

# Foil Bearing lubricated with contaminated air : a numerical analysis

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**Abstract**– This paper presents an original theoretical investigation of the steady-state and dynamic performance characteristics of foil journal bearings lubricated with contaminated air taking into account couple-stresses due to the presence of small amount of natural or artificial pollutant substances. The substance can be solid particles (e.g. dust, ash, pollen, smoke) liquid droplets, or gases.

To determine the aerodynamic pressure and the power loss, the governing modified Reynolds' equation, and the viscous dissipation term appearing on the RHS of the modified energy equation are derived using the V. K. Stokes micro-continuum theory.

According to the obtained results, the effects of couple stresses on the steady-state and dynamic behavior of foil self-acting gas bearings are significant and cannot be overlooked. The results also show that the dynamic deformation of the bump foil should be included in calculating the dynamic performance characteristics of foil journal bearings.

**Keywords**-- couple-stress fluid, foil bearing, modified Reynolds' equation, dynamic deformation, micro-continuum theory, analytical perturbation method

## I. INTRODUCTION

In classical aerodynamic and elasto-aerodynamic lubrication analyses published in technical literature since the sixties, gaseous lubricants are usually assumed to behave as nonpolar (i. e. Newtonian) fluids with no distributed couples per unit area on the boundary surfaces and no distributed body couples per unit volume or per unit mass, but only external surface traction forces and body forces. Many practical lubrication applications may be found where the Newtonian fluid model is not a satisfactory engineering approach to better describe the behavior of lubricated mechanical systems such as gas and liquid lubricated bearings. External couple distributions could result from the action of an external magnetic field on magnetized particles of the material or the action of an electric field on polarized matter. Historically, couple stresses were included in continuum mechanics by Voigt [1-2], but these formulations were not much applied. In solid mechanics, Mindlin and Tiersten [3], and Koiter [4] have considered elastic materials with couple stress. In fluid mechanics, the use of micro-continuum theory is more suitable for the theoretical study of such fluids because the application of classical Navier-Stokes and energy equations to describe their motion leads to erroneous results. The V. K. Stokes micro-continuum theory [derived from the couple-stress linear elasticity formulation of Mindlin and Tiersten [3]] is the

simplest theory of fluids proposed in the technical literature since the sixties, which allows the polar effects such as the presence of couple-stresses and body couples in addition to the body forces and surface forces. In this work, we theoretically investigate the steady-state and dynamic performance characteristics of a first generation foil journal bearing by using the Vijay Kumar Stokes micro-continuum theory. Besides, we try to determine how these characteristics vary with some operating conditions, such as the rotor rotational speed as well as the whirl frequency for various values of couple stress. The dynamic coefficients will be determined from the linearized dynamic theory using an analytical perturbation process to consider dynamic deformations of the bump foil.

## II. MODIFIED REYNOLDS EQUATION

For inverse aerodynamic lubrication problem (i.e. when the static external load is imposed), the motion of the rotor in a compliant self-acting gas film bearing is described in the frame  $(X, Y)$  related to the applied load  $W$  where the origin is located at the bearing center as illustrated in figure 1. The gas film pressure can be integrated over the surface of the bearing to obtain the lift force components:

$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = \int_{-L/2}^{L/2} \int_0^{2\pi} p \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} R d\theta dz \quad (1)$$

Where  $p$  is the gas film pressure,  $R$  is the shaft radius,  $L$  is the bearing length, and  $\theta$  is the circumferential co-ordinate as indicated in figure 1.

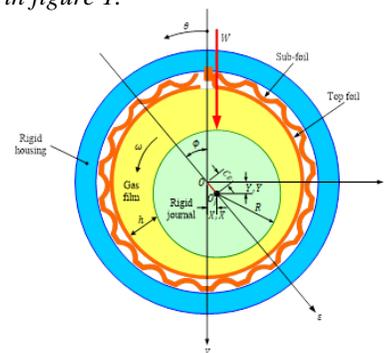


Fig. 1 First generation gas foil bearing cross-section schematic, and gas-film and bump foil structure models.

The film pressure is obtained from the solution of the modified Reynolds' equation derived from modified Navier-Stokes and continuity equations for a non-Newtonian couple stress fluid by using the V. Kumar Stokes micro-continuum theory:

Digital Object Identifier (DOI):

<http://dx.doi.org/10.18687/LACCEI2019.1.1.505>

ISBN: 978-0-9993443-6-1 ISSN: 2414-6390

$$\nabla \cdot (\rho \mathbf{Q}) = -\frac{\partial}{\partial t}(\rho h) \quad (2)$$

Where  $\rho$  is the fluid mass density,  $h$  is the film thickness,  $t$  is the time variable, and

$$\mathbf{Q} = \begin{Bmatrix} Q_x \\ Q_z \end{Bmatrix} = -\frac{G(h,l)}{12\mu} \begin{Bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial z} \end{Bmatrix} + \begin{Bmatrix} \frac{h}{2} \omega R \\ 0 \end{Bmatrix} \quad (3)$$

$Q_x$  and  $Q_z$  being the volume flow rate components per unit length in  $x$  and  $z$  directions, respectively.

