Fee Schedule Design when Debt Collection is Price Sensitive

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Abstract– When payment for a good (or service) is not required to be done immediately after its delivery, collection in full may not be possible. This is critical for post-paid billing systems such as mobile phone, water, electricity, and other utility service providers. In order to maximize their revenue, firms of these characteristics must not focus solely on increasing sales; instead, more importantly, they should focus on increasing collection. Thus, pricing should consider not only the good's demand elasticity, but also market's payment capacity—or its willingness to pay. We propose a linear programming model that can be used to maximize a firm's revenue collection over a one-period decision horizon. Our model works by segmenting customers based on their consumption level and assumes that customer's willingness to pay is similar within each segment. Prices are, then, found for each customers' segment. A case study is provided and its solution analyzed to develop further insights about the model.

Keywords— Pricing, willingness to pay, collected revenue

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Abstract– When payment for a good (or service) is not required to be done inmediately after its delivery, collection in full may not be possible. This is critical for post-paid billing systems such as mobile phone, water, electricity, and other utility service providers. In order to maximize their revenue, firms of these characteristics must not focus solely on increasing sales; instead, more importantly, they should focus on increasing collection. Thus, pricing should consider not only the good's demand elasticity, but also market's payment capacity—or its willingness to pay. We propose a linear programming model that can be used to maximize a firm's revenue collection over a one-period decision horizon. Our model works by segmenting customers based on their consumption level and assumes that customer's willingness to pay is similar within each segment. Prices are, then, found for each customers' segment. A case study is provided and its solution analyzed to develop further insights about the model.

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I. Introduction and Literature Review

When payment for a good (or service) is not done immediately after its delivery, it may not be possible to fully collect the debt. According to Zeng et al. in [1] the two main factors for invoice-payment risk are customer's financial capabilities and their willingness to pay. In their work, willingness to pay is defined as the maximum amount that someone is willing to pay to purchase a good or service.

Under these circumstances if a firm wants to find a price for a good or service that maximizes its revenue, the pricing model should not focus solely on increasing demand. The reason for this is that selling a good or service may not result in revenue if the customer decides not to pay the bill. The previous observation becomes more critical when the billing process follows a post-paid billing system, as in the case of mobile phone, water, electricity, and other utility service providers.

Moreover, the behavior of customers for post-paid goods is not exclusively related to the *per unit fee*, as in a regular price elasticity demand relationship. This has been confirmed empirically in the work presented by Inglesi-Lotz and Blignaut in [2], where a panel data study was conducted to assess electricity price elasticity in different sectors. The study concludes that the industrial sector was the only sector showing price elasticity for electricity; in sectors such as agriculture, transportation, and mining, the price of electricity did not affect its consumption.

The effect of price for post-paid goods or services can be understood by noting that the purchase decision and payment decisions are taken at different moments in the acquisition process, as shown in Figure 1. In pre-paid billing systems, a customer decides to purchase a good just after the price is known, hence that decision is directly affected by price elasticity of demand. On the other hand, in post-paid billing systems the customer decides to pay just after the good has been consumed, and the bill received. As a result, the decision to disburse is affected by the customer's willingness to pay. Our model aims to define a price that increases a firm's *collected* revenue, therefore, focusing on the payment decision made by the customer.
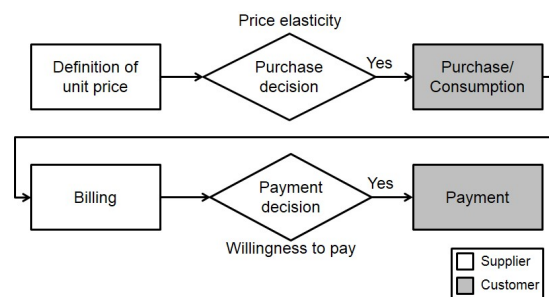

Fig. 1: Acquisition process for post-paid goods.

