Evaluation of the drinking water consumption in Lima city using the grey clustering method

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Abstract—OMS indicated that a person should consume 100 liters of water per day to satisfy their needs. In some districts of Lima, the consumption of this resource is much higher, while elsewhere the consumption is under of the recommended. In this study, we realized an evaluation of the consumption of drinking water using the grey clustering method, which is based on the grey systems theory. The case study was carried out in 32 districts of Lima, Peru. As a result, a map on the situation of drinking water consumption was obtained. The results revealed that the districts that consume more water are San Isidro, Miraflores, La Molina, San Borja, and Lince. In addition, the districts that consume less water were Ancon, Puente Piedra, Villa el Salvador, and Independencia. Moreover, the results of the study could help to the central government or the authorities of districts to make the best decision about the equitable distribution of water to all districts of Lima city. The Grey clustering method showed interesting results and could be applied to other problems of the country, since this method considers the uncertainty into its analysis.

Keywords—Drinking water consumption, The grey clustering method, Water computacional modeling.

I. INTRODUCTION

Water is essential for life and all people must have a satisfactory supply (sufficient, safe and accessible), as an important social aspect [1] [2]. Improving access to safe drinking water could provide tangible benefits for health [3]. In addition, For the ONU, drinking water and sanitation are essential to life and health, considered that the realization of the human right to water is essential to healthy living which respects human dignity [4].

In Peru, a constitutional reform law was enacted in 2017, recognizing the right of every person to progressively and universally access drinking water, prioritizing human consumption over other uses and promoting sustainable water management [5].

The Peruvian state has a company to supply drinking wáter, which is called SEDAPAL, this company provides potable water and sewerage services to the urban sector of the city of Lima, reported that an average Peruvian consumes a maximum of 163 liters of water per day, although the OMS indicates that a person has to consume 100 liters per day, also announced that the water deficit in Lima is 5.3 m^3 per second and that San Isidro consumes 447 liters of drinking water per day, a figure that exceeds by 400% the estimate recommended by OMS [6]–[9].

In this study, we analyzed drinking water consumption in 32 districts of the city of Metropolitan Lima, applying the grey clustering method.

The grey glustering method is based on the grey systems theory, initially developed by Deng in 1985. Grey systems focus on the study of problems involving small samples or limited information; this fact makes the grey systems theory have a wide range of applicability, such as the evaluation of websites [10], transport management [11] or safety management [12].

In addition, the grey grouping method that could be applied by means grey incidence matrices or whitening weight functions. In this paper, we use center-point triangular whitening weight functions (CTWF), since it is applied mainly to check if the objects of observation belong to predetermined classes, called grey classes [13], as shown by the studies on a management of the water [14], or in innovation strategies [15]. Therefore, the specific objective of this study is to classify the districts of the Lima city, according to the level of drinking water consumption in the districts of Lima applying the CTWF method.

In this paper, Section II provides details of the CTWF method. In Section III, the case study is described, followed by the results and the discussion in Section IV. The conclusions are provided in Section V.

II. METHODOLOGY

First, a set of m objects, a set of n criteria, a set of s grey class, and a set of sample values xij (i=1, 2,…, m; j=1, 2,…, n), are assumed. Next, the steps of the CTWF method are presented as follows [16]:

Step 1: The criteria are divided into s grey classes, and then their central points are determined (λ_1, λ_2 …. and λ_s).
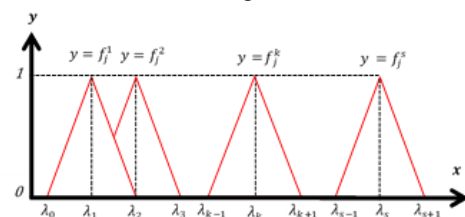**Step 2:** Grey classes 0 and (s + 1) are added, with their central-points λ_0 and λ_{s+1}, see Fig. 1.

Fig. 1. CTWF [13].

For the kth grey class, k=1, 2,…, s, of the jth criterion, j=1, 2,…, n, for an observed value x_{ij}, the CTWF is calculated using Eq. 1.

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$$f_j^k(x_{ij}) = \begin{cases} 0, & x \notin [\lambda_{k-1}, \lambda_{k+1}] \\ \dfrac{x - \lambda_{k-1}}{\lambda_k - \lambda_{k-1}}, & x \in [\lambda_{k-1}, \lambda_k] \\ \dfrac{\lambda_{k+1} - x}{\lambda_{k+1} - \lambda_k}, & x \in [\lambda_k, \lambda_{k+1}] \end{cases} \qquad (1)$$

Step 3: The global clustering coefficient σ_i^k for the object i, i = 1, 2, ..., m, with respect to the grey class k, k = 1, 2, ..., s, is calculated by the Eq. 2.

$$\sigma_i^k = \sum_{j=1}^{n} f_j^k(x_{ij}) \cdot \eta_j \qquad (2)$$

Where $f_j^k(x_{ij})$ the CTWF and η_j is the weight of criterion j.

