

An approach to enhance the teaching of undergraduate engineering introductory physics courses

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ABSTRACT

Based on the analysis of published results by researchers working in *Physics Education Research*, we'll be arguing that one of the major difficulties to overcome in the teaching of introductory physics courses for engineers could be associated to the lack of a consistent and coherent methodological teaching framework which integrates both conceptual and mathematical reasoning aspects, in a systemic way of thinking. In fact, a large body of published research shows that students have strong difficulties in expressing, interpreting, and operating physical results in mathematical terms. We will be presenting a plausible six steps problem-solving strategy, that could be applied to tackle the aforementioned difficulty.

Keywords: Physics learning, teaching of physics, mathematical reasoning, student performance

1. INTRODUCTION

We might all agree that the role of mathematics in the education of students majoring in engineering is vital, not only in order to understand the fundamentals that sustain many of the mathematical models used in engineering, but to enhance students' analytical reasoning skills. It is also an undeniable true that it is in physics classes where students can start to apply what they have learned in their math classes and to find new non-formal approaches to performing computations.

The importance of being able to express, interpret and manipulate physical results in mathematical terms was also stressed by the great physicist Lord Kelvin "I often say that when you can measure something and express it in numbers, you know something about it. When you can not measure it, when you can not express it in numbers, your knowledge is of a meager and unsatisfactory kind. It may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the state of science, whatever it may be." (Hewitt, 1993) Freeman Dyson was more eloquent "...mathematics is not just a tool by means of which phenomena can be calculated; it is the main source of concepts and principles by means of which new theories can be created."(Dyson, 1964)

Now, being physics intrinsically a quantitative based subject, much of the recent research favors an overemphasis on qualitative (conceptual) physical aspects (Mualem and Eylon, 2007; Hoellwarth et al., 2005; Sabella and Redish, 2007; Walsh et al., 2007), while standard mathematical abilities, which are crucial for understanding physical processes, are not stressed, or even taught, because, rephrasing a passage from a recent editorial(Klein, 2007), they interfere with the students' emerging sense of physical insight. Consequently, physics instructors face the problem of finding suitable advice on how to approach the teaching of physics in the most efficient way and an answer to the question of how much time should be spent on intuitive conceptual reasoning and how much time in developing quantitative reasoning. Thus, the panorama regarding the learning of physics is even more dramatic on the side of the students, as for a typical course work for students majoring in engineering they usually must take more than one physics class. It could happen that in one term his/her physics instructor may emphasize quantitative reasoning over conceptual analysis, and in another term the respective instructor could rather accentuate conceptual learning over quantitative analysis, likely causing confusion on students, leading them to wonder which emphasis is correct.

2. ABOUT THE PROBLEM

It is not difficult to find published results in Physics Education Research where it is shown directly or indirectly the inability of students to express, interpret, and manipulate physical results in mathematical terms. That is, students shows a clear deficiency in their training to exploit the mathematical solution of a problem (which usually is obtained mechanically or by rote procedures) to enhance their knowledge regarding conceptual physics (Hammer, 1996; Sabella and Redish, 2007; Rimoldini and Singh, 2005; Meredith and Marrongelle, 2008). In fact, after compiling and analyzing students' performance on some tests dealing with rotational dynamics computations, it has been found that most of the students, including an honor class, failed to perform standard computations of rotational inertia on simple geometries involving the computation of one dimensional uncomplicated integrals (Rojas, 2008).

More important, the compilation and analysis of published excerpts of student's responses to interviews conducted by some researchers, using the *think-aloud* methodology to further understand students' way of reasoning while solving physics problems, shows that students lack of a structured methodology for solving physics problems (Hammer, 1996; Rimoldini and Singh, 2005; Sabella and Redish, 2007; Walsh et al., 2007).