The effect of trade credit within a supply chain has been study by Fabbri and Klapper in [3], where they offer insights of how market power and financial constraints affects a firm's financial decisions and credit conditions for its customers. However, the effect of debt delinquency was not studied.

Credit policies have also been included in lot-sizing models, like in the one presented by Shinn in [4], where the retailers profit depends on the product price and credit terms offered by the seller. Abad and Jaggi in [5] presented a model where the unit price and the credit length are the decision variables, and analyze the model under the assumption of cooperative and non-cooperative structures. Chang et al. in [6] included a permissible delay in payment based on the order quantity in their lot-sizing model, where the market demand was price sensitive.

The previous models assume that payment will occur in a specified period of time. However, in practice, the payment may never occur, and therefore the actual profit is reduced.

We propose a one-period pricing model that can be used to maximize a firm's revenue, assuming that the firm offers a post-paid billing system to its customers. As in practical scenarios, customers may decide to pay the entire amount billed or only a portion of it. This decision depends on the total indebted amount and customer's financial capability. In

this paper, we are not including the consequences of debt delinquency for customers—such as, service interruption or late payment fees—since we are only focusing on the firm's revenue during one period.

Our model can be used, not only within commercial environments, but also from a social perspective. Policy makers interested in defining the price for social goods or services, may offer price differentiation by taking into account the willingness to pay of social sectors that can only assume low debt levels. As a result, the model can maximize the actual revenue, and reduce the negative impact of outstanding debt and service disruption.

The remainder of this paper is organized as follows. Section II describes the main assumptions about customer's consumption and payment behavior. Section III presents the pricing model, where the revenue function is maximized. Section IV shows numerical results obtained from the model implementation, and Section V summarizes our work and discusses future research directions.

II. CUSTOMER'S CONSUMPTION AND PAYMENT DESCRIPTION

In this section we give a mathematical description of customer's consumption and payment behavior for a particular good (or service). In what follows we will use the term *product* implying, either, good or service.

The quantity q consumed of a product is considered as a random variable with a probability distribution F_q, and expectation defined as $E[q] = \mu$.

We define $f(q)$ as the fee charged for the qth unit consumed. Fee $f(q)$ may be affected by internal or external factors. For example, when the consumption of a good needs to be regulated, $f(q)$ can be increasing in q. On the other hand, if economies of scale are present, then $f(q)$ can be decreasing in q.

Fee $f(q)$ is not necessarily a continuous function. Particularly, in this paper we are interested in defining a fee schedule, where $f(q)$ is defined as a stepwise function and customers are grouped into subsets R_m's, for $m \in \mathcal{M} = \{1, \ldots, M\}$. In order to create the groups, we define breakpoints $r_m \in \mathbb{R}^+$, where $r_{m-1} < r_m$. A client consuming $r_{m-1} < q \leq r_m$ is then assigned to subset R_m. In Appendix A we show a procedure for obtaining breakpoints r_m from data. Figure 2 shows an example of such a fee schedule.
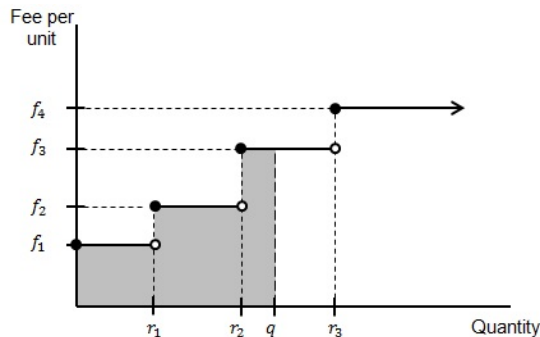

Fig. 2: Stepwise fee function.

The total amount billed to a customer consuming q units is thus given by

$$B(q) = (q - r_{m-1})f_m + \sum_{j=1}^{m-1}(r_j - r_{j-1})f_j,$$

for $q \in R_m$. In Figure 2, the shaded area corresponds to the total amount billed $B(q)$, for $q \in R_3$.

