MATHEMATICAL OPTIMIZATION OF CONSTRUCTIVE PARAMETERS IN THE BEHAVIOR OF VERTICAL AXIS WIND TURBINES WITH STRAIGHT BLADES

Manuel F. Mejía De Alba
Ingeniero Mecánico, M.Sc.
Research engineer - Senior lecturer
Universidad Central
mmejiaa2@ucentral.edu.co

Luis Eduardo García Fernández
Ingeniero Mecánico, M.Sc.
Research engineer – Senior lecturer
Universidad Libre
luise.garciaf@unilibrebog.edu.co

Mónica A. Gutiérrez
Mechanical engineer
moniangual@gmail.com

ABSTRACT

Due to researches developed on wind Energy, it has been possible to note the influence of different constructive characteristics (Alignment angle, height, radius and solidity) over the power coefficient Cp. This paper proposed an optimization method, which objective is maximizing the Cp integrating all geometric characteristics mentioned before. The importance of this work is based in the fact that the most part of results of research actions on this field have been obtained through wind tunnel experiments, in which each parameter is individually evaluated. The computational models developed (CFD) have been focused in demonstrate their validity comparing their results with the dates obtained experimentally. Nevertheless, it is hard to find a research which its objective has been determine the influence of different constructive characteristics, studying its behavior simultaneously. The calculation procedure was based on selecting geometrical parameters to the optimization model. Considering that Cp is different for each TSR value and it is the parameter to optimize. The objective function was defined as the area below the curve Cp vs TSR (Tip Speed Ratio). As starting point were taken the parameters cited by several authors (relation HR, solidity S, alignment angle γ,). The results provide evidence of studying simultaneously the influence of constructive characteristics on which Cp depends, allow reach the same performance as the horizontal axis wind turbine and besides overcome problems in the starting, which is the main cause for disinterest in investigating vertical axis turbines.

KEYWORDS: Wind turbine, vertical axis, constructive characteristics, aerodynamic airfoil.

RESUMEN

Debido a las investigaciones desarrolladas en energía eólica, se ha podido ver la influencia de diferentes características constructivas (ángulo de alineamiento, altura, radio y solidez) sobre el coeficiente de potencia C_p. Este artículo propone un método de optimización, cuyo objetivo es maximizar el C_p integrando todas las características geométricas mencionadas anteriormente. La importancia de este trabajo está basada en el hecho que la mayor parte de los resultados de las acciones de investigación en este campo han sido obtenidos mediante experimentos en túneles de viento, en los cuales cada parámetro es individualmente evaluado. Los modelos computacionales (CFD), se han concentrado en demostrar su validez al comparar sus resultados con los datos obtenidos experimentalmente. Sin embargo, es difícil encontrar un estudio cuyo objetivo sea determinar la influencia de las diferentes características constructivas, estudiando su comportamiento simultáneamente. El procedimiento de cálculo fue basado en la selección de parámetros geométricos para la optimización del modelo. Considerando que Cp es diferente para cada valor de TSR y este es el parámetro a optimizar. La función objetivo fue definida como el área bajo la curva Cp vs TSR. Como punto de partida se tomaron los parámetros citados por diversos autores (relación HR, Solidez S, Angulo de alineamiento γ). Los resultados obtenidos dan evidencia de la influencia del estudio simultaneo en las características constructivas de las cuales el Cp depende permiten alcanzar valores similares de eficiencia a los obtenidos en turbinas eólicas de eje horizontal y además se superan problemas de arranque, los cuales son la principal causa del desinterés de investigación en turbinas de eje vertical.

PALABRAS CLAVES: Turbina eólica, eje vertical, características constructivas, perfil aerodinámico.

1. INTRODUCCION

At present, the use of fossil fuels has led to environmental problems such as greenhouse gases and acid rain, which are related to harmful emissions processes obtained from traditional energy generation. As an alternative solution, interest has focused on the generation of renewable energy. Within these wind generation has great potential. This resource has been exploited for several centuries in applications such as navigation, milling grain and pumping water; from the middle of the twentieth century have developed applications that seek to generate electricity from this resource.

The equipment used for this purpose is called wind turbines and rely on aerodynamic principles to operate. The vertical straight bladed turbines operation based on the principle of aerodynamic lift, are known as Darrieus turbines, vertical axis are considered due to its rotation axis is located vertically. They are characterized by get going with winds from any direction without being redirected. In addition, its constructional arrangement allows mounting of the generation and transmission systems in the soil. However are machines that have less performance than a horizontal axis, although this is offset, for ease of operation at low speeds [13].