These findings can not be surprising at all. In fact, none of the most commonly recommended physics textbooks (i.e. (Halliday et al., 2000; Tipler and Mosca, 2003; Serway and Jewett, 2003)) make use of a consistently and clear problem-solving methodology when presenting the solution of the textbook worked out illustrative examples. Moreover, the lack of a coherent problem-solving strategy can also be found in both the student and instructor manual solutions that usually accompany textbooks. Generally, standard textbooks problem-solving strategies encourage the use of a formula based scheme as compiled by the formulae summary found at the end of each chapter of the text, and this strategy seems to be spread out even in classroom teaching (Hamed, 2008). Consequently, students merely imitates the way in which problems are handled in the textbooks.

3. A SYSTEMIC PROBLEM-SOLVING TECHNIQUE

Thus, a moment of thought about the above summarized difficulties leads us to postulate the necessity of a systemic (Bunge, 2000; Bunge, 2004) approach which, from an operational point of view, could help instructors and students to achieve a better performance in the process of teaching and learning physics. Now, learning to approach problems in a systemic way starts from teaching and learning the interrelationships among conceptual knowledge, mathematical skills and logical reasoning (Heron and Meltzer, 2005). The problem arisen after the opening of the Millennium Bridge can further illustrate the needs for teaching and learning based on a systemic approach which recognizes the interrelatedness of every aspect of a physical process (physics, mathematics, and engineering design) (Strogatz et al., 2005).

Earlier work on the importance and necessity of a problem-solving strategy can be found in the work of the great mathematician George Pólya (Pólya, 1973; Pólya, 1963a; Schoenfeld, 1985; Schoenfeld, 1987; Schoenfeld, 1992; Lederman, 2009), who made emphasis on the relevance of the systematicity of a problem-solving strategy for productive thinking, discovery and invention.

Inspired on Pólya's ideas, we are presenting a six steps problem-solving strategy that could be applied to tackle the afore mentioned problems in the teaching and learning of physics. Based on the experience of applying the strategy in our own teaching activities, we have found that it is extremely useful in teaching both conceptual and quantitative reasoning explicitly. Moreover, it is just a matter of time to combine this problem-solving approach with some fruitful ideas that have been advanced about how to properly address the design of instruction, so the involved learning cognitive mechanism of the students are triggered, leading to a more effective teaching outcomes (Hestenes, 2003; Dunn and Barbanel, 2000; Reif, 1981; Reif and Scott, 1999; Redish and Steinberg, 1999; Scherr, 2007).

3.1 A six steps problem-solving technique

1. **Understand the problem:** some considerations to develop at this step involves drawing a figure and asking questions like: What is the unknown? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? That is, in this stage students needs to actually be sure to what the problem is. In addition to making drawings to get a grasp of the problem, students might need to reformulate the problem in their own words, making sure that they are obtaining all the giving information for solving the problem. This is a crucial step in the sense that *if one does not know where are we going, any route will take us there*.

2. **Provide a qualitative description of the problem:** in this stage students need to think and write down the laws, principles, or possible formulations that could help them to solve the problem. For instance students need to consider any possible frame work of analysis that could help them to represent or describe the problem in terms of the principles of physics (i.e. Newtons law, energy conservation, momentum conservation, theorem of parallel axis for computing inertia moment, non-inertial reference system, etc.) If necessary, the drawings of the previous step could be complemented by the corresponding problem free-body and/or vector diagram.

3. **Plan a solution:** some considerations to have in mind in order to develop this step involves looking at the unknown and trying to think of a familiar problem having the same or a similar unknown. Some questions to be ask are like Have you seen this before? Or have you seen the same problem in a slightly different form?. Once the student have as many possibilities to approach the problem, he/she only needs to pick one strategy of solution and write down the corresponding mathematical formulation of the problem, avoiding as much as possible to plug numbers in the respective equations. Also, they need to think whether the information at hand would be enough to get a solution (i.e. if a set of algebraic equations is under or over determined, or if the number of provided boundary conditions is enough to solve a differential equation).

4. **Carrying out the plan:** at this stage the student will try to find a solution to the mathematical formulation of the problem sketched according to the previous steps, and perhaps they will need to go back in order to find a easier mathematical formulations of the problem. This can be facilitated if the students have writing down alternatives of solution as they were suppose to do on item 2.

