Determination of frequency assignment for a public transport system using a mathematical model.

Abstract— **The actual quantity of private vehicles emits an excessive sum of carbon dioxide this is why is important that the passengers uses the public transportation. It is necessary to offer a service equal or better than the private one. To do so, it has to be established a design of the public transport system and with this, a mathematical model that allows to simulate the system. In this paper we propose a mathematical model to estimate the frequency setting of the buses.**

I. INTRODUCTION

Public transportation is a key aspect of urban transport infrastructure providing economic, social and environmental benefits. With the increasing pressure upon global carbon dioxide emissions the role of public transport has been given a renewed focus [5], [6]. A focus were public transport systems (PTS) have to lure the passengers much more than before. There have been various approaches over the time like trains, public buses, Integrated Public Transport Systems (IPTS) among others.

The design of an efficient model of PTS is an urgent matter to encourage users away from the private transportation onto it. This design has to be done in such way that the overall time spent in the public transport of a user to get from his departure point to the final is less than the one on a private one, this is what we'll call a good service. If any PTS doesn't provide a good service, the passengers wont use it and the system it self wont be self sustainable.

A. *How to provide a good service?*

As mentioned before, PTS has to be designed in such way that they provide a good service. They way to do this is to divide the problem in reduced ones. This is well known as the Urban Transit Network Design Problem (UTNDP).

There are five subproblems of the [1]: network design; frequency setting; timetable development; bus scheduling and driver scheduling, with each problem requiring the output of the previous. In this paper we'll focus on the second, frequency setting narrowed for ITS. By doing an implementation of these subproblems, it's possible to provide a good service.

As mentioned before, there are various approaches over the time like trains, public buses, IPTS among others. But, there are cities where a train its not possible to implement, the public bus system is over saturated or heavily delayed and there's not water bodies enough to cover the whole city. In this cities the most accurate PTS is the IPTS.

The IPTS model such as Transmilenio consists in having a main road to be used specifically for the IPTS's buses which helps to minimize the in-vehicle travel time. Although this helps to minimize the overall time spent, the waiting time is an important matter for the users to choose the public transport over the private one. However, assigning specific roads to IPTS's buses can increase the in-vehicle time of the private transport users since it decrease the number of roads it could travel on but this intrinsically augments the quantity of users that would prefer the public transport. Additionally, extensive waiting times on IPTS can lead to a conglomeration of users which evolves to a system that facilitates pickpocketing, human harassment and even user protests if the previous two are frequent.

We consider that the waiting time is an important matter to the user since in IPTS this is the time that the user doesn't know he'll spend, this is why we think we'll have to minimize it. In order to do this, we'll consider how to determine the frequency setting of buses that are sent to transportation by taking in account the number of users waiting for a specific bus.

This paper is organized as follows: Section II shows a background of the ITS in Colombia. Section III discusses how the problem have been attacked. Section IV introduces our approach. Section V define the results got. Finally, Section VII presents the conclusions and future work.

II. BACKGROUND

Several cities has implemented ITS like Transmilenio (Bucaramanga, Barranquilla, Cartagena, Cali, Medelln and Risaralda). The study case we approach in this paper is Cartagena's ITS which is the Transcaribe. In this ITS the routes planning and frequency setting were determined 10 years before it was working so the studies made didn't apply very well when it started working. Currently, there are several routes and frequency assigned in Cartagena's ITS.

III. RELATED WORK

In this section we want to look at how the scientific community has approached the PTS. Specifically the six sub problems that [1] proposed. What they proposed are a series of subproblems that determines the steps to follow when designing a public transport system.

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1) Network design: The the network design subproblem goal is to set all the routes that the buses will follow.

With the timetable, it's be possible to schedule the buses according with the network design, [8] does it by searching the shortest path repeatedly in a timetable based public transport time-space network graph. It concerns on the edges of the graph, individual preferences of the travelers and departure time from the origin. This method takes into account bus and train transports and a series of functions to determine the choice sets output. Then evaluates the choice sets by using their size, by asking if it generated at least one path having a high similarity to a corresponding observed path and the ability to generate choice sets which facilities stable parameter estimates in the estimation of route choices. These choice sets are used to develop the network design.

2) Frequency setting: The frequency determines how often the buses will go on transit.

Schele [9] proposed a non-linear programming solution for the frequency assignment problem. He proposed to solve the problem by taking in account the capacity of the buses attempting to minimize the general travel time (walking, in vehicle and waiting time), assuming that the network routes are already established and only included the most relevant vertices which reduced the size of the problem. The proposed model was tested on the town of Linping with 6 routes.

