Effect of Curvature on the Natural Frequency of a Riveted Plate

Malik Hocine
Mechanical Engineering Technology Program, Vaughn College of Aeronautics and Technology, Flushing, NY, USA, malik.hocine@vaughn.edu

Faculty Mentors:

Dr. Yougashwar Budhoo and Dr. Hossein Rahemi
Department of Engineering and Technology, Vaughn College of Aeronautics and Technology, Flushing, NY, USA, Yougashwar.budhoo@vaughn.edu

ABSTRACT

An analytical approach for finding the natural frequency of simply supported rectangular plate has been presented already (M.K Baharami, M. Loghami and M. Pooyanfar, 2008) as the complexity of the plate increases, so does the analytical solution. In some cases a closed-form analytical solution may not even be available; hence researchers seek a numerical approach tackle the problem. The objective of the project is to find the natural frequency of a curved rectangular plate using CATIA V5. The approach was further extended to solve for the natural frequency of a curved plate riveted around its edges and also investigate the variation of natural frequency as the "angle of curvature" changes.

It was found that the natural frequency of a curved plate varies linearly with the change in angle of curvature.

Keywords: Riveted plates, Curved plates, and Natural Frequency

1. INTRODUCTION

In vibration analysis, the natural frequency of a system is very important. If a system is given an initial disturbance and then left to vibrate on its own without any external force acting on it, the frequency at which the system oscillates is known as the natural frequency. If a system experiences an external excitation which coincides with its own natural frequency, the dynamic response of the system will be very large. This condition is known as resonance and should be avoided to prevent failure of the system.
Plates elements used in structural applications as membranes, shells or shear panels are in some cases riveted on to the structure. Solving for the natural frequency of these riveted plates by assuming that they are simply supported or fixed may not yield the most accurate solutions. Therefore the aim of this paper is to investigate the effect of rivets on the natural frequency of a plate in comparison to a simply supported plate. In addition, investigation is made into the effect of angle of curvature on the natural frequency of a riveted plate, and an equation is developed to relate them. The numerical tool used for this investigation is CATIA V5 (Nader G. Zamani, 2009).

2. MODELING

The dimension of the plate to be used in this investigation is given in fig 1. The material used for computational purposes is aluminum.

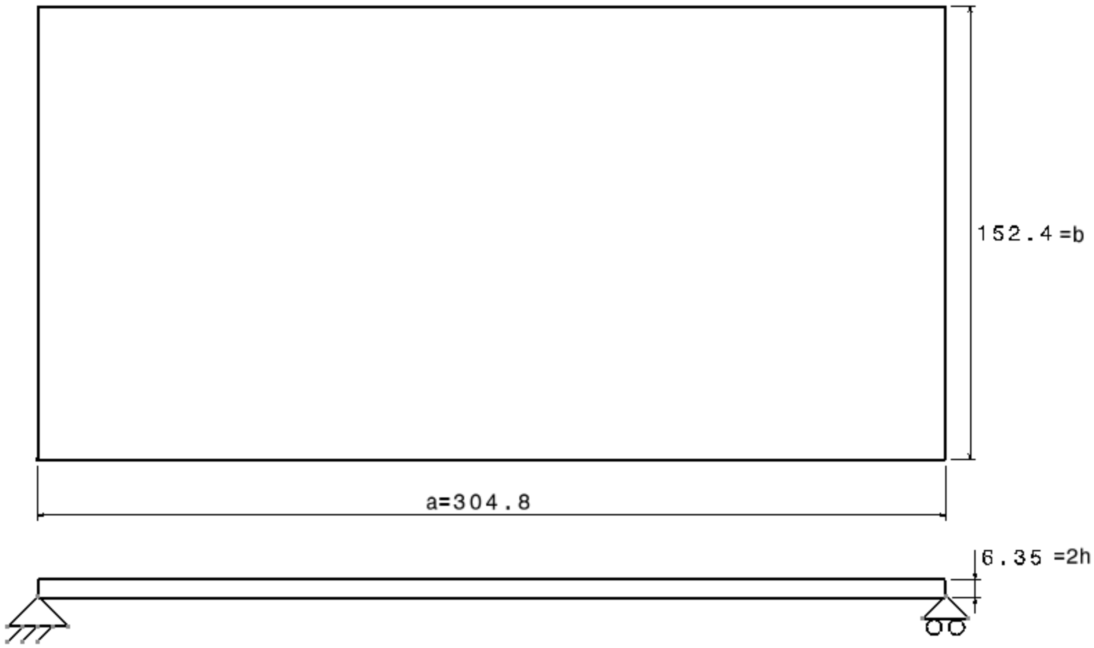

Figure 1: Rectangular plate simply supported (dimensions are in mm)