We assume that the amount paid by a customer depends on the amount billed $B(q)$. The proportion of the bill payed is a random quantity $\lambda_q \in [0, 1]$, with expectation $E[\lambda_q] = \pi_q$. Note that π_q depends on q.

The collected amount C_q for a consumption of q units is, thus, given by

$$C_q = B(q)\lambda_q,$$

where $0 \leq C_q \leq B(q)$. Due to its construction, note that C_q is a random variable.

For a particular value of q, the expected collected amount $E[C_q|q]$ is given by

$$E[C_q|q] = B(q)\pi_q.$$

Assuming that there are N customers whose consumption levels are independent, we may express the set of individual consumptions q_n for $n \in \mathcal{N} = \{1, \ldots, N\}$ as an i.i.d. random sample $Q_N = \{q_1, \ldots, q_N\}$, where $q_n \sim F_q$.

The total amount collected from all customers within a sample Q, defined as $T(Q)$, is given by

$$T(Q) = \sum_{q \in Q} C_q$$
$$= \sum_{q \in Q} B(q)\lambda_q.$$

We may compute the expectation of the total amount collected, given a random sample Q_N, $E[T(Q)|Q = Q_N]$, as

$$E[T(Q)|Q = Q_N] = \sum_{n \in \mathcal{N}} E[C_q|q = q_n]$$
$$= \sum_{n \in \mathcal{N}} B(q_n)E[\lambda_{q_n}]$$
$$= \sum_{n \in \mathcal{N}} B(q_n)\pi_{q_n}.$$

In this paper, we are interested in defining a fee schedule model that maximizes the expected total amount collected $E[T(Q)|Q = Q_N]$, assuming that the individual consumptions are known. The expression for the unconditional expectation $E[T(Q)]$ when the individual consumptions are unknown is discussed in Appendix B.

III. FEE SCHEDULE MODEL

In this section we present a linear programming model for the design of a fee schedule that maximizes our expected total amount collected and that includes a price differentiation based on volume consumed. The model considers a product with inelastic demand.

In our model we will consider that the product fee is increasing in the volume consumed. This can be the case for

scarce products, when the supplier is interested in moderating customer's consumption. Note, however, that if a supplier is willing to offer price reduction due to economies of scale, our model can be easily modified to consider such decreasing fees.

We want to maximize the total amount billed, denoted by $z = E[T(Q)|Q = Q_N]$. With this in mind, we define our linear programming model as

$$\max_{f_m} \quad z \tag{1a}$$

$$\text{s.t.}$$

$$z \geq K + k \sum_{q \in Q_N} q, \tag{1b}$$

$$\sum_{q \in R_m} B(q)\pi_q \geq z \cdot \alpha_m, \quad \text{for } m \in \mathcal{M} \tag{1c}$$

$$f_m - f_{m-1} \geq \delta_L^m, \quad \text{for } m \in \mathcal{M} \tag{1d}$$

$$f_m - f_{m-1} \leq \delta_U^m, \quad \text{for } m \in \mathcal{M} \tag{1e}$$

$$f_m \in \mathbb{R}^+, \quad \text{for } m \in \mathcal{M}. \tag{1f}$$

Here the parameters are:

N Number of customers

q_n Quantity of product consumed by customer $n \in \mathcal{N}$. We assume that $q_n \leq q_{n+1}$ for $n \in \mathcal{N}$

K Fixed cost incurred by the supplier

k Variable cost per unit, incurred by the supplier

M Number of subsets of customers aggregated by consumption

α_m Fraction of the total amount collected from customers in subset R_m for $m \in \mathcal{M}$

δ_m^u Upper bound for the difference in the fee between customers in subset $m - 1$ and m, for $m \in \mathcal{M}$

δ_m^l Lower bound for the difference in the fee between customers in subset $m - 1$ and m, for $m \in \mathcal{M}$

π_q Expectation of the proportion of the bill payed by a customer that consumes q units of product

r_m Break point defining subsets R_m, such that $q \in R_m$ iff $r_{m-1} < q \leq r_m$, for $m \in \mathcal{M}$. We define $r_0 = 0$, and $r_M = \max_{n \in \mathcal{N}} q_n$,

and the variables are:

f_m Fee per unit of product consumed for customers with consumption level $q \in R_m$, where $m \in \mathcal{M}$. We define $f_1 \geq \underline{f}$, and $f_M \leq \overline{f}$.

