

# **Problem Based Learning in Multivariable Differential and Integral Calculus for engineering course**

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## **ABSTRACT**

Problem Based Learning Active (PBL) is usually a methodology applied to small class. This article presents adapted Problem Based Learning Active (PBL) activities for large class applied on Multivariable Differential Calculus in Classical Calculus II courses for second year engineering students.

**Keywords:** Engineering Education, Multivariable Differential and Integral Calculus, PBL

## **RESUMEN**

Problemas de Aprendizaje Activo (PBL) es por lo general una metodología aplicada a clase pequeña. En este artículo se presenta problema Problemas de Aprendizaje Activo (PBL) para grandes clase aplicada sobre cálculo diferencial multivariable en clásicos cursos de Cálculo II para estudiantes de segundo año de ingeniería.

**Palabras claves:** Enseñanza de la Ingeniería, cálculo diferencial y integral multivariable, PBL

## **1. INTRODUCTION**

Since 2007, the School of Engineering Maua, in order to promote changes in the learning of their students, implemented an Institutional Project, called Project Evolution. This project was based on the concepts of project-based learning in general and proposed that activities being performed by students in groups to:

- promote greater integration between disciplines;
- develop skills of analysis and systemic view;
- promote communication skills and speech, as well as interpretation of contextualized problems;
- promote active student learning through projects.

Since then the project was implemented in Calculus II course (Baracat et al, 2008), (Baracat et al, 2012a), (Baracat et al, 2012b), (Baracat et al, 2012c). The goal is to present a work proposal coupled with active learning seeking a better understanding of the concepts of Multivariable Differential Calculus: Contour lines, Maximum Elevation and Depression points, Double integrals and Triple integrals with and without change of variables, Line Integrals, Conservative fields and potential functions, Flux Calculation, Stokes' Theorem, First Order and Second Order Differential Equations by the students, as well as their interpretation in the application of practical

situations, while intended to develop skills and competencies necessary for professional life of Engineering (Engineering Graduate Education And Research , 1985), (Engineering Education, 1995), , Learning And Understanding, 2002),( The Engineer of 2020, 2004), (Cardoso, 2012), such as the understanding of different cultures, foreign language skills, skills in oral and written expression, among others.

Students in basic sciences in engineering education needs to be continually encourage to improve theirs learning process. Many books brings examples of activities for single variable Differential and Integral Calculus, but there are not so many references for Multivariable Differential and Integral Calculus for engineering course. This work shows a Problem Based Learning Active (PBL) (Wood , 2000), (Seng , 2001),( University of Nottingham , 2003), (De Graaff and Kolmos,2003), (Prince , 2004), (Case, 2008), (Du et al, 2008),( Morell, 2012) for Multivariable Calculus using applied problems (as its interpretation and application of practical situations such as those presented in Fluid Mechanics, Strength of Materials, Electromagnetics, Heat Transfer and Physics) and computational tools.

The studied group consists of about 1,000 students per year splited between daytime and nighttime classes, with almost 90 students in theory classes and 45 students in exercises classes.

## **2. DESIGN OF PBL PROJECT**

During the past two decades we have been researching the calculus curriculum reform. Most surveys of this reform have been made in undergraduate teaching and other in high school. The research found that calculus students have a primitive concepts understanding of functions, trigonometric functions, continuity, limits, etc. (Ferrini-Mundy and Lauten, 1993), (Tall, 1996). They also noted that students have cognitive difficulties in algebraic and graphical functions representations (Schnepp and Nemirovsky, 2001). There is equally a difficulty in previous concepts such as trigonometry or algebra (Gomes and Vicente, 2005). Some researchers propose different approaches to teaching the calculus principles (Frid, 1994).

Researches show that the cognitive difficulties to the calculus learning arise from two distinct aspects:

- a) first related to the linguistic and representational aspects.
- b) second related to intuition.

Since the calculus courses are based on symbolic manipulation, it is not surprising that linguistic or representational factors give rise to cognitive difficulties. If we consider that students want to understand and make sense of the content they are studying, their intuitions play a key role to construct and to structure these concepts. Researchers suggest that the cognitive generalization processes and flexibility offered by an approach Krutetskian are extremely helpful (Norman and Prichard, 1994).

It appears that there is need to establish connections between different representations (concrete, visio-spatial, graphical, algebraic, numeric, etc.) to help students to understand the calculus concepts. The graphic visualization is an important tool to develop this understanding. One of the guiding principles of the Harvard Consortium Calculus text is the multiple representations "whenever possible topics should be taught graphically, numerically and analytically. The goal is to produce a course where the three pillars support the students' view on different angles" (Hughes-Hallett , 1990).

