# Design of Cam-Follower Mechanism using VAP Function in the Displacement Law

Héctor Fabio Quintero, Ph.D.[1], Libardo V. Vanegas-Useche, Ph.D.[1], and Magd M. Abdel-Wahab, Ph.D.[2]
[1]Universidad Tecnológica de Pereira, Colombia, hquinte@utp.edu.co, lvanegas@utp.edu.co
[2]Ghent University, Belgium, magd.abdelwahab@ugent.be

***Abstract– This paper uses a new mathematical function, VAP function, to define the displacement law of a cam-follower mechanism. The VAP function is based on an exponential function; it has been devised so that the dependent variable increases and decreases almost linearly from its maximum and minimum values, and so that, at the same time, the function is smooth. As a case study, a disc cam with a translating roller follower is considered, using two displacement laws: based on a harmonic function and based on a VAP function. The cam-follower mechanism designed with the VAP function has smaller follower accelerations, as well as lower pressure angles than the one with harmonic function.***

***Keywords-- VAP function, harmonic function, displacement law, cam-follower mechanism, pressure angle.***

## I. INTRODUCTION

Cams are widely used in many types of machines because they make it possible to obtain an unlimited variety of motions. Many different types of cam profiles are designed and manufactured depending on machine requirements [1]. There are a high number of ways to express a cam profile mathematically. The functions of standard motions include harmonic, cycloid, trapezoidal, polynomial, etc. Depending on condition, these mathematical methods may be directly applied, modified, or combined into piecewise functions [1].

Spline curves have been successfully applied in cam design [2,3]. In [4], an improved approach for designing flexible cam profiles is considered by using smoothing spline curves; the method considers displacement, velocity, and acceleration.

In this article, we utilise a mathematical smooth function that may be adjusted to produce a small, almost minimum range of its derivative. Originally, this mathematical function was devised to obtain small driven shaft accelerations in a noncircular gear pair system [5].

## II. VAP FUNCTION

The concept from which the function has been conceived is illustrated by means of Fig. 1. The variable of interest, $y$, varies from known minimum, $y_{\min}$, and maximum, $y_{\max}$, values and may be characterised by the mean, $y_m$, and alternating, $y_a$, components. In order to attain the minimum range of its derivative ($y'_{\max} - y'_{\min}$), the function has to be a triangle wave. However, this is not smooth and produces abrupt changes in its derivative.

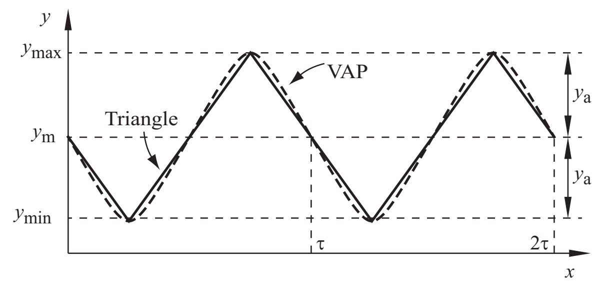


Fig. 1 Triangular and VAP function

In order to smooth the curves, an exponential function has been devised. In the new function, the slope is slightly greater than that of a triangle wave and is virtually constant, excepting that around $x = \tau/4$, $\tau/2$, $3\tau/4$, etc. it rapidly becomes zero. The slope is zero at these values in order that $y$ and $y'$ are smooth. This function is attained by [5]:

$$y(x) = y_m + \frac{2y_a}{1-b} h_1(x)\left(1 - b\, e^{\frac{1-b}{b}[2h_2(x)-1]}\right). \tag{1}$$

where

$$h_1(x) = \frac{1}{\pi}\arcsin\left(\sin\left(\frac{2\pi x}{\tau}\right)\right). \tag{2}$$

and

$$h_2(x) = \frac{1}{\pi}\arcsin\left\{\sin\left[\arccos\left(\cos\left(\frac{2\pi x}{\tau}\right)\right)\right]\right\}. \tag{3}$$

In (1), the "smoothness parameter" $b$ defines the shape of the curve and has to be in the interval (0,1). The closer the parameter is to zero, the more similar the function is to a triangle function, but it becomes less smooth.

The shapes of the auxiliary functions $h_1(x)$ and $h_2(x)$ are shown in Fig. 2.

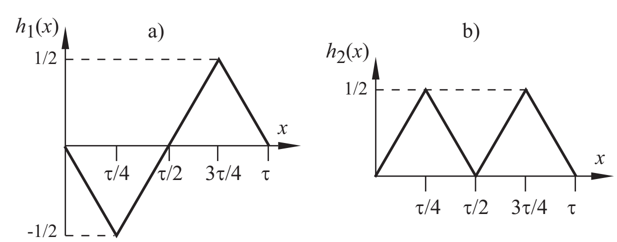


Fig. 2 Auxiliary functions a) $h_1(x)$, b) $h_2(x)$

## III. Case Study

As a case study, two disc cams with radial roller follower mechanisms are designed using as their displacement motions: i) a harmonic function, and ii) a VAP function with $b$ = 0.2. The radius of the cam prime circle is $R_0$ = 20 mm, the radius of the follower roller is $R_r$ = 10 mm. The follower rises 15 mm in the first 150º of cam revolution, dwells for 90º, and then falls in the remaining 120° rotation of the cam. Fig. 3 shows the two displacement motions.