We want to maximize the revenue obtained by the supplier after debt collection, defined in Section II. The constraint defined in (1b) guarantees that the total amount collected covers the fixed and variables costs for the firm. Note that, if (1b) is not satisfied, the firm will have a deficit and may decide not to supply the product. Constraint (1c) makes sure that the percentage of the total revenue collected from each subset of customers is at least equal to α_m. Note that α_m can be set equal to zero, when there is no lower bound in the revenue that has to be collected from a subset R_m. The set of constraints (1d) and (1e) guarantee that the increase in the product fee does not exceeds the upper and lower bound

defined by the supplier. Finally (1f) define the model variables as continuous and non-negative.

IV. NUMERICAL TEST

In this section we provide a numerical case study for designing the fee schedule of an electricity provider firm. This case study shows the implementation of the proposed model. Additionally, we provide an analysis of the case study and its solution to develop further insights about the model. The case study uses monte-carlo simulation to generate a dataset of monthly consumptions.

A. Generating the Dataset

We generate $N = 1,000$ data points corresponding to individual electricity consumption q_n, given in kWh. An exponential distribution was used to generate the amount consumed monthly, with mean value 289.25 kWh/month.

The consumption range is subdivided into four subsets $R_1, \ldots, R_4$, as shown in Figure 3. The first subset R_1 is defined to contain the bottom 40% of the customers in terms of their consumption. The second subset R_2, is defined to contain the next lowest 35% of the customers in terms of their consumption. Likewise, for R_3 and R_4 we used the next 20%, and 5%, respectively.
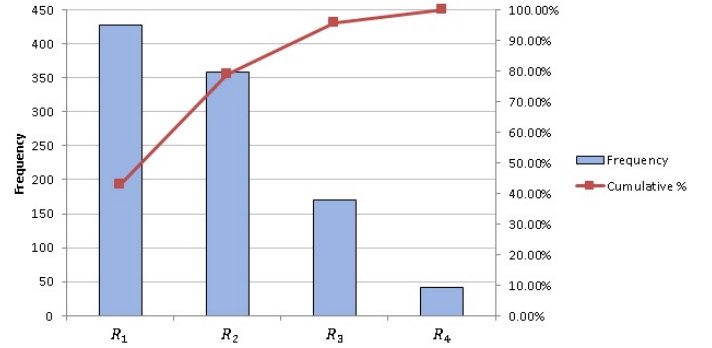

Fig. 3: Histogram of generated consumptions.

Points r_m were defined analytically as the 0.4, 0.75, 0.95, and 1 quantile of the exponential distribution, respectively. The resulting r_m points, as well as the corresponding consumption within each subset R_m, are shown in Table I.

TABLE I: Generated consumption

m	r_m	# of customers in subset	Consumption within subset
1	153	429	30,387.39
2	416	359	93,524.05
3	899	170	99,804.07
4	$\max_i\{q_i\}$	42	46,929.03

B. Definition of Model Parameters

In this section we present the parameters used in our numerical case study. The parameters used were chosen to emulate a realistic scenario.

For overhead costs K we used $\$ 11,934.73$, and for variable costs k we used 0.038 \$/kWh. Using these parameters, the total

cost for generation and distribution is obtained as $K + k \sum_i q_i$ which is \$ 22,316.65.

Table II shows the numerical values used for the difference of the fees δ_m^u and δ_m^l, the fraction α_m of the total amount collected for customers in subset R_m, as well as the proportion of the bill paid π_m for each customer in subset R_m.