One of the most important points in Calculus research shows is that there should be greater emphasis on the conceptual learning using multiple representations and the connections before students immerse themselves in symbolic manipulations. There is a need to seek explanations and illustrations about the physical or geometrical tools that are developed, conceptually illustrating its applications and interpretations in the engineering field. "Calculus needs to be studied across many years as is done with geometry" (Kaput , 1994).

The proposed approach is based on the following key assumptions about learning and teaching:

- Conceptual learning leads to cognitive acquisition development of formal procedural operations. Lev Vygotsky argues that the development of formal operations depends on creating a successful learning environment, which includes the use of conceptual tools (Vygotsky , 1987). "The development of students procedural skills in the calculation is derived from a conceptual understanding of big calculus ideas."
- The general ideas to specific procedures is a key methodological tool for the design of a rigorous math curriculum conceptual (Davydov, 1990). Thus, the development of calculus understanding conceptual principles must be made from multivariable calculus concepts to single-variable principles.
- The cognitive-visual conceptualization (CVC) through the use of modeling and technology plays a critical role in learning the calculus basics principles.

But, how to motivate students to learn multivariable calculus?

The answer was writing scripts in a PBL model (Wood , 2000), (Seng , 2001),( University of Nottingham , 2003), (De Graaff and Kolmos,2003), (Prince , 2004), (Case, 2008), (Du et al, 2008),( Morell, 2012) that captivate most of them. So the principal idea was to use TV serials as a background script. In our case, in the first year of the project, was used "Lost" TV serial. So students lost in an island must use calculus knowledge to solve engineering problems.

So the following strategies were used:

- Team work
- Several and growing problems to solve
- Seek new knowledge
- Decision maker
- Real problems
- Open Script with different kind of solutions (sequential scripts with a background story)
- Integration with others basic sciences

The Calculus II Course is an annual course with two exposure fronts. Each has weekly classes during 100 minutes and they have an independent nature. For example, while one of them has derivation concepts the other develops integrals concepts. The classes are conceptual expositions and always look for to support the analysis on applied problems in Physics, Fluid Mechanics, Strength of Materials, Geometry, Heat Transmission, which serve as a link between what is being developed in Calculus with the technical knowledge basis already acquired in previous years. We apply individual student evaluation bimonthly to evaluate knowledge, mainly analyzing the important calculus concepts assimilation. In these evaluations are not allowed to consult notes or electronic calculators usage.

Theory Classes has nearly 90 students and exercises classes nearly 45 students. So the chosen solution was work during the exercises classes with small groups of students (5 students by group). The activities had to be done, by students, in extra-class time and they could look for helping during the last minutes of each class or during a specific week time schedule from teachers.

The full process have:

- 5 scripts for semester (almost one each 2 weeks)
- scripts with previous and future knowlegde (concepts that the students will have in future classes)
- oral presentation, with help of slides, at the end of semester
- evaluation process (at the end of each semester) : teacher (40%); himself and others team mates (30%); reports (30%)

### 3. RESULTS

The proposed background themes was changing during the last years: "The Perfect Wave Pursuit" (2007), "The Overcoming Challenges and Perfect Balance Pursuit" (2008), "The Bermuda Economic Progress Survey" (2009) , "The Sustainability Pursuit" (2010 );.... so the students always have new challenges. A briefly example is present in APPENDIX A.

Average, since this methodology was applied failures in Calculus II have decrease about 10%.

It could be observed that student performance get better after the project was included in course program. Teachers must have to adapt in the beginning but today it is usually executed by full teacher's team. The principal problem was the different approaches when making questions for evaluate student's efforts, but the problems were solved by preparatory teacher's reunion in the beginning of each semester. Students also, complains in the beginning because the dedicated time need to work in the project, but at the end they understand better and see the good results in grades and behavior.

### 4. CONCLUSIONS

This experience was used since 2007 and the approval rates have growing up, also others skills are developed like relationship skills, critical analyses and text abilities. Writing scripts that are connected in some way to TV programs that already are watched by students and applying concepts shows to be a good combination for those who intend to do an adapted PBL activity.

### APPENDIX A

The 2010 project included the Sustainability Pursuit. This is the description of a series of events that occurred during a visit by a group of students from a traditional engineering school to a farm that was structured to exploit natural resources for self-sustainability. Everything goes naturally to the moment they are harvested by a heavy rain that disables your location tool and destroys the trail by which they came. Conclusion: get lost in the Amazon jungle and are without communication with the rest of the world.