The plate shown in fig 1 above was analyzed analytically to obtain its natural frequency. The frequency was then used a baseline line value to check the accuracy of the numerical results. A thorough mesh refinement and convergence study was done to obtain the 'best' suited mesh size for the numerical study.

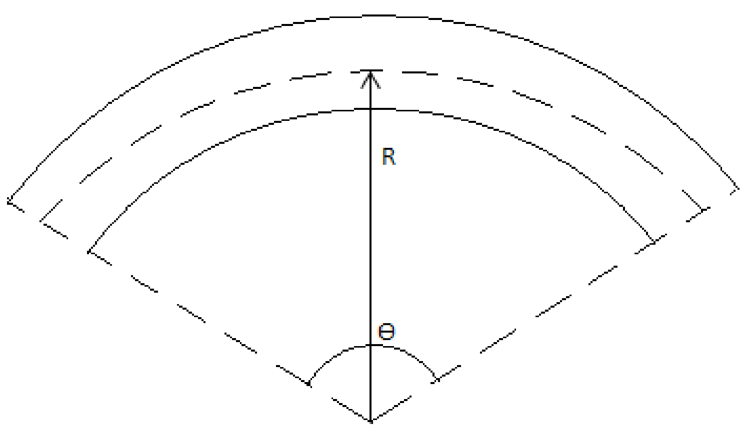

Figure 2: Curved Plate

The figure 2 above shows the schematic of a curved plate used to investigate the effect of natural frequency as a function of 'angle of curvature'. In this paper, "θ'' is referred to as the angle of curvatures. This angle was changed from 5 to 305 degrees to study the effect it has on the natural frequency of the plate.

2.1 Analytical Solution

The governing equation of an isotropic plate is given by equation (1)

$$D\nabla^2\nabla^2\omega = -2\rho h\ddot{\omega} \qquad (1)$$

The analytical solution to the natural frequency of a simply supported plate [1] which is given in equation (2)

$$\omega_{mn} = \sqrt{\frac{D\pi^4}{2\rho h}}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right) \qquad (2)$$

Where

$D = \dfrac{Eh^3}{12\left(1-\nu^2\right)}$ *Bending Rigidity*

E = *Elastic Modulus*

ρ = *Density*

ν = *Poisson's ratio*

Using Values for aluminum in equation (2) the bending rigidity was found to be

$$E = 2x10^{11}\ \frac{N}{m^2}$$

$$\rho = 7860\frac{kg}{m^3}$$

$$\nu = 0.266$$

$$D = \frac{2\times10^{11}\times\left(3.175\times10^{-3}\right)^3}{12\left(1-0.226^2\right)} \qquad \rightarrow D = 574.05Nm$$

Letting m=n=1, the fundamental frequency is given by:

$$\omega_{11} = \left(10.76+43.05\right)\sqrt{\frac{574.05\times3.14^4}{2\times7860\times\left(3.175\times10^{-3}\right)^3}}$$

$$\underline{\overline{\omega_{11}=1799.28}}HZ$$

2.2 Mesh Convergence Study

A smaller mesh typically results in a more accurate solution. However, as a mesh size is reduced, the computation time increases. It is ideal that the mesh will balance computational time and acceptable results.

A simple supported aluminum plate was modeled in CATIA with a large mesh size. Fig 3 below shows the model of the plate. The numerical solution of the natural frequency was found and compared to the analytical results.