TABLE II: Used parameters

m	δ_m^l	δ_m^u	α_m	π_m
1	-2	2	0.2	0.4
2	-2	2	0.6	0.9
3	-2	2	0.5	0.8
4	-2	2	0.3	0.7

We used an inverted "U" shape for the parameters π_m, as shown in Figure 4, assuming that customers in the low range of consumption will be more likely to have financial difficulties, and therefore, the proportion of the bill paid will be low; while customers in the high range of consumption defaults often.
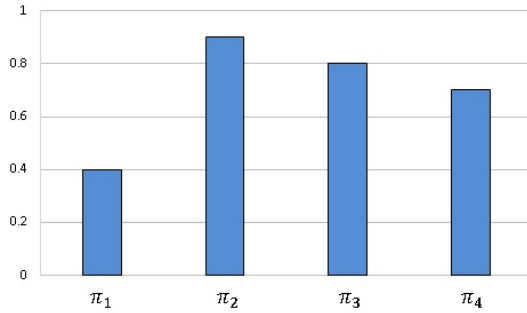

Fig. 4: Proportion π_m of bill payed for each subset R_m.

C. Results and Analysis

The linear programming model, for these particular choice of parameters, proofs to be feasible and provides the fee schedule f_m, for $m = 1, \ldots, 4$. The results are presented in Table III.

TABLE III: Results

m	f_m	Total Billed for R_m	Total collected from R_m
1	1.08	\$ 32,818.38	\$ 13,127.35
2	3.08	\$ 178,022.24	\$ 160,220.02
3	2.00	\$ 223,861.02	\$ 179,088.82
4	2.00	\$ 81,484.34	\$ 57,039.03
	Total	\$ 516,185.99	\$ 409,475.23

The total aggregated amount billed to all the customers was \$ 516,185.99. Because of the willingness to pay, only \$ 409,475.23 was collected. It can readily be checked that the amount collected, \$ 409,475.23, covers the generation and distributions costs \$ 22,316.65 by a large margin.

V. CONCLUSIONS AND FUTURE DIRECTIONS

Pricing models often focus on the effect of price on the product demand, without taking into consideration the actual amount collected. The effect of debt delinquency becomes more critical under post-paid billing systems, where the customer may decide not to pay the bill in full, even after the product has been consumed.

We propose a pricing model that finds the optimal fee schedule to increase a firm's collected revenue. Our model assumes that customers are aggregated by consumption and bill payment behavior, and finds the optimal unit fee for each subset of customers.

By including information about the expected proportion of the bill payed by the customers, our model allows the seller to define fees that guarantee that amount collected will be at least equal to the total cost. Also, in a way, this model can ensures a fair fee definition, since subset of customers with known risk of debt default can be identified and controlled.

As future research, we would like to consider the values for the breakpoints r_m, that define the fee function, as model variables. In this way, the model will define how the customers will be aggregated. Also, we would like to extend our model to a finite number of periods, and therefore, include the collected amount from customers with late payments. A multi-period model will further allow us to evaluate the efficiency of our pricing decisions, making sure that initial investment costs and operational cost are covered over the considered time horizon.

APPENDIX A
ESTIMATION OF THE PROPORTION PAYED BY EACH CUSTOMER

Here we will show a procedure to estimate the proportion payed π_q of the total bill $B(q)$. The procedure assumes that we have a dataset containing historical billed amounts $B(q_n)$ and their corresponding collected amounts C_{q_n}, for $n \in \mathcal{N}$.

Our estimation approach seeks to minimize the divergence between observed and predicted values. To measure this divergence we use the sum of the absolute differences, and formulate the following optimization problem

$$\min_{\pi_{q_n}} \quad \sum_{q \in Q_N} |C_q - E[C_q]|$$
$$\text{s.t.:} \quad 0 \leq \pi_{q_n} \leq 1, \qquad \text{for } n \in \mathcal{N},$$

where $E[C_q] = B(q)\pi_q$.

