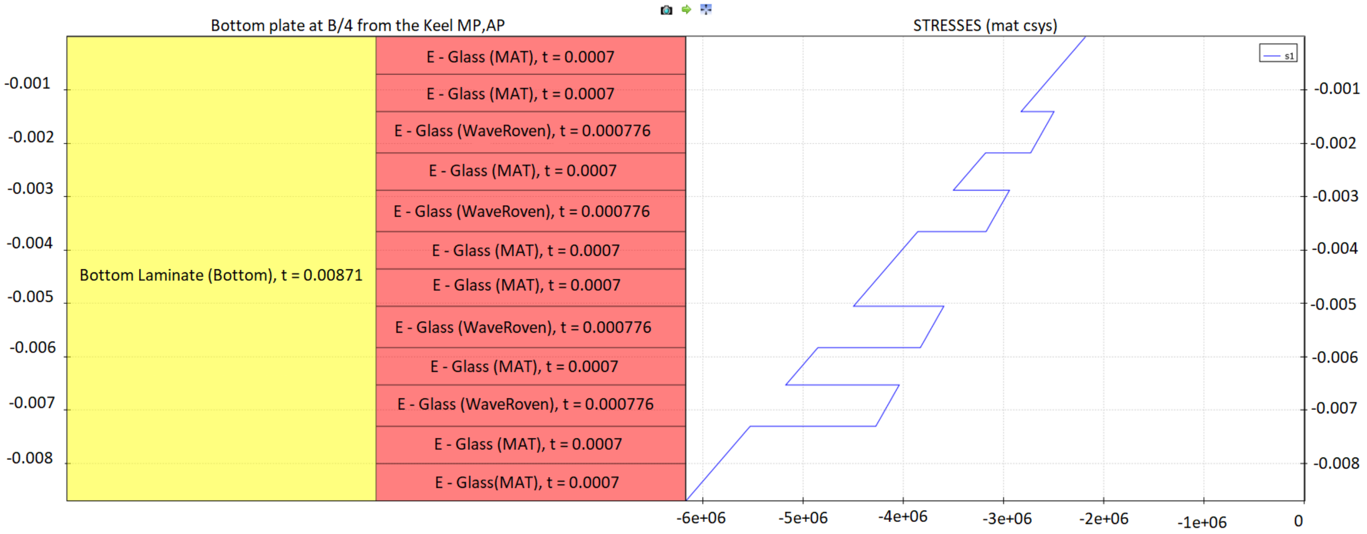

Fig. 21 Distribution of longitudinal normal stresses of a bottom panel, load condition 2, N/m²

CONCLUSIONS AND RECOMMENDATIONS

This paper presents a method to obtain an optimized structure module built with fiberglass reinforced plastic for a high-speed small craft for minimum weight. The optimization algorithm combines integer and real variables, and the restrictions include the recommendations from ISO standard. The laminate includes some recommendations from local practice, which allows simplification of the optimization process. Sides members of the hull structure include the same number of transverse frames, but are allowed to select the required number of side girders and define their own laminate. Final optimal structure shows reduction in weight, by selecting a higher number of transversal frames together with an increase in its height; in the present case, optimal structure does not include longitudinal girders. This higher number of frames reduces the shortest sides of the plating panels, and as a consequence, decreases the required thickness. Also, greater height of the frames allows to reduce the thickness of its laminate.

The structure module is finally analyzed with finite elements and with its ends categorized as clamped. This introduces extra rigidity in the supporting areas, and as a result artificial concentration factors of the stress. It is recommended to compliment the analysis of the whole boat structure, which includes modules of different lengths. Also, it is recommended to apply a similar procedure to the aft region where the weight of the outboard engines is structurally very demanding.

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APPENDIX

PROPERTIES OF FIBERS

	Fiber Content	**Young's Modulus [N/mm^2]**	**Ultimate tensile strength [N/mm^2]**	**Ultimate compressive strength [N/mm^2]**	**Ultimate flexural strength [N/mm^2]**	**Intralaminar shear strength [N/mm^2]**
CSM	0.3	6400	85	117	152.18	17.25
WR	0.48	13240	182.92	144	222.66	14.1

PROPERTIES OF CHANUL WOOD FOR CORE OF STIFFENERS

Property	**Value**	**Units**
Density	870	kg/m^3
Isotropic elasticity		
Young modulus	1.724E+10	Pa
Poisson ratio	0.229	
Compressibility modulus	1.06E+10	Pa
Shear modulus	7.01E+09	Pa
Isotropic stress limits		
Yield stress in tension	1.55E+08	Pa
Yield stress in compression	8.06E+07	Pa

OPTIMIZATION FOR SPEED OF 30 KNOTS

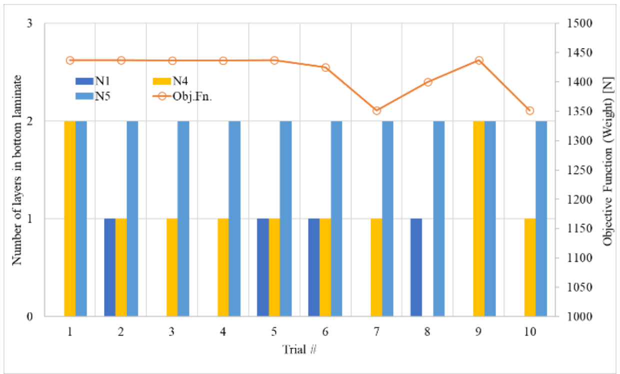

Fig. 22 Optimum laminate of bottom plating.

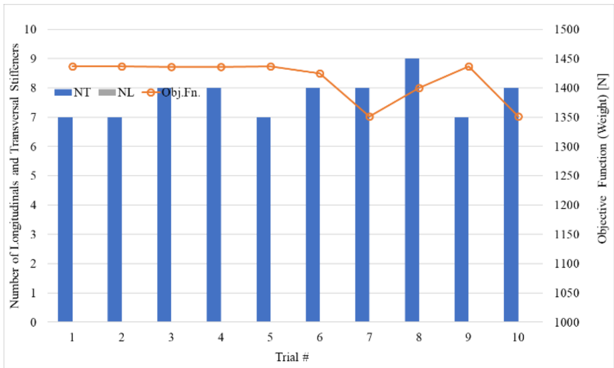

Fig. 23 Number of transverse frames.

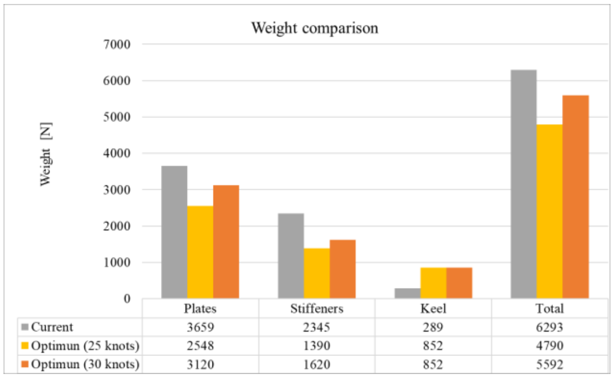

Fig. 24 Resulting weight of the module.