Huang et al. [4] proposed a bi-level formulation for assigning the frequency setting. The upper level concerned in two factors: the network cost which is the passengers' expected travel time that is based on uncertain bus passenger demand and operating costs and the network robustness, this is indicated bu the variance in passengers' travel time. The lower level, he calculated the proportional flow eigenvalues using the optimal strategy transit assignment model.

3) Timetable Development: Parbo, Jens and Nielsen, Otto Anker and Prato, Carlo Giacomo [7] calculated an optimized timetable by using a transit network already established. They calculated a weight of the transfer waiting time, optimized this weight using a public assigment model to evaluate the behavior changes of the passengers, imposed a new timetable using the changes evaluated in the optimization and repeat this process until there wasn't any optimization achieved. This approach was applied in a large scale network route in Denmark. Although it was applied in all transit lines of Denmark, only the buses timetables were changed due to it's optimization. It took 5 iterations to converge, the transfer waiting time weight was improved in 5% and the general travel cost was reduced in 0.3%.

Song, Baoyang and Wynter, Laura citesong2017real proposed a modern method to determine a real-time timetable using a WiFi network on every train line. They do it using a spectral clustering method based on derived commuter trajectories. The data they proposed to collect is a record for every device which has minimum fields including an anonymous identifier (MAC address), a timestamp and location. The probe time varies on every device.

4) Bus Scheduling: Wagale, Makrand and Singh,[10]

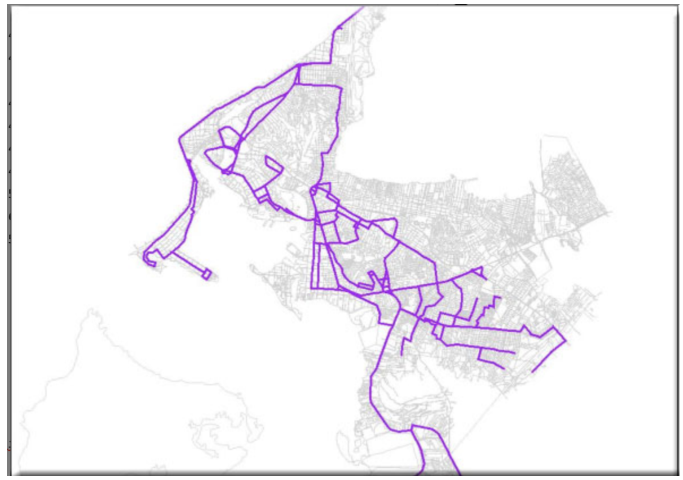

Fig. 1. Transcaribe's network road.

5) Driver Scheduling: Han and Wilson [3] had the objective of minimizing the occupancy at the most loaded point on any route in the network. They added a constraint where there is a maximum number of buses to reduce the size of the problem. The passenger assignment problem was attacked as follows: if a there's only one route for an origin-destination pair available, this route was used. However, if there is a set of routes available the demand was shared between them using frequency sharing [2]. The methodology for this solution is to calculate a lower bound by iteratively assigning passenger flows and rout frequencies until a convergence was achieved. Then. they formulated a surplus allocation problem by linear constraints. The proposed model was tested on an instance of 6 vertices and 3 routes.

IV. Our approach

In this section, we'll explain how we got the formulas to estimate the current state of Transcaribe.

A. Road network

Transcaribe already has the network routes defined. These are shown in Fig.1. Since they're already established, we'll have to work with these routes but to narrow the problem, we'll focus only in one route.

B. Frequency estimation

To get the formula for frequency setting, we need to calculate the departure time first (Dt). To do this, we had to estimate the departure time which is calculated with the travel time of the current hour (t_h), the number of buses assigned for the route (b) and the enlistment time of the bus (Et). This enlistment time is the one that every bus takes from the time it arrives to the final point to the time it's ready to transportation.

$$Dt = \frac{\frac{\sum_{i=0}^{b} Et_i}{b} + t_h}{b} \tag{1}$$

$$Wtu_i = [St + b_i * (\frac{Et_i + t_h}{b})] - Atu_i \tag{2}$$

Now we can estimate the frequency (F), we took the departure time (Dt) the quantity of time in minutes (t) of the range that F will be calculated and the number of busses asigned to that route.

$$F = \frac{Dt * t}{b} \tag{3}$$

C. Variable estimation

To estimate the maximum capacity per hour for the route (C_h), we used the frequency (F) and the maximum capacity of each bus (C_b):

$$C_h = C_b F \tag{4}$$

D. Estimating waiting time

To estimate the waiting time (W_t), we used the time when the user arrives to the bus stop (A_t) and the departure time (Dt):

$$W_t = Dt - A_t \tag{5}$$

For the proposes of this paper, we'll assume that A_t is 0 since we can't know exactly when the user will arrive to the bus stop. This lead us with the maximum waiting time which is Dt.