This problem can be cast as a linear programming problem as

$$\min_{\pi_{q_n}} \quad \sum_{n \in \mathcal{N}} \varepsilon_n^+ + \varepsilon_n^-$$
$$\text{s.t.:} \quad C_{q_n} - B(q_n)\pi_{q_n} \leq \varepsilon_n^+, \quad \text{for } n \in \mathcal{N}$$
$$B(q_n)\pi_{q_n} - C_{q_n} \leq \varepsilon_n^-, \quad \text{for } n \in \mathcal{N}$$
$$\pi_{q_n} \leq 1, \qquad \text{for } n \in \mathcal{N}$$
$$\pi_{q_n}, \varepsilon_n^+, \varepsilon_n^- \geq 0, \qquad \text{for } n \in \mathcal{N},$$

and can be trivially solved by

$$\pi_{q_n} = \frac{C_{q_n}}{B(q_n)}, \qquad \text{for } n \in \mathcal{N}.$$

Note that, since $C_{q_n} \leq B(q_n)$, then $0 \leq \pi_{q_n} \leq 1$.

However, having the value of π_{q_n} for each customer n with a specific consumption level q_n is not useful when we are trying to define a general fee schedule.

We now consider the setting in which customers have been classified into different subsets R_m, for $m \in \mathcal{M}$, depending on their amount consumed. If customers' willingness to pay is similar within each subset R_m, we may then assume that there is a single π_m for clients in R_m. The resulting optimization problem is now

$$\min_{\pi_1 \ldots, \pi_M} \quad \sum_{n \in \mathcal{N}} \varepsilon_n^+ + \varepsilon_n^-$$

$$\text{s.t.:}$$

$$C_{q_n} - B(q_n)\pi_m \leq \varepsilon_n^+, \text{ for } m \in \mathcal{M}, \text{ and } \{n : q_n \in R_m\}$$

$$B(q_n)\pi_m - C_{q_n} \leq \varepsilon_n^-, \text{ for } m \in \mathcal{M}, \text{ and } \{n : q_n \in R_m\}$$

$$\pi_{m-1} r_{m-1} = \pi_m r_{m-1}, \text{ for } m \in \mathcal{M} \setminus \{1\}$$

$$\pi_m \leq 1, \text{ for } m \in \mathcal{M}$$

$$\pi_m \geq 0, \text{ for } m \in \mathcal{M}$$

$$\varepsilon_n^+, \varepsilon_n^- \geq 0, \text{ for } n \in \mathcal{N}.$$

The problem above can be used to estimate the expected payed amount, π_m, for customer within each subset R_m, for $m \in \mathcal{M}$.

APPENDIX B
UNCONDITIONAL EXPECTATION OF THE TOTAL AMOUNT COLLECTED

We denote a random sample containing the consumption of N customers by $\Omega = \{q_1 \ldots, q_N\} \sim F_q^N$. We find the unconditional expectation $E[T(\Omega)]$ as

$$E[T(\Omega)] = E\left[E[T(\Omega)|\Omega = Q_N]\right]$$

$$= \sum_{q \in Q_N} E\left[E[C_q|\Omega = Q_N]\right]$$

$$= \sum_{q \in Q_N} E\left[E[B(q)\lambda_q|\Omega = Q_N]\right]$$

$$= E\left[\sum_{q \in Q_N} B(q)\pi_q|\Omega = Q_N\right]$$

$$= N[B(\mu_q)\pi_{\mu_q}] + N \cdot COV[B(q), \pi_q].$$

We consider that there is a dependency between the amount billed to a customer and the proportion of that bill actually payed, i.e., $B(q)$ and π_q are regarded as dependent random variables. In general, however, we are not able to attribute $COV[B(q), \pi_q]$ a particular value, since the exact relationship between $B(q)$ and π_q cannot be generalized. Furthermore, even if the value for $COV[B(q), \pi_q]$ can be found, its value may be different among different regions. This is attributed to the non-monotonic shape of π_q as the one shown in Figure 4.

ACKNOWLEDGMENT

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