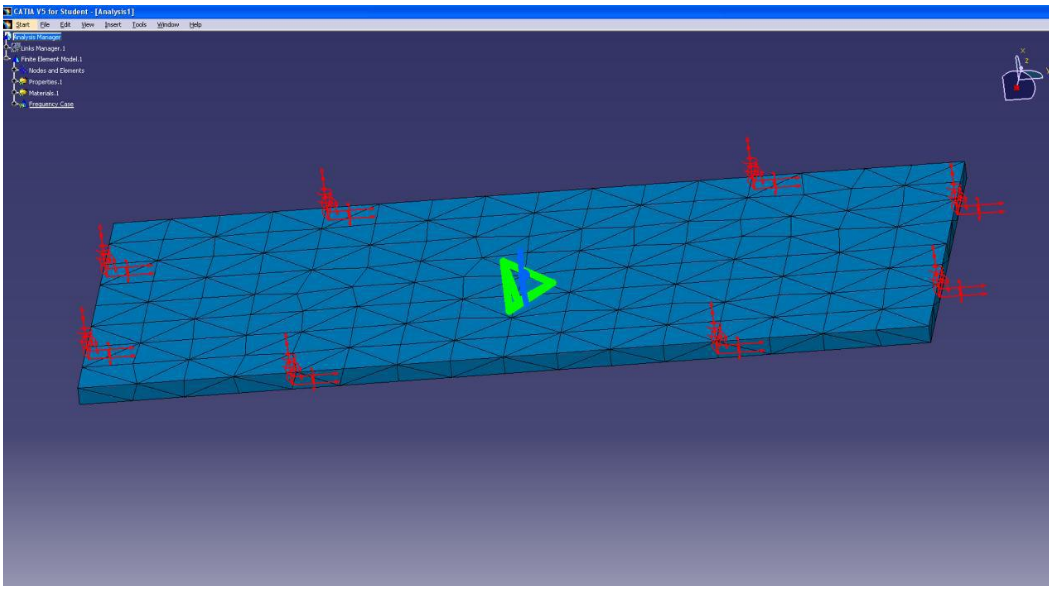

Figure 3: Rectangular plate simply supported

The mesh size was then reduced to study its effect on the frequency. The results are plotted in fig 4.

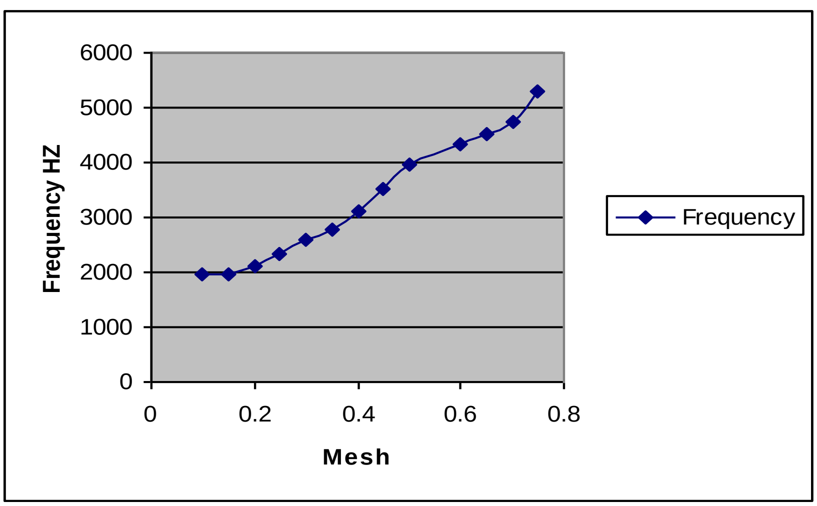

Figure 4: effect of mesh size on the natural frequency

It can be seen from the plot that the mesh size has a great impact on the numerical results, but as the mesh size approach 0.15, the effect becomes minimal. Therefore in a real world application, using the mesh size of 0.15 or 0.1 will not have a large effect on the numerical results, however it was found that the computational time increased by more than 100% . Since a large computational time is not an issue in this project. The mesh size used in this paper was 0.1 since it gives the best results in comparison to the analytical solution. The comparison is given in table 2 below. Based on the results given in table 2, it will be assumed that CATIA provides an acceptable approximation of the natural frequency of simply supported plate. Based on this assumption, it will further be assumed that making the plate more complex by changing the boundary conditions from simply supported to riveted, and also bending the plate, the numerical results obtained will be acceptable.