In Eq. (3), the function  $G$  is defined by

$$G(h,l) = h^3 - 12l^2 \left[ h - 2l \operatorname{Tanh}\left(\frac{h}{2l}\right) \right] \quad (4)$$

Where  $l = \sqrt{\frac{\eta}{\mu}}$  and  $\eta$  is the material constant responsible for the couple-stress property.

According to equation (3) and (4), Equation (2) becomes

$$\frac{\partial}{\partial x} \left[ \frac{\rho G(h,l)}{12\mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{\rho G(h,l)}{12\mu} \frac{\partial p}{\partial z} \right] = \frac{\omega R}{2} \frac{\partial(\rho h)}{\partial x} + \frac{\partial(\rho h)}{\partial t} \quad (5)$$

Where  $\mu$  is the fluid absolute viscosity, and  $\omega R$  is the linear velocity of the journal surface.

For a compliant cylindrical bearing, the film thickness  $h$  is defined by the following expression:

$$h(\theta, z, t) = C + X(t)\cos\theta + Y(t)\sin\theta + \mathfrak{L}p(\theta, z, t) \quad (6)$$

Where  $C$  is the bearing radial clearance,  $X$  and  $Y$  are the Cartesian co-ordinates of the shaft center, and

$\mathfrak{L} = \frac{2s}{E} \left( \frac{\ell}{t_b} \right)^3 (1 - \sigma^2)$  is the scalar compliance operator of the bump foil in (m/Pa) according to the Heshmat's analytical model [5-6] by modelling the corrugated sub-foil as a simple Winkler elastic foundation with isotropic stiffness.

The boundary conditions associated to the Reynolds equation (5) may be classified as follows :

Boundary conditions related to the environment in which the system operates:

$$p(\theta, z = \pm \frac{l}{2}, t) = p_a \quad \text{at the bearing edges} \quad (7a)$$

Periodicity condition:

$$p(\theta=0, z, t) = p(\theta=2\pi, z, t) \quad (7b)$$

- Boundary conditions related to lubricant flow (depression phenomenon):

$$\begin{cases} p(\theta_{sub}, z, t) = p_a \\ \frac{\partial p}{\partial \theta}(\theta_{sub}, z, t) = \frac{\partial p}{\partial z}(\theta_{sub}, z, t) = 0 \end{cases} \quad (9c)$$

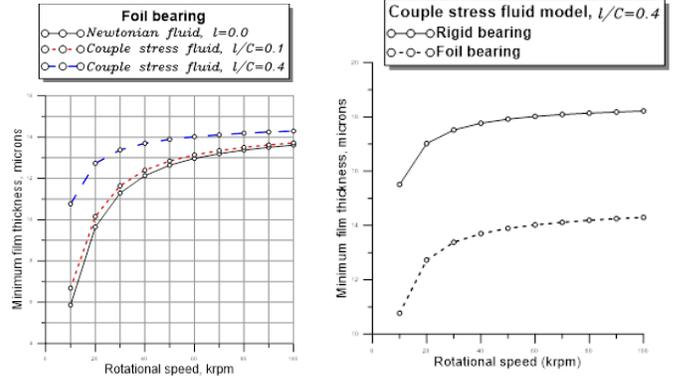
At these conditions, we can add for aligned journal bearings the following condition:

$$\frac{\partial p}{\partial z}(\theta, z=0, t) = 0, \quad \text{at the bearing centerline.} \quad (7d)$$

### III. SOME RESULTS

Figure 2 shows the minimum film thickness over a range of rotor rotational speeds for a rigid bearing and a foil bearing subjected at the same static load  $W_o$ . As revealed in this figure, the minimum film thickness calculated for both bearings is widely enhanced by the presence of couple stresses even for low rotational speeds. For the same speed, the value of the minimum film thickness for a rigid bearing is greater

than that for a foil bearing and the increase is more pronounced for high values of  $l$  which can be regarded as the characteristic size of solid particle present in the lubricant as clearly depicted in figure 3. The increase of the minimum film thickness with the rotor speed is mainly due to the decrease of the operating eccentricity since the system operates with imposed load (inverse problem) and not with imposed eccentricity (direct problem).



Figures 2 (left) and 3 (right)

### IV CONCLUSION

For foil journal bearings lubricated by contaminated air modelled as a couple stress or polar fluid, some following conclusions were reached:

1. When the bearing operates at imposed load, the static peak pressure falls with increases of the couple stress parameter.
2. The couple stresses produce higher gas-film thicknesses than those predicted by the nonpolar theory especially for the compliant foil bearings even for low rotational speeds.
3. The side leakage flow decreases with increasing the couple stress parameter.
4. The power loss is not affected by the couple stresses in both rigid and compliant bearings

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