E. Data survey

To define the time spent to get from the initial point to the end, we used the Google traffic API by taking the estimated range time it shows when departing at a certain hour in a business day and the real time. Then, we took the percentage errors of the lower limit, the average limit and the upper limit. Calculated the average and used the one that has the minimum error to be use it for further investigation. This results are shown in Table I.

V. Results

In this section, we'll show the results got with the equations exposed previously with the study case route. This route has six buses assigned and the travel times are shown in Table I.

In table II we show the frequency got with Equation 3.

In table III we show the results got with Equation 4

To ensure that this is how Transcaribe's model works, we waited for an hour (10:00 to 11:00) in one bus stop for one week and witnessed that the frequency was within a rage of 10 b/h and 12 b/h which is what we estimated with the previous results.

VI. Future work

For future work we want to adjust the model proposed and do another iteration of data gathering. This would improve the model estimation.

VII. Conclusion

We saw that in it's first iteration, the model doesn't adjust to the reality. Adjusting the model proposed could improve the results.

TABLE I
GOOGLE TRAFFIC TIMES

Hour	Estimated time (mins)	Real time (mins)	%error min	%error average
9:00	12 to 16	13	7,7	7,7
10:00	12 to 16	14	14,3	0,00
11:00	12 to 16	13	7,69	7,69
12:00	12 to 16	13	7,69	7,69
13:00	12 to 16	13	7,69	7,69
14:00	12 to 16	14	14,29	0,00
15:00	12 to 18	14	14,29	7,14
16:00	12 to 18	13	7,69	15,38
17:00	12 to 20	16	25,00	0,00
18:00	14 to 20	18	22,22	11,11
19:00	12 to 18	15	20,00	6,67
20:00	12 to 16	14	14,29	7,14
average		14,17	13,57	6,5

TABLE II
FREQUENCY ESTIMATION

Hour	Time in route (mins)	%Freq. estimation (mins)
9:00	26	4,3
10:00	28	4,7
11:00	26	4,3
12:00	26	4,3
13:00	26	4,3
14:00	28	4,7
15:00	28	4,7
16:00	26	4,3
17:00	32	5,3
18:00	36	6,0
19:00	12	5,0
20:00	28	4,7

TABLE III
TRANSCARIBE'S MAXIMUM PASSENGER CAPACITY

Hour	Freq. estimation (mins)	Max. capacity per hour (passengers)
9:00	4	1300
10:00	5	1400
11:00	4	1300
12:00	4	1300
13:00	4	1300
14:00	5	1400
15:00	5	1400
16:00	4	1300
17:00	5	1600
18:00	6	1800
19:00	5	1500
20:00	5	1400

References

[1] Avishai Ceder and Nigel HM Wilson. Bus network design. *Transportation Research Part B: Methodological*, 20(4):331–344, 1986.

[2] Claude Chriqui and Pierre Robillard. Common bus lines. *Transportation science*, 9(2):115–121, 1975.

[3] Anthony F Han and Nigel HM Wilson. The allocation of buses in heavily utilized networks with overlapping routes. *Transportation Research Part B: Methodological*, 16(3):221–232, 1982.

[4] Zhengfeng Huang, Gang Ren, and Haixu Liu. Optimizing bus frequencies under uncertain demand: case study of the transit network in a developing city. *Mathematical problems in Engineering*, 2013, 2013.

[5] Matthew P John. *Metaheuristics for Designing Efficient Routes & Schedules For Urban Transportation Networks*. PhD thesis, Cardiff University, 2016.

[6] Natalie Mueller, David Rojas-Rueda, Xavier Basagaña, Marta Cirach, Tom Cole-Hunter, Payam Dadvand, David Donaire-Gonzalez, Maria Foraster, Mireia Gascon, David Martinez, et al. Urban and transport

planning related exposures and mortality: a health impact assessment for cities. *Environmental health perspectives*, 125(1):89, 2017.

[7] Jens Parbo, Otto Anker Nielsen, and Carlo Giacomo Prato. User perspectives in public transport timetable optimisation. *Transportation Research Part C: Emerging Technologies*, 48:269–284, 2014.

[8] Thomas Kjær Rasmussen, Marie Karen Anderson, Otto Anker Nielsen, and Carlo Giacomo Prato. Timetable-based simulation method for choice set generation in large-scale public transport networks. *European Journal of Transport and Infrastructure Research*, 16(3):467–489, 2016.

[9] Siv Schéele. A supply model for public transit services. *Transportation Research Part B: Methodological*, 14(1):133–146, 1980.

[10] Makrand Wagale, Ajit Pratap Singh, Ashoke K Sarkar, and Srinivas Arkatkar. Real-time optimal bus scheduling for a city using a dtr model. *Procedia-Social and Behavioral Sciences*, 104:845–854, 2013.