	Frequency
Analytical	1800 HZ
Numerical	1954.28 HZ

Table 2: natural frequency of a simple supported rectangular plate

Fig 5 below shows the simple supported plate with the actual mesh size that was used in this paper

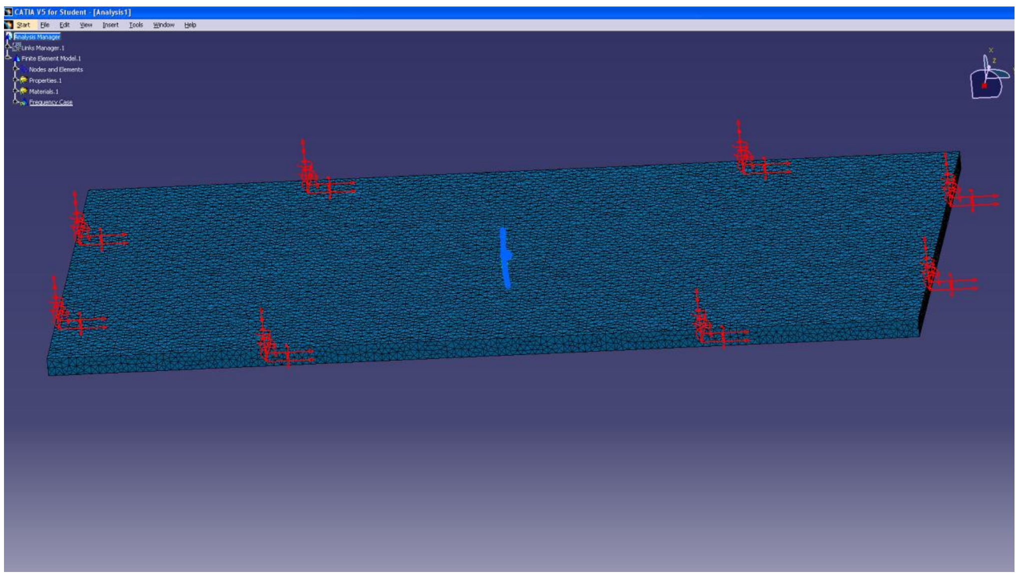

Figure 5: Natural frequency of the rectangular plate at mesh size 0.1

2.3 Natural Frequency of Riveted Plate

The motivation for fixing the end of the plate with rivets is by looking at the skin of an aircraft wing and fuselage. Every aircraft uses rivets to hold down the plates on its hull. Rivets were used in the project to mimic an actual aircraft wing. In this project, a rivet is placed at a spacing of 3 inches. Fig 6 below shows a model of the plate with rivets. The plate is attached with the rivets, and the rivets are fixed in space. In CATIA, the contact definition between the rivet and inner wall of the holes of the plate is modeled as rigid. This means that the inner wall of the plate which is in contact with the rivet is not allowed to slide.