5. **Verify the internal consistency and coherence of the used equations:** at the moment of finding a solution of the involved mathematical equations, students need to verify whether the equations are consistent with what they represent (i. e. are the equations dimensionally correct? Do they represent a volume or a surface?). Though this seems to be an unnecessary step, experience shows that students too often do not verify the internal consistency and coherence of the equations they solve. And this mistake is also found to be performed by textbook writers, as discussed in a recent editorial (Bohren, 2009). After verifying that no inconsistencies are found in the mathematical solution of the problem, students could then plug numbers in the obtained results to find, whether required or not, a numerical solution which in turn could be used in the next step to further evaluate the obtained result. In the next section, by means of a illustrative example we will show how the right answer of the illustrative posed problem could be obtained, even though the internal consistency of a used equation is not right. Let's mention that, in order to further show students the necessity of continuously verifying the consistency of the used equations, one could resort to the naive problem involving the wrong proof that $1 = 2$: starting with $x(x-x) = (x-x)(x+x)$, after (**wrongly**) dividing by $(x-x)$ in both sides of this identity leads to $x = 2x$, which in turns leads to $1 = 2$.

6. **Check and evaluate the obtained solution:** once a solution has been obtained, its plausibility needs to be evaluated. Some questions could be asked in this regards: can the results be derived differently? Can the result or the method be applied to solve or fully understand other problems? Can the solution be used to write down the solution of a less general problem? Can the solution be used to further understand the qualitative behavior of the problem? Is it possible to have a division by zero by changing a given parameter? Does it makes sense?, and so forth.

In the next section will examine further these steps by applying them to solve an illustrative example.

4. ILLUSTRATIVE EXAMPLE

In the following example we will present an approach on how to introduce students in the use of our proposed six steps problem-solving strategy. It is pertinent to point out that, by proper training, students could absorb the steps of our problem-solving strategy in such a way that they could perform the corresponding operations mentally, naturally, and vigorously.

Problem: About its central axis, find the moment of inertia of a thin hollow right circular cone with radius R , lateral length L , and mass M uniformly distributed on its surface with density σ , as shown in Figure 1.

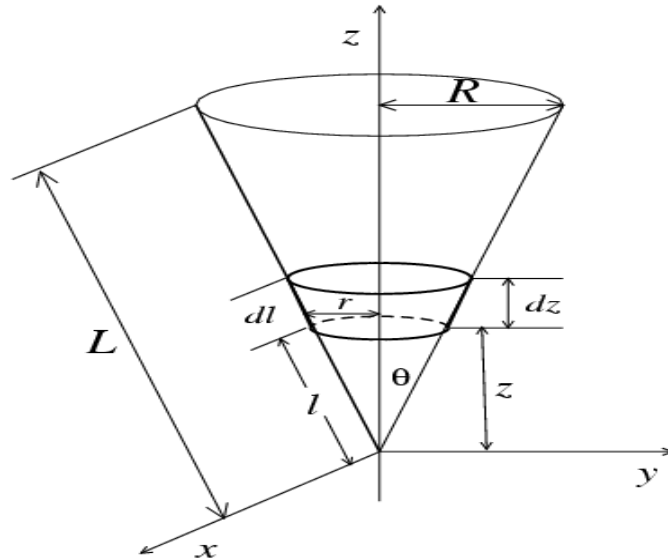


Figure 1: A hollow right circular cone with radius R , lateral length L , and uniform mass M . The cone's high is $H = \sqrt{L^2 - R^2}$. The figure also shows, at the lateral distance l , measured from the cone's apex at the origin of the coordinate system, an infinitesimal ring of lateral length dl and radius r . Two useful geometrical relations among some of the dimensions shown in the figure are $r = (R/L) l$ and $r = (R/H) z$.