Below is present an example of background story of proposed scripts:

1st script: Geographic data stored on the laptop from searches made by Google earlier show a kind of relief. It is up to students within a set of functions that identify stipulated foot closest to the relief they had. They must also submit their contours and interpret what it means closeness and distance between contour lines.

2nd script: From the contours and set each of the cardinal points (axes XOY) each member of the group takes a particular direction from a pre-set point. It should describe what occurs in terms of the topography of land from the predetermined point in each direction considered. They also must determine the terrain slope from two contours considering a starting point about one and a direction of travel.

3rd script: Estimate from the contours of the X and Y coordinates of the points of highest elevation and greater depression. From the function used in the 1st script estimate the elevation and depression of their quotas. Make the calculation using Maxima and Minima. Determine the area of planting more suitable for a certain type of vegetables.

4th script: The group finds a small stream and is studying the possibility of installing a small hydroelectric plant. For both must estimate the flow stream with the help of a timer and a small container shaped circular paraboloid. It is estimated width of the bed and pick up a certain reference region whose size coincides with the nozzle diameter of the circular paraboloid. Measure the time to reach a certain dimension in the paraboloid (determination of volume per double or triple integral with change of variables). Proportionality is determined by the volume of water passing through the stream and from the measured time has the flow stream. From the gap between two points stream can determine the type of turbine, hydraulic power fall, and therefore the turbine to be installed (provided that establishes a certain hydraulic performance) the same as the power of the electric generator.

5th script: The group plans to build a board with pieces of stem shrubs. Remember the formula for resistance of materials related to moment of inertia. From the estimate of the weight they must support should determine the dimensions of the rectangular plank so that it supports the desired load (determining the moment of inertia and stresses acting against allowable stresses). (Calculation of moment of inertia by double integral).

6th script: Depending on the digit obtained from the composition of enrollment figures asked to find clean energy sources including: Solar, Wind, Hydro, Bagasse or Tide.

The group notes the existence of a depression in the ground that can be leveraged for the accumulation of water. With the help of the wind wants to build a windmill that can move barrels that are in the area of the farm to bring clean water to a water tower strategically located to supply the village. It is the group size the amount of barrels to water supply can be properly scaled to the daily per capita consumption. (Flow Calculation).

7th script: Sets up a route on the topography of the site conducted by one of the group members. From the mass of the integral for determining line integral, the work done by shifting your weight on well considered. Describe this work is tough or motor. Asked to determine the length of the various sections of the route using the length of curves Calculus I and length of curves given by parametric equations. Given the speed of movement of the human being asked to determine the time of this trajectory. Use software to perform the calculation and compare the results with algebraic. (Full line).

8 script: Built the windmill and inserted into the casks for the transportation of water into a cylindrical vessel which measures the elevation of the water level versus time accumulated, it is requested to determine the water flow inside this container. Perform calculations for triple integral, through algebraic and using software. Compare the results. Check if this flow is sufficient to supply the village consisting of a number of occupants. (The water consumption including hygiene and personal needs).

9th script: The group plays ball made from socks on the shores of a lake. It is known the diameter of the ball. Suddenly she falls into the lake and sinks. Using Gauss' theorem to demonstrate the Archimedes principle. Apply Gauss's theorem to calculate the flow of an electrostatic field created by an electrostatic charge located at a certain point in space. What conclusion do you get? The result was expected in the context of physics? (Flow Calculation).

10th script: With some casks built up by introducing shower holes in the bottom of vats. Water is obtained from the large reservoir and is accumulated inside the barrels. The amount of water is sufficient for a bath. Shows the equation that governs the flow of water through a hole (vena contracta). It is a first-order differential equation. Asked to solve it algebraically and with the help of a software. Compare the results. Provide a graphical solution of the problem using Excel. Interpret the result graph in the physical context of the problem.

11th script: is allowed to fall a certain ball solid material with known dimensions and density within a tank of oil (specific gravity known). About the ball three forces act, the weight force, the buoyant and viscous friction. Derive the differential equation of motion (linear differential equation of second order). Asked to solve the differential equation algebraically. Ditto with the help of a software. Compare the results between the two techniques. Provide the law hourly velocity and displacement. Make a graphical representation of these solutions and present a physical interpretation of the results in the context of this problem.

12th script: Is the outcome of the chapters, where the team that was lost is found by a rescue team.

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