Step 4: If $\max_{1 \leq k \leq s}\{\sigma_i^k\} = \sigma_i^{k^*}$, it is decided that object i belongs to the grey class k*.

III. CASE STUDY

The assessment of drinking water consumption was carried out in the Lima city, where we evaluated 32 districts, as shown in Fig. 2.
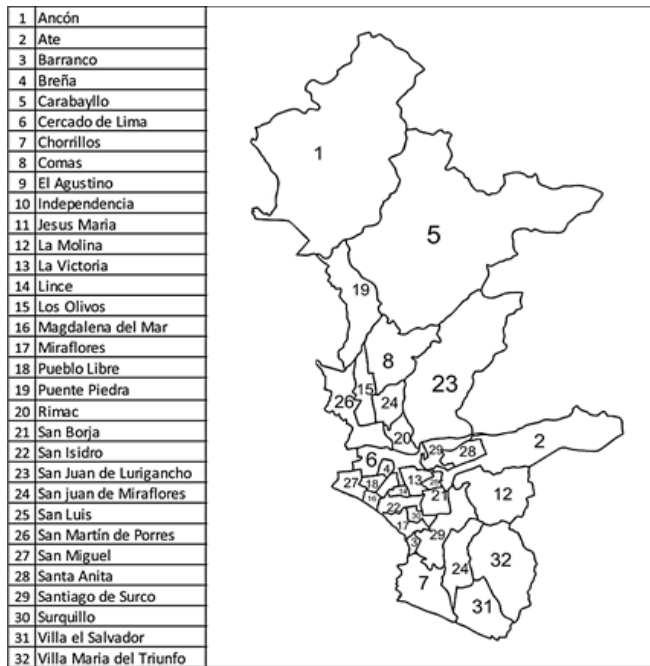

1	Ancón
2	Ate
3	Barranco
4	Breña
5	Carabayllo
6	Cercado de Lima
7	Chorrillos
8	Comas
9	El Agustino
10	Independencia
11	Jesus Maria
12	La Molina
13	La Victoria
14	Lince
15	Los Olivos
16	Magdalena del Mar
17	Miraflores
18	Pueblo Libre
19	Puente Piedra
20	Rimac
21	San Borja
22	San Isidro
23	San Juan de Lurigancho
24	San juan de Miraflores
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26	San Martín de Porres
27	San Miguel
28	Santa Anita
29	Santiago de Surco
30	Surquillo
31	Villa el Salvador
32	Villa Maria del Triunfo

Fig. 2. Districts of the city of Lima.

A. Description of the context

In 2017, the distribution of drinking water in the city of Lima increased unlike in previous years, 85% already have water and only the upper areas of the city of Metropolitan Lima are missing. In this study, we evaluated the total drinking water consumption of the SEDAPAL report from Peru, we obtained results of water consumption in each district of the city of Lima in the year 2017 [7]. The information is presented in Table I.

TABLE I.   DRINKING WATER CONSUMPTION BY DISTRICT

District	L /P /D	District	L /P /D
1	64.0	**17**	395.2
2	90.1	**18**	202.8
3	218.7	**19**	73.5
4	176.1	**20**	111.7
5	77.0	**21**	248.1
6	182.5	**22**	447.5
7	106.4	**23**	88.6
8	89.9	**24**	98.4
9	102.0	**25**	181.5
10	76.7	**26**	110.3
11	222.9	**27**	186.2
12	258.6	**28**	99.2
13	168.9	**29**	225.3
14	240.1	**30**	182.2
15	124.6	**31**	75.1
16	216.7	**32**	78.7

B. Calculations using the CTWF method

The calculations for the case study, based on the CTWF method, are preceded as follows.

Step 1: The scope of drinking water consumption figures [64; 447.5] was divided proportionally into five intervals or grey classes, in order to satisfy the need to reflect the characteristics of the specific problem under study [17]. The central points λ_1, λ_2, λ_3, λ_4, and λ_5, of the five grey classes are shown in Table II.

TABLE II.   CENTRAL POINTS OF GREY CLASSES IN THE CASE STUDY

Very low class (λ_1)	low class (λ_2)	Medium class (λ_3)	High class (λ_4)	Very high class (λ_5)
k=1	k=2	k=3	k=4	k=5
103	180	257	334	411

Step 2: The grey classes extended in two directions, adding the grey classes "extra low" and "extra high", respectively, with their central-points λ_0 and λ_6; as shown in Table III and Fig. 3.