Within the design of the constituent parts of a wind turbine rotor design plays a key role in determining the efficiency of the team, is why it is important to understand the influence of construction parameters (angle, height, radius, profile aerodynamic and strength) in the power coefficient Cp, which is the percentage of wind energy extraction can achieve the turbine. In the case of vertical axis wind turbine blade straight, most extraction has been 39% in the VAWT 360, tested in Britain 20 years ago [2], one of its biggest drawbacks is that have a low yield TSR low which does not allow the TSR accelerate to higher energy extraction. This has caused disinterest in turbines such as, the horizontal axis has achieved an efficiency of up to 48% and has no problem at start [3].

Despite these disadvantages, studies have been developed which assesses each of the constructive and operating parameters, in order to overcome this situation and take advantage they have on the horizontal axis, among which are the following: easy access to electrical and mechanical maintenance and repair, there is not need a positioning device relative to the wind direction and efforts on blades are smaller due to in vertical turbines these are not found under the cantilever [3].

Among the main research efforts are: the development of computational models to predict the rotor power coefficient at different TSR (Howel and Qin [4]), the evaluation of the number of blades recommended considering construction costs, strength / weight cyclic variations of the forces acting on them and influence on *Cp*, (Kirke [3], Paraschivoiu [6], and M. El-Samanoudy, et al [8]), as well as the implementation of experiments and mathematical models to predict the behavior of different airfoils (Kirke [3] M. El-Samanoudy, et al [8] and Mejia et al. [11]).

Now it is proposed a mathematical optimization procedure integrates several building parameters in order to observe its influence on the power coefficient *Cp*.

2. OPTIMIZATION

As mentioned above, the power coefficient *Cp* is the efficiency with which the turbine extracts energy from the wind, and as such depends on the constructive characteristics of the rotor. The calculation procedure for the design is shown in Diagram 1, the highlighted items are selected geometric parameters for the optimization model. As can be seen, the *Cp* depends on the design stage, and therefore each of these parameters. Given that the *Cp* is different for each *TSR* is the parameter which is to be maximized, the objective function was defined as the area under the curve *Cp* vs *TSR* as shown in Equation 1.

$$F.O. = \int_0^{TSR_{máx}} Cp\ dTSR \tag{1}$$

Where:

F.O.: Objective function
C_p : Power coefficient.
TSR: Tip Speed Ratio.

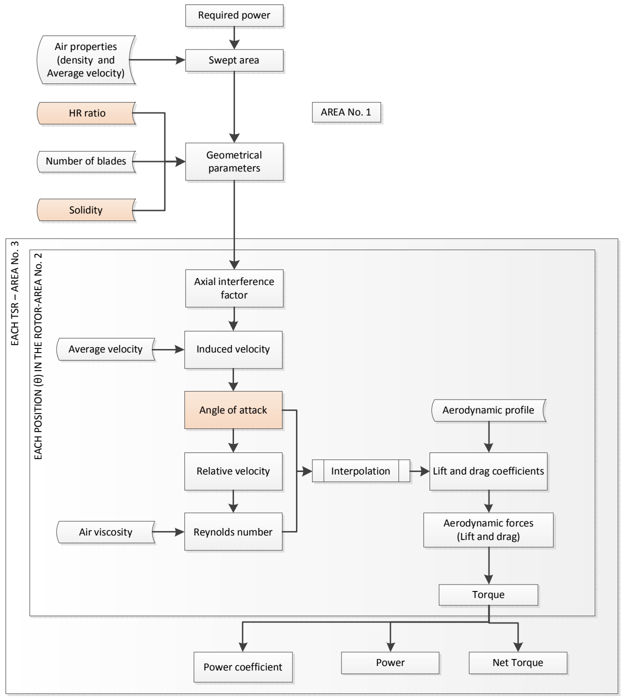

Diagram 1. Flowchart of the calculation process of the turbine.

2.1 Parameters to optimize

The model optimization was performed taking as its starting point the constructive parameters recommended by different authors. These parameters and their corresponding values are listed below:

Ratio HR: It is the ratio between the height and radius of the impeller, the value of this parameter is set after getting swept area. To vertical axis turbines, this area is rectangular and corresponds to the projection of a cylinder in the axial plane of the turbine. The swept area is determined by Equation 2.

$$A_b = 2\,R\,H \tag{2}$$

Where:

A_b: Swept area.

R: Rotor radius.

H: Rotor height.