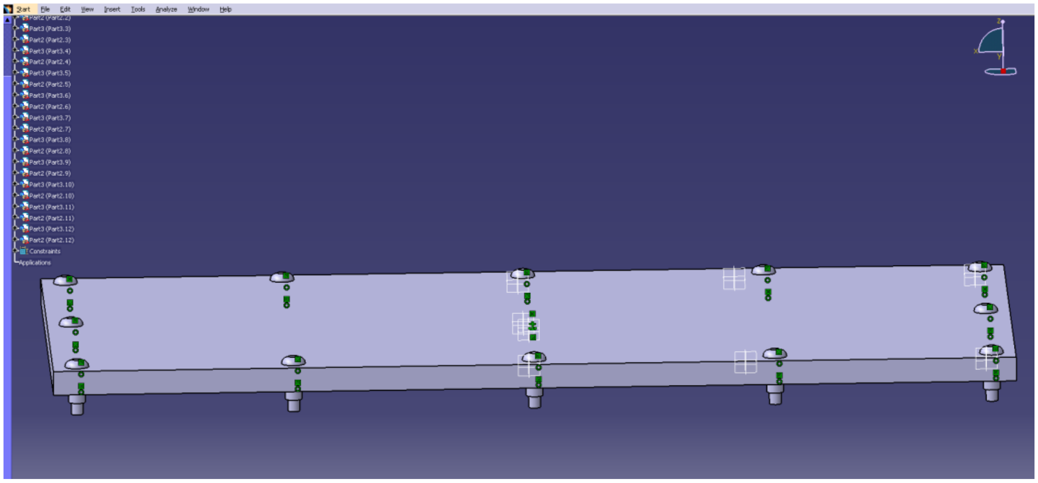

Figure 6: Natural Frequency of a riveted plate

Table 3 below shows a comparison of the results for the fundamental frequency. It is very important to observe the difference in the natural frequency of a plate which is simple supported as compared to one that is held in place with rivets. The difference in frequency is approximately 20% when comparing the two numerical results.

Frequency	
Analytically	1799.28 HZ
Numerical-simply supported plate	1954.28 HZ
Numerical-riveted plate	1556.30 HZ

Table 3: Frequency of the riveted plate compared to other result

2.4 Natural Frequency of a Curved Plate

A curved plate has been used in this project to mimic the curvature of the skin an aircraft wing and fuselage. A variation of different angle degrees has been used to simulate aeronautical authenticity. Angle of curvature was varied from 0 degrees, which is considered as a flat plate to a plate with a curvature of 305 degrees.

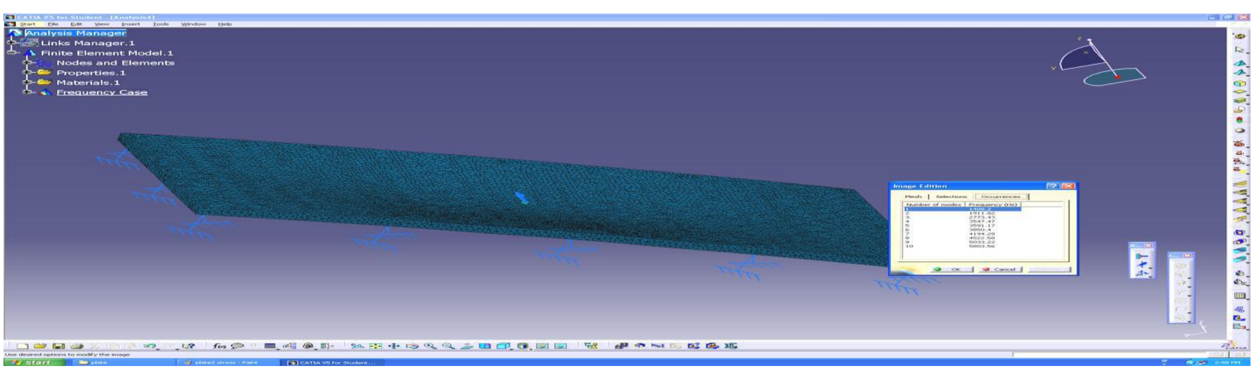

Figure 7: model of a flat plate

In fig 7, it can be seen that the rivet was eliminated and replaced by fixed conditions in the holes of the plate. In CATIA fixing the holes or adding a rigid contact condition between the inner wall of the holes and rivet is equivalent. Due to complexities that arise in contact between the rivet, washer and plate, it is simpler to replace the rivets with a fix condition.