TABLE III.   CENTRAL POINTS OF EXTENDED GREY CLASSES

Central points of extended grey classes						
λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6

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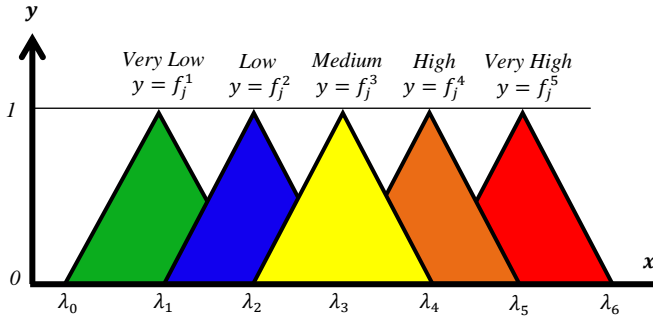
Fig. 3. CTWF for the case study.

Now, the values presented in Table III were substituted in Eq. 1, to obtain the CTWF of the five grey classes. The results are shown in Eq. 3-7:

$$f_1^1(x) = \begin{cases} 0, & x \notin [26, 180] \\ \dfrac{x-26}{77}, & x \in [26, 103] \\ \dfrac{180-x}{77}, & x \in [103, 180] \end{cases} \quad (3)$$

$$f_1^2(x) = \begin{cases} 0, & x \notin [103, 257] \\ \dfrac{x-103}{77}, & x \in [103, 180] \\ \dfrac{257-x}{77}, & x \in [180, 257] \end{cases} \quad (4)$$

$$f_1^3(x) = \begin{cases} 0, & x \notin [180, 334] \\ \dfrac{x-180}{77}, & x \in [180, 257] \\ \dfrac{334-x}{77}, & x \in [180, 334] \end{cases} \quad (5)$$

$$f_1^4(x) = \begin{cases} 0, & x \notin [257, 411] \\ \dfrac{x-257}{77}, & x \in [257, 334] \\ \dfrac{411-x}{77}, & x \in [334, 411] \end{cases} \quad (6)$$

$$f_1^5(x) = \begin{cases} 0, & x \notin [334, 488] \\ \dfrac{x-334}{77}, & x \in [334, 411] \\ \dfrac{488-x}{77}, & x \in [411, 488] \end{cases} \quad (7)$$

Step 3: Then, from Table I, the CTWF values for each district were calculated using equations 3-7. The results are shown in Table IV.

TABLE IV. VALUES OF CTFW FOR EACH DISTRICT

District	1	2	3	4	5	6	7	8
$f_j^1(x)$	0.49	0.83	0,00	0.05	0.66	0,00	0.96	0.82
$f_j^2(x)$	0,00	0,00	0.5	0.94	0,00	0.97	0.04	0,00
$f_j^3(x)$	0,00	0,00	0.5	0,00	0,00	0.03	0,00	0,00
$f_j^4(x)$	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
$f_j^5(x)$	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
District	9	10	11	12	13	14	15	16
$f_j^1(x)$	0.99	0.66	0,00	0,00	0.14	0,00	0.72	0,00
$f_j^2(x)$	0,00	0,00	0.44	0,00	0.86	0.22	0.28	0.52
$f_j^3(x)$	0,00	0,00	0.56	0.98	0,00	0.78	0,00	0.48
$f_j^4(x)$	0,00	0,00	0,00	0.02	0,00	0,00	0,00	0,00
$f_j^5(x)$	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
District	17	18	19	20	21	22	23	24
$f_j^1(x)$	0,00	0,00	0.62	0.89	0,00	0,00	0.81	0.94
$f_j^2(x)$	0,00	0.70	0,00	0.11	0,12	0,00	0,00	0,00
$f_j^3(x)$	0,00	0.29	0,00	0,00	0.88	0,00	0,00	0,00
$f_j^4(x)$	0.21	0,00	0,00	0,00	0,00	0,00	0,00	0,00
$f_j^5(x)$	0.79	0,00	0,00	0,00	0,00	0.53	0,00	0,00
District	25	26	27	28	29	30	31	32
$f_j^1(x)$	0,00	0.91	0,00	0.95	0,00	0,00	0.64	0.68
$f_j^2(x)$	0.98	0.09	0.92	0,00	0.41	0.97	0,00	0,00
$f_j^3(x)$	0.02	0,00	0.08	0,00	0.59	0.03	0,00	0,00
$f_j^4(x)$	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
$f_j^5(x)$	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Step 4: From the Table IV, $\max\limits_{1 \le k \le s}\{\sigma_i^k\}$ was calculated for each district, the results are shown in Table V:

TABLE V. VALUES OF σ_i^k FOR EACH DISTRICT

1	2	3	4	5	6	7	8
0.49	0.83	0.50	0.94	0.66	0.97	0.96	0.82
Very Low	Very Low	Medium	Low	Very low	Low	Very Low	Very Low
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
0.99	0.66	0.56	0.98	0.86	0.78	0.72	0.52
Very Low	Very Low	Medium	Medium	Low	Medium	Very Low	Low
17	**18**	**19**	**20**	**21**	**22**	**23**	**24**
0.79	0.70	0.62	0.89	0.88	0.53	0.81	0.94
Very High	Low	Very Low	Very Low	Medium	Very High	Very Low	Very Low
25	**26**	**27**	**28**	**29**	**30**	**31**	**32**
0.98	0.91	0.92	0.95	0.59	0.97	0.64	0.68
Low	Very Low	Low	Very Low	Medium	Low	Very Low	Very Low

IV. RESULTS AND DISCUSSION

The results and the discussion, according to the specific objective in this study, are presented as follows:

A. The Case Study

Fig. 4, constructed from Table V, shows the following results:

First, in 2017, the districts that consume the most drinking water in Lima were San Isidro and Miraflores. 447.5 Lt. and 395.2 Lt. respectively.

Second, in 2017, the districts with the less water consumption in Lima were Ancon, Ate, Carabayllo, Chorrillos, Comas, Independencia, Los Olivos, Rimac, San Juan de Lurigancho, San Juan de Miraflores, San Martin de

Porres, Santa Anita, Villa El Salvador, and Villa María del Triunfo.

Third, in 2017, the districts that were within the mean were Barranco, Jesus Maria, La Molina, Lince, San Borja, and Santiago de Surco. All results are presented in Fig. 4.
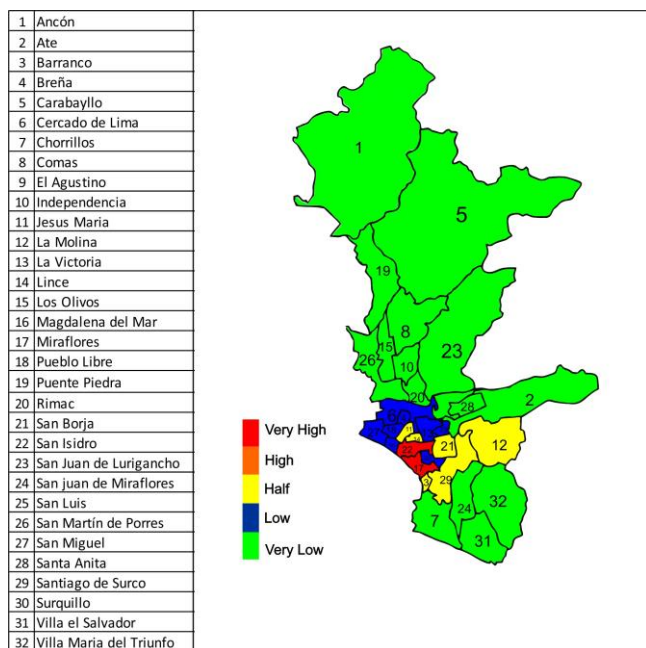

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13	La Victoria
14	Lince
15	Los Olivos
16	Magdalena del Mar
17	Miraflores
18	Pueblo Libre
19	Puente Piedra
20	Rimac
21	San Borja
22	San Isidro
23	San Juan de Lurigancho
24	San juan de Miraflores
25	San Luis
26	San Martín de Porres
27	San Miguel
28	Santa Anita
29	Santiago de Surco
30	Surquillo
31	Villa el Salvador
32	Villa Maria del Triunfo

Fig. 4. Drinking water consumption level by districts.

B. The grey clustering method

An advantage of the grey clustering method is that it considers uncertainty within its analysis [13], as some classical approaches to multi-criteria analysis, such as Delphi[18] or the hierarchical analytic process (AHP) [19], do not consider the uncertainty within their analysis, due to the fact that the importance of the degrees of the criteria is precisely presupposed [20]. Therefore, it could be argued that the grey clustering method could be applied to different problems, which have a high level of uncertainty.

V. CONCLUSIONS

The grey clustering method determined and classified the districts with a high and lower drinking water consumption level in the Lima city. In addition, the results obtained in this work could help to the central government, the local authorities of the districts, or SEDAPAL to make the best decision about the level of drinking water consumption.

As a major advantage, the grey clustering method could be more effective than other classical multi-criteria methods, since it considers the uncertainty within its analysis. Moreover, as a major limitation, methods based on grey systems theory are not widely disseminated compared to others approaches based on multi-criteria analysis or statistical models. In addition, the grey clustering method could be improved and simplified by implementing a computer system.

In future studies, the grey clustering method could be applied to other types of social problems, in which the number of criteria or variables involved could be

determined according to the particularities of each social type problem.

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