Graphic-analytical Method for the Kinetostatic Analysis of a Fourth-class Assur Group

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Abstract – This paper presents a method for the kinetostatic analysis of a fourth-class Assur group. The solution method uses two Assur special points. The study case considers a mechanism with 1 DOF and a fourth-class Assur group. Since there are no solutions for fourth-class structural groups in the literature, this method develops a complete modular procedure for the kinetostatic analysis of mechanisms, with the methodological advantages that this type of solution offers.

Keywords – kinetostatic analysis, fourth-class Assur group, structural analysis, modular analysis, special Assur point.

I. Introduction

In 1914 L.V. Assur proposed and developed a method for the formation of mechanisms as a consecutive superposition of kinematic chains that have determined structural properties. A structural Assur group can be defined as a kinematic chain with a number of degrees of freedom equal to zero, and it cannot be divided into simpler kinematic chains with a number of degrees of freedom equal to zero [1, 2].

The kinematic analysis of a mechanism can be carried out by the same sequence of the mechanism formation [3, 4, 7]. The force analysis sequence begins with the last group added to the mechanism, and it ends with the driving links. The analytical method for the kinetostatic analysis is found widely [5, 6]. The solution for the second and third class Assur groups using graphical methods can be found in the academic literature.

A fourth-class Assur group consists of two ternary links (with two internal joints and an external joint) and two binary joints (both are internal joints) [7]. From the literature review, it is concluded that the kinetostatic solutions for the fourth-class Assur groups practically do not exist in the specialized literature. This fact makes it difficult to develop a complete modular method for this kind of task [6]. This work aims at contributing with one solution to the aforementioned problem.

II. Methodology

We use a graphical-analytical method and the *special Assur points* to determine the reactions in the joints of the fourth-class Assur groups.

- The first step consists of determining the special Assur points for the binary links of the group with internal joints (referred as closure links).
- The components of internal forces can be obtained, such as an existing component with the direction from the internal joint to the special point, called here *normal*, and the other perpendicular to that one, called here as *tangential*.
- The moment equilibrium equations around special points can be obtained from the closure links.
- Afterwards the equilibrium equations for moments for each one of the two links with external joints, around the external joints of the group, are required.
- In four of the moment equations, the magnitude of the four tangential components of the reactions on the internal joints are unknown. Therefore, the linear equation system can be solved.
- In order to determining the respective normal components, the graphical method has to be used. The equilibrium equation of forces for every link with internal joints should be raised.
- The reactions on external joints can be determined using the force diagram of each dragging link.

III. Case study

We consider a mechanism of 1 DOF with structural formula $\mathrm{R-(RRP-RRR)}$. Fig. 1 shows mechanism with a fourth-class Assur group: links 2, 3, 4 and 5. The links with external joints (dragging links) are links 2 and 4 with joints B and G, respectively, as union joints to the base mechanism. Links 3 and 5 are those links that join the two ternary links with external joints. Geometric and inertial parameters (Par.) are registered in Table I.

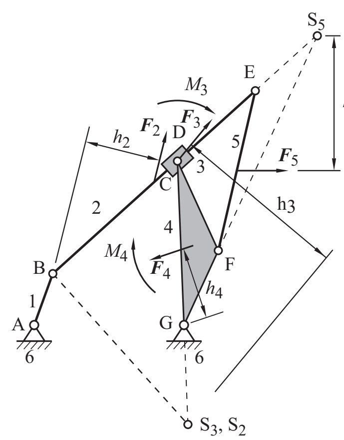

Fig. 1 Forces acting on the mechanism

The equation of moments for links 3 around point S_3 is:

$$M_3 + M_{S3}(\boldsymbol{F}_{23}) + M_{S3}(\boldsymbol{F}_3) + M_{S3}(\boldsymbol{F}_{43}^{t}) = 0 \qquad (1)$$

Considering as positive the clockwise moments, we obtain:

$$M_3 - F_{23} \cdot h_{23} + F_{43}^{t} \cdot L_{DS3} + F_3 \cdot h_3 = 0 \qquad (2)$$

TABLE I

GEOMETRIC AND FORCE PARAMETERS OF MECHANISM

Par	Value (mm)	Par.	Value (mm)	Par.	Value (N)
L_{AG}	275	L_{BS3}	248,060	F_2	(10, 40)
L_{AB}	100	L_{DS3}	481,985	F_3	(25, 30)
L_{BE}	500	L_{FS5}	436,292	F_4	(–30,–10)
L_{EF}	300	L_{ES5}	155,112	F_5	(40, 0).
L_{GD}	300	h_2	139,783	Par.	Value (Nm)
L_{FG}	150	h_3	323,089		
L_{FD}	180	h_4	132,458	M_3	3
		h_5	146,102	M_4	2

Equation (3) can be obtained for link 5.