To determine the relation *HR* is not easy, as they must take into account other variables that modify together with the height and radius affecting the performance of the rotor. Bastianon [9] recommends a ratio *HR* = 2.

S Solidity: It is the ratio of the swept area and the area occupied by the turbine blades. Can be used as an indicator of the amount of material required to construct the rotor. For constant section blades is defined as:

$$S = \frac{N\,c\,H}{A_b} = \frac{N\,c}{2\,R} \tag{3}$$

Where:

N: Number of blades.
c: Blade chord.

Some authors have established empirical relations and graphs to determine the solidity at optimal *TSR* [9] [11], Kirke [3] recommends a value of *S* = 0.2.

Angle alignment: It is formed between the tangent of the circle describing the motion of the turbine and the airfoil chord [9]. That is, a value of $\gamma = 0°$, means that the airfoil chord coincides with the direction of the tangential velocity. Of this parameter depend directly aerodynamic forces generated on the profile, because it modifies the effective angle of attack which does the same. In Equation 4 and Figure 1 show the relationship between the angles.

$$\alpha' = \alpha - \gamma \tag{4}$$

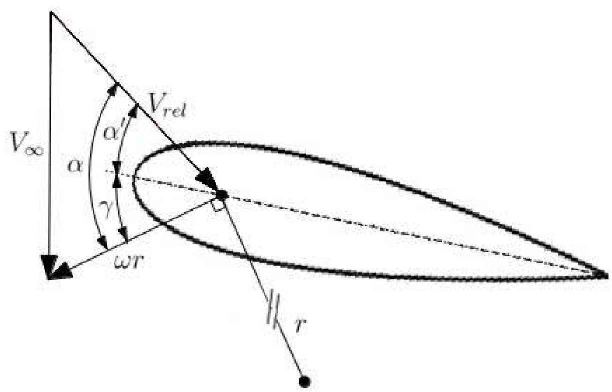

Figure 1. Speeds triangle representation.

2.2 Fixed parameters

The parameters discussed below are not defined as optimization variables in the model, nevertheless, it is worth defining because they also influence the efficiency of the turbine and its structural behavior:

Airfoil: Generally symmetrical airfoils are used, however, can also be used as the curved avoid problems in startup and increase efficiency of the rotor, its disadvantage is that the drag and lift coefficients are not within range Reynolds numbers and angles of attack required for the calculation of these coefficients depend on the aerodynamic forces generated by the profile (see diagram 1). Mejía et al. [11] validated a method for obtaining the coefficients missing, which was used to complete the S1210 curved profile data and to compare their behavior with symmetrical NACA 0018 profile. Best performance was obtained with the profile S1210 and he was the one selected.

Number of blades: The amount of energy produced by a wind turbine depends on the wind speed and the wiping area, not the number of blades. Turbines with a single blade can be as efficient as those having two or more blades and can extract the same amount of wind kinetic energy into mechanical energy and turn the shaft [4], therefore, this parameter have no significant influence on the *Cp*. Nevertheless, with 1 or 2 blades are significant variations in size and net force on the main axis of the rotor, causing undesired vibrations in it and reducing its life. When using 3-blade rotor is balanced to avoid the aforementioned problems, therefore, 3-blade rotor is selected in this study.

2.3 Optimization constraints

Defined these parameters, we proceeded to determine the constraints of the optimization process, these are:

Cp** $_{TSR\ 0,\ 1\ and\ 2}$* $\geq$ ***0: This constraint is defined because the starting issues of vertical axis wind turbines arise when *Cp* is negative, this means that the net torque is exerted in the opposite direction of the movement expected rotor. At present this phenomenon, the turbine starts but will not accelerate their movement to achieve maximum extraction *Cp*.

Although one can assume that this problem is overcome by using curved profiles, we wanted to ensure that this condition is met, since it depends on the rotor to get maximum performance. This usually presents *TSR* = 0, 1 and 2.

$-90° \leq \gamma \leq 90°$: This range was determined taking into account the extent feasible can have the alignment angle.

$RPM_{Cp\ max} \geq 100$: The dimensions of the turbine indirectly dependent rotor speed. As the speed increases the radius decrease and deviate from the nominal value of the generator selected. This restriction is defined in order that revolutions are as close to the nominal generator, looking as simple as possible in the transmission system. Given that the radius and height of the turbine must allow easy implementation of the rotor and, if possible, to prevent transmissions between the rotor and the generator.

HR ratio > 0* and *Solidity S > 0: It was wanted to ensure that the HR and solidity ratio S were positive. Due to *Cp* is being maximized the radius tends to be the greatest possible and therefore the ratio HR and strength tend to zero.