Solution:

1. **Understand the problem:** "It is foolish to answer a question that you do not understand. ... But he/she should not only understand it, he should also desire its solution." (Pólya, 1973) Following Pólya's comment, before attempting to solve this problem, students need to have been exposed to a basic theory on computing moment of inertia (I). Particularly, students need to be familiar with the computation of I for a thin circular ring about its main symmetric axis. To further understand the geometry of the present problem, students could, for example, be talked about the shape of an empty ice cream cone. After some discussion, a drawing better than the one shown in Figure 1 could be presented on the board. Let's mention that additional ways of presenting each step in meaningful ways can be found in (Pólya, 1973; Heller et al., 1992).

2. **Provide a qualitative description of the problem:** In this step one could further motivate the discussion by associating the computation of I with rotational motion quantities (i.e. kinetic energy, angular momentum, torque, etc.). One can even motivate the qualitative discussion by considering the hollow cone as a first crude approximation of a symmetric top or of a cone concrete mixer. The drawing of Figure 1 could even be made more explicative.

3. **Plan a solution:** "We have a plan when we know, or know at least in outline, which calculations, computations, or constructions we have to perform in order to obtain the unknown. ... We know, of course, that it is hard to have a good idea if we have little knowledge of the subject, and impossible to have it if we have no knowledge. ... Mere remembering is not enough for a good idea, but we can not have any good idea without recollecting some pertinent facts." (Pólya, 1973) Accordingly, at this stage instructors could point out the

superposition principle to solve the problem by slicing the hollow cone in a set of small, infinitesimal, rings distributed along the symmetrical axis of the cone. Thus, each infinitesimal ring will have in common the same rotational axis about which the moment of inertia of them is already known $dI = r^2 dm = r^2 \sigma dS$, where r , dm , and dS respectively represent the radius of each infinitesimal ring, the mass of each infinitesimal ring, and the surface of each infinitesimal ring.

4. Carrying out the plan: To carried out the plan, it won't be a surprise to choose the wrong dS . In fact, it is not difficult, at first sight, having students choosing wrongly (see Figure 1): $dS = 2\pi r dz = 2\pi (R/H) z dz$, which lets to $S = \pi RH$, as the hollow cone surface (this result is of course wrong). Using this surface element, the moment of inertia (or rotational inertia) for the small ring about its axis takes the form $dI = r^2 \sigma dS = 2\pi\sigma(H/R)r^3 dr$ (see Figure 1), which lets to $I = 2\pi\sigma(H/R)(R^4/4) = (1/2)(\sigma S)R^2 = MR^2/2$, as the required moment of inertia of the hollow cone (which is the right answer). It is not difficult to get students performing this sort of wrong computations, and they become uneasy when trying to convince them that in spite of having found a right result, it is specious because it was obtained via a wrong choice for dS . Eventually students might agree on the incorrectness of their procedure if asked to compute explicitly the cone's mass.

5. Verify the internal consistency and coherence of the used equations: "Check each step. Can you see clearly that the step is correct? Can you prove that it is correct? ... Many mistakes can be avoided if, carrying out his/her plan, the student check each step." (Pólya, 1973) Steeping on our teaching experience, it is too easy for students to perform without hesitation the just aforementioned wrong computations, as presented in the previous step. And it is not easy to get students to realize their mistake. For God's sake, they have computed the right answer!!!: that is, for a hollow thin cone, rotating about its symmetric axis, $I = MR^2/2$!!!. How in the world a mistaken procedure could have lead to a right answer !!! In this situation, to make aware students of their mistake, the easy way is the experiment. Instructors could unfold several hollow cones to actually show the students that the respective surface is $S = \pi RL$, instead of the wrongly obtained $S = \pi RH$. Accordingly, we hope to have provided enough evidence for the need to, explicitly and repeatedly, mention to the students the need to check each computational step, including checking for dimensionality correctness. In this case, the right approach is to consider $dS = 2\pi r dl = 2\pi(R/L)dl$, which yield $S = \pi RL$, the right answer for S (the cone's lateral surface). This choice for dS lets to $dI = r^2 \sigma dS = 2\pi\sigma(L/R)r^3 dr$ (see Figure 1), which yields $I = 2\pi\sigma(L/R)(R^4/4) = (1/2)(\sigma S)(R^2) = MR^2/2$, the right answer for a hollow cone's rotational inertia I about its symmetrical axis.