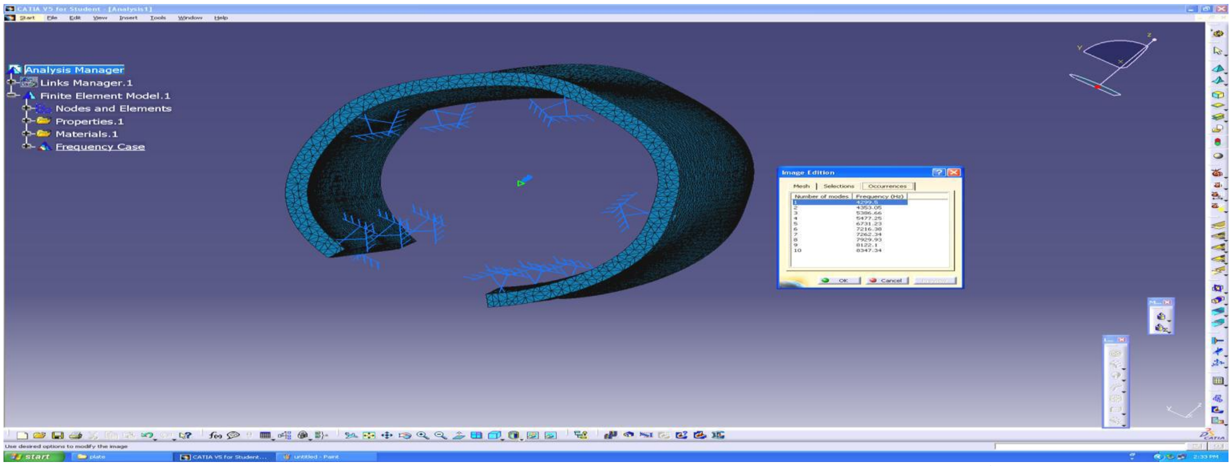

Figure 8: model of a plate with a curvature of $\theta = 305^0$

Table 4 shows the summary of the results of the fundamental frequency as the angle of curvature changes. To visualize the results a graph was plotted and shown in fig 9.

Angle of curvature	Frequency
0 degree	1408.20 Hz
5 degrees	1451.85 Hz
105 degrees	2461.22 Hz
205 degrees	3376.68 Hz
305 degrees	4299.50 Hz

Table 4: Frequency versus angle of curvature

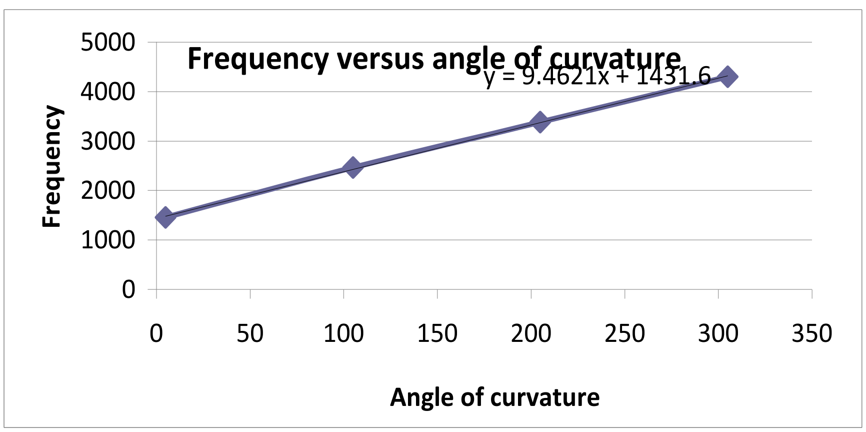

Figure 9: Fundamental frequency versus angle of curvature

From the graph a best fit line was added, and it can clearly be seen that there is a linear relation between angle of curvature and the fundamental natural frequency. This relation can be expressed in the equation (3) given below.

$$\omega = 9.462\theta + 1430 \quad (3)$$

3.0 Conclusion

The natural frequency depends on factors such as geometry and material properties. The natural frequency of a system can be altered. This can be achieved by changing any of these factors.
From this research was found that the natural frequency of a plate depends on the boundary conditions. It was seen that replacing a simple-supported plate with rivets reduced the fundamental frequency by approximately 20%. It was also found that CATIA can be used to find the natural frequency of a simply supported plate with acceptable results providing that the proper mesh size is being used. A very interesting finding from this research is that the fundamental frequency varied linearly with the angle of curvature can be represented by eq. (3)

References

M.K Baharami, M. Loghami and M. Pooyanfar *"Analytical solution for free vibration solution to Kirchhoff plate from wave approach"* World academy of science, engineering and technology, 2008

Nader G. Zamani, "*CATIA V5 FEA Tutorials 18*" Southern Utah University, 2009

AUTHORIZATION AND DISCLAIMER