3. ANALYSIS OF VARIABLES TO OPTIMIZE

In general, the ideal behavior of *Cp* is when its value does not change significantly in the range of TSR broadest possible meaning that is less sensitive to the natural variation of the wind.

As mentioned above, there are analyses where variables are considered in an isolated way. Below is the influence of these parameters on the extraction coefficient *Cp*.

3.1 Ratio HR

The influence of ratio *HR* is not as significant as that of the other variables, as can be seen in Figure 2. Nevertheless, it can be seen that the greatest *Cp* relationship occurs when *HR* is smaller, this is because it decreases as the radius increases, as well as the torque generated and the power extracted by the rotor.

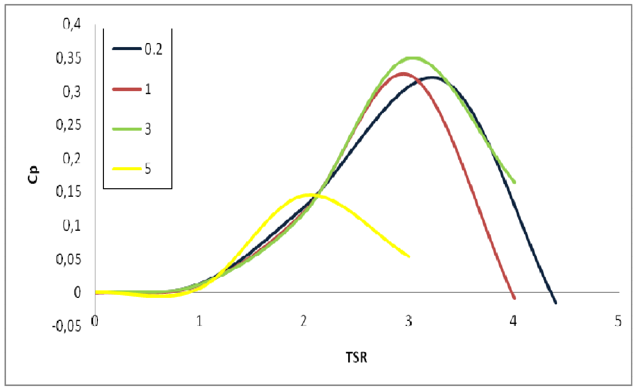

Figure 2. *HR* influence on *Cp*. The colors indicate different values of *HR* considered.

3.2 Alignment angle γ

As seen in Diagram 1, the alignment angle directly dependent drag and lift coefficients. If the angle increases in magnitude it approach to the aerodynamic stall, drag is considerable and the tangential force, torque and *Cp* decrease. It should find the angle that allows maximum closer to the lift in most positions. Figure 5 shows the influence of the alignment angle on *Cp*.

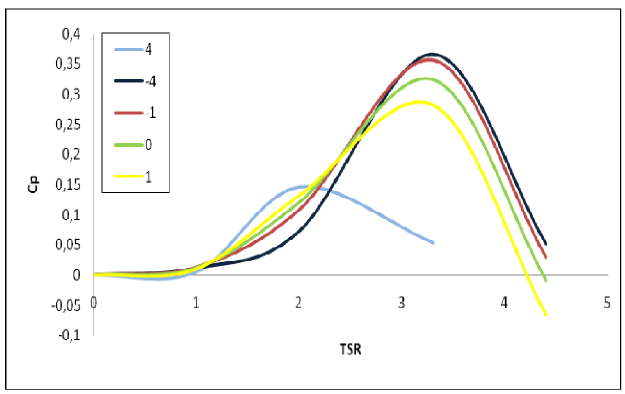

Figure 3. influence on Cp. The colors indicate the different values of the angle considered.

3.3 Solidity *S*

Figure 4 shows the graph of *Cp* vs *TSR* with different strength values. As it increases, the area occupied by the blades in the rotor is increased, and the degree of braking therein. This does not allow to maximize the extraction of energy due to the drag becomes relevant. At values less than S = 0.1, the negative torques that are generated when the blade is downstream are significant and the net torque is lower, the advantage present in these *TSR* is that the sensitivity to changes in wind is very low.

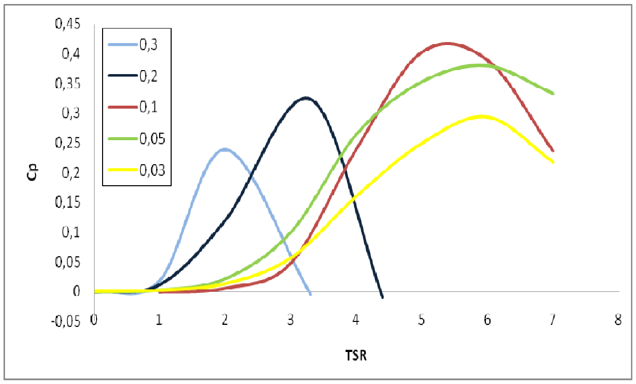

Figure 4. Influence of S on Cp. The colors indicate different values of S considered.

4. OPTIMIZATION RESULTS

After define constraints and variables to consider and observe the influence of them on the *Cp*, it is proceeded with mathematical optimization using the Matlab software. The following table shows the results for the case of a vertical turbine of 350 W.