Considering that it is not hard to find stories on reported wrong results due to wrong or incomplete computations (Strogatz et al., 2005; Veysey and Goldenfeld, 2007), this problem could also be used as an example of how computations of a physical quantity (the surface of a cone shell) can be used to judge a mathematical result (the wrong value for S) that is used in subsequent computations yielding a right answer.

6. Check and evaluate the obtained solution: "Some of the best effects may be lost if the student fails to reexamine and to reconsider the completed solution." (Pólya, 1973) After gaining confidence on the obtained solution of the problem, it is necessary to spend sometime in evaluating its plausibility. Examining the solution of our problem one could ask: it is not striking that the rotational inertia for a hollow cone about its symmetric axis is the same as for a solid disk having the same uniformly distributed mass M and radius equal to the cone's base? Does not it a counter example for the statement that rotational inertia only depends on how the mass is distributed around the axis of rotation? Furthermore, if for some reason the wrong choice for the dS was not caught in the previous step, it could be detected if analyzing the case of having a non constant σ . A further interpretation of the result can be found at (Bolam and Wilkinson, 1961).

5. DISCUSSIONS

This article presents a six step problem-solving strategy, aiming to approach three major problems in the learning and teaching of introductory engineering physics courses: 1) the demand of the physics instructors for effective teaching strategies that could help in the teaching of intuitive conceptual and quantitative reasoning, and how to teach both aspects holistically 2) the students' need for suitable methodology that could help students to fill the textbooks' gap on enhancing their mathematical reasoning abilities, which are essential for reinforcing students' knowledge of conceptual physics, and 3) a deficiency in the teaching of physics leading to students not being taught a coherent physics problem-solving strategy that enables them to engage in both mathematical and conceptual reasoning.

At this point one could recall a particular point of view that the great mathematician Pólya stressed very much in his writings about the art of teaching and learning, which, in some sense, can be considered as an "axiomatic thought" about the art of teaching and learning. He was emphatic on the fact that "*for efficient learning, the learner should be interested in the material to be learnt and find pleasure in the activity of learning.*" In other words, inspiration to learn is without doubt a necessary condition in order to have an efficient and effective teaching and learning environment.

This, of course, is by no means a new discovery, and, paraphrasing Schoenfeld (Schoenfeld, 1977), some ideas to circumventing few of the barriers between the dedicated instructor and his/her students' attitudes in "learning" the subject that is being taught has been set forward in (Schoenfeld, 1977; Ehrlich, 2007; Duda and Garrett, 2008). Nevertheless, one should keep in mind that "we know from painful experience that a perfectly unambiguous and correct exposition can be far from satisfactory and may appear uninspiring, tiresome or disappointing, even if the subject-matter presented is interesting in itself. The most conspicuous blemish of an otherwise acceptable presentation is the 'deus ex machina'." (Pólya, 1963b)

6. OVERALL CONCLUSIONS

The large number of published "Comments on ..." and "Reply to ..." articles in which much of the discussion is about the incorrectness of the physical interpretation of a concept or an idea, are indicative that the qualitatively understanding of the concepts of physics is a very elusive task, where even experienced researchers can fail to grasp. In spite of this evidence, much of the recent publications in Physics Education Research favor conceptual learning over quantitative reasoning. Consequently, both instructors and students find it very difficult to find suitable teaching and learning literature emphasizing both aspects. As a result, students of general physics courses have strong difficulties in answering correctly questions involving uncomplicated computations, like comparing the kinetic energy of simple rotating objects. Consequently, in this article we presented a problem-solving strategy that can be used to simultaneously incorporate the teaching and learning of conceptual physics and mathematical reasoning. There is evidence that using problem-solving strategies in combination with active learning methodologies could improve significantly the performance of students in this regards (Heller et al., 1992), a fact that we have experienced in the teaching of physics to students majoring in Biology.

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