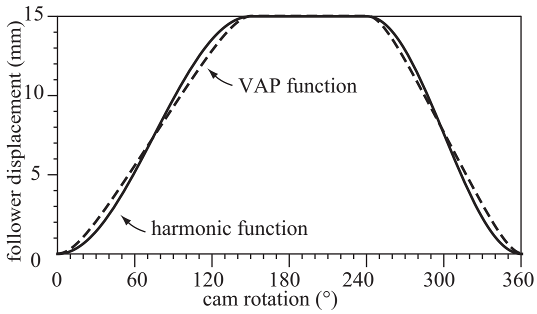


Fig. 3 Displacement law

The cam profiles require to deliver the harmonic motion and the VAP motion of the follower are shown in Fig. 4 and Fig. 5, respectively. The profiles were determined using analytical methods based on the principle of kinematic inversion, imagining the cam to be stationary and allowing the follower to rotate opposite to the direction of cam rotation.

The pressure angle is the angle between the direction of the path of the follower and the normal to the pitch curve through the centre of the cam follower. Neglecting friction, this normal is collinear with the contact force between the cam and the follower. Fig 6 shows the pressure angle of both designed cam-follower mechanisms. The pressure angles of the cam-follower mechanism with VAP function are lower than their counterparts of the cam-follower mechanism with harmonic function.

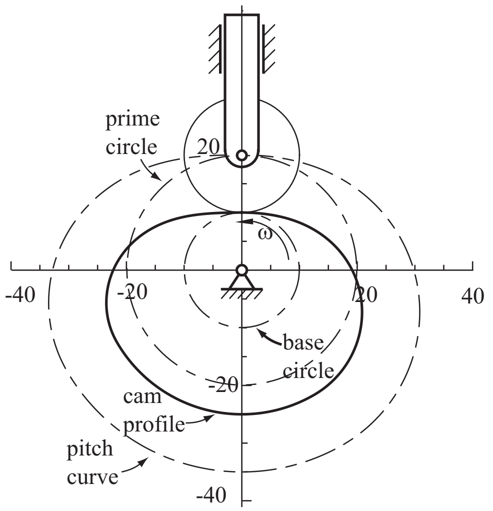


Fig 4. Cam profile with harmonic function

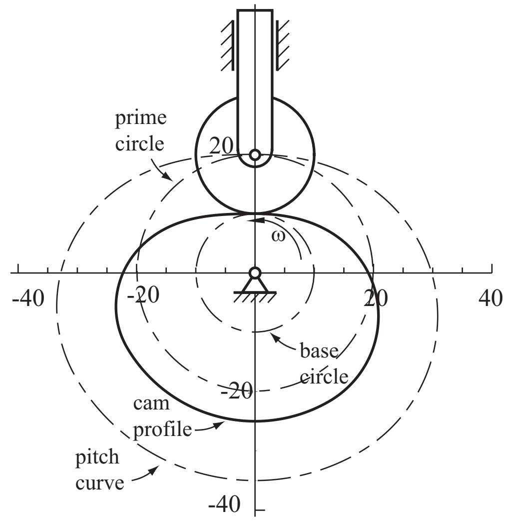


Fig 5. Cam profile with VAP function

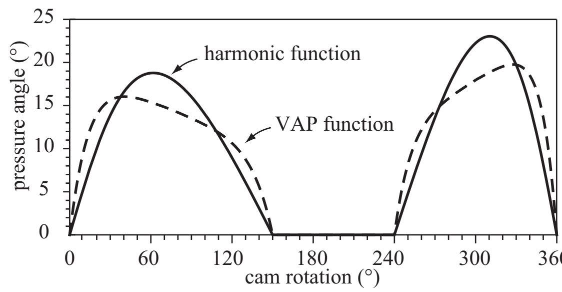


Fig.6 Comparison of pressure angle

## IV. Conclusions

The VAP function allows to obtain displacement motions of cam-follower mechanisms. This function depends on a smoothness parameter, $b$, which enables to modify the displacement law in the design process. According to the results, not only are the accelerations of the follower reduced with the VAP function, but also the pressure angles of cam-follower mechanism are lower than those of the cam-follower mechanism with harmonic function.

## References


[1] P. W. Jensen, *Cam Design and Manufacture*, 2nd ed., Marcel Dekker, 1987.

[2] H. Qiu, C. J. Lin, Z. Y. Li, H. Ozaki, J. Wang, and Y. Yue, "A universal optimal approach to cam curve design and its applications", *Mechanism and Machine Theory*, vol. 40, no. 6, pp. 669-692, 2005.

[3] S. P. Mermelstein and M. Acar, "Optimising cam motion using piecewise polynomials", *Engineering with Computers*, vol. 19, pp. 241-251, 2003.

[4] V. T. Nguyen and D. J. Kim, "Flexible cam profile synthesis method using smoothing spline curves", *Mechanism and Machine Theory*, vol. 42, no. 7, pp. 825-838, 2007.

[5] L. V. Vanegas, M. M. Abdel-Wahab, and G. A. Parker, "Design of noncircular gears to minimize shaft accelerations", *Proceedings of the 8th Biennial ASME Conference on Engineering Systems Design and Analysis*, Torino, Italy, July 4–7, pp. 421-430, 2006.