Parameters	Obtained value
HR ratio	3.527
Alignment angle	-0.276
Solidity	0.105

Table 1. Results obtained after optimizing the model.

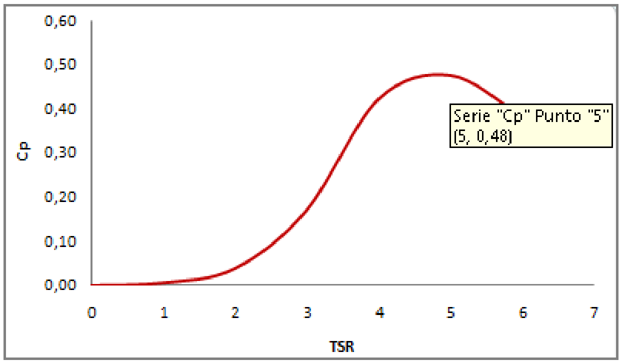

Figure 5. Optimized turbine efficiency.

5. CONCLUSIONS

The optimization model proposed as study simultaneously the influence of the structural characteristics of which depends on *Cp*, allow see that by obtaining the optimal values of these features can achieve the theoretical percentage of wind energy extraction reaches in horizontal axis turbines and also overcomes the problems in the boot up, which is the main cause of disinterest in investigating vertical axis turbines.

The results obtained with this optimization model can be used as starting point for any investigation concerning vertical axis wind turbine with straight blades, regardless of wind conditions or the power rating of the particular application.

The proposed model allows to reach an optimal configuration and compare it with experimental or computational models, It has a good cost / benefit ratio reflected in saving time, money and other resources.

According to the results obtained in this study, which show that the efficiency of vertical axis turbines with straight blades can reach values similar to those obtained for horizontal axis turbines could foster the construction and implementation of vertical axis turbines using its advantages, especially in rural areas that need low power installations.

6. REFERENCES

[1] P. Deglaire, et. al. "Analytical solutions for a single blade in vertical axis turbine motion in two-dimensions". *European Journal Of Mechanics B/Fluids,* vol. 28, pp. 506–520, 2009.

[2] C.A. Morgan, P. Gardner, I.D. Mays, M.B. Anderson, "The demonstration of a stall regulated 100 kW vertical axis wind turbine," *EWEC*, pp. 645, 1989.

[3] B. Kirke, "Evaluation of self-starting vertical axis wind turbines for stand-alone applications". Ph.D. dissertation, Fac. of Engineering and Information Technology, School of Engineering, Griffith University , Australia, 1998.

[4] R. Howell, N. Qin, "Wind tunnel and numerical study of a small vertical axis wind turbine", *Renewable Energy* vol. 35, pp. 412 - 422, 2010.

[5] L.B. Wang, L. Zhang, "A potential flow 2-D vortex panel model: Applications to vertical axis straight blade tidal turbine", *Energy Conversion and Management,* vol. 48, pp. 454 - 461, 2007.

[6] I. Paraschivoiu, *Wind turbine design: with emphasis on Darrieus concept*, Canada: Polytechnics international press, 2002.

[7] M. Islam and D. Ting, "Aerodynamic models for Darrieus-type straight-bladed vertical axis wind turbines," *Renewable and Sustainable Energy Reviews* vol. 12, pp. 1087 - 1109, 2008.

[8] M. El-Samanoudy, et al., "Effect of some design parameters on the performance of a Giromill vertical axis wind turbine," *Ain Shams Engineering Journal*, vol. 1 No. 1, pp. 85 - 95, 2010.

[9] R. Bastianon, *Energía del viento y diseño de turbinas eólicas*, Buenos Aires: Tiempo de cultura ediciones, 2000.

[10] M.S. Selig et al., *Summary of Low-Speed Airfoil Data, vol. 1, Vol. 2, Vol.3*, Virginia Beach: SoarTech Publications, 1995.

[11] M. Mejía, et al. "Metodología de obtención de los coeficientes de sustentación y arrastre para un rango amplio de números de Reynolds y ángulos de ataque para aplicaciones en turbinas eólicas," *Avances,* vol. 8 No.14, pp. 53 – 60, 2011.

[12] L. E. García, et al. "Procedimiento de cálculo de turbinas eólicas de eje vertical de alabes rectos," *Revista de Investigación - Universidad de América*, vol. 4 No.1, pp 49-60, 2011.

[13] J. Valderrama, "Información tecnológica," *Centro de información tecnológica*. vol. 8 No 3, La Serena, Chile 1997.

Authorization and Disclaimer