$$M_{S5}(\boldsymbol{F}_{25}^{t}) + M_{S5}(\boldsymbol{F}_5) + M_{S5}(\boldsymbol{F}_{45}^{t}) = 0 \qquad (3)$$

Considering positive the clockwise moment, it has:

$$F_{25}^{t} \cdot L_{ES5} - F_5 \cdot h_5 + F_{45}^{t} \cdot L_{FS5} = 0 \qquad (4)$$

Figure 3 shows the forces acting on links 2 and 4. The equation of moments for link 2 around the external link B is:

$$M_B(\boldsymbol{F}_2) + M_B(\boldsymbol{F}_{32}) + M_B(\boldsymbol{F}_{52}^{t}) = 0 \qquad (5)$$

Considering negative the clockwise moments:

$$F_2 \cdot h_2 - F_{32} \cdot h_{23} - F_{52}^{t} \cdot L_{BE} = 0 \qquad (6)$$

Equilibrium equation of moments for link 4 around the external joint G can be obtained:

$$M_4 + M_G(\boldsymbol{F}_4) + M_G(\boldsymbol{F}_{34}^{t}) + M_G(\boldsymbol{F}_{54}^{t}) = 0 \qquad (7)$$

Considering positive the clockwise moments, it has:

$$M_4 - F_4 \cdot h_4 - F_{34}^{t} \cdot L_{DG} + F_{54}^{t} \cdot L_{FG} = 0 \qquad (8)$$

System of equations (2), (4), (6), and (8) can be solved.

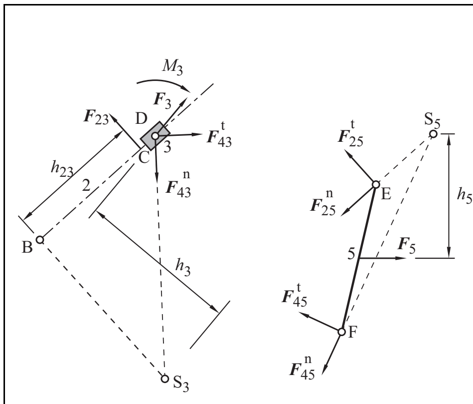

Fig. 2 Forces acting on links 3 and 5

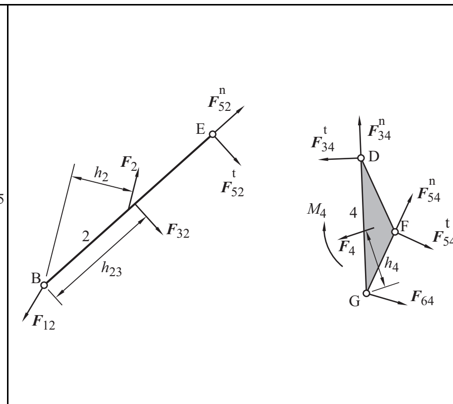

Fig. 3 Forces acting on link 2 and 4

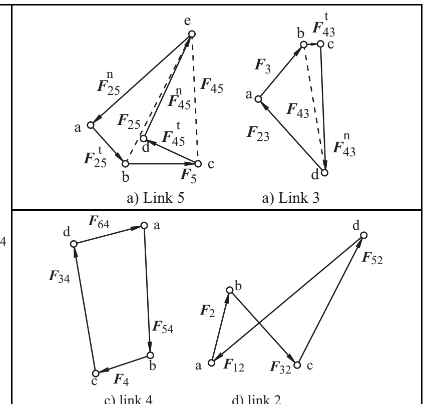

Fig. 4 Force diagrams of links 5, 3, 4, and 2

To determine the respective normal components, the equations for link 3 and 5 are written.

$$\boldsymbol{F}_{23} + \boldsymbol{F}_3 + \boldsymbol{F}_{43}^{t} + \boldsymbol{F}_{43}^{n} = 0 \quad (9); \qquad \boldsymbol{F}_{25}^{n} + \boldsymbol{F}_{25}^{t} + \boldsymbol{F}_5 + \boldsymbol{F}_{45}^{t} + \boldsymbol{F}_{45}^{n} = 0 \quad (10)$$

From the force diagram of link 3 and link 5, the values of F_{23}, F_{45} and F_{25} can be obtained.

The forces equations for link 2 and 4 are respectively:

$$\boldsymbol{F}_2 + \boldsymbol{F}_{32} + \boldsymbol{F}_{52} + \boldsymbol{F}_{12} = 0 \quad (11); \qquad \boldsymbol{F}_{34} + \boldsymbol{F}_4 + \boldsymbol{F}_{54} + \boldsymbol{F}_{64} = 0 \quad (12)$$

Figure 4 shows the force diagram of links 3 and 5. Fig. 4.c and Fig. 4.d shows diagrams based on (11) and (12) respectively. Taking the lengths, multiplying them by the force scale factor, the rest of the reactions can be obtained:

$$F_{12} = (-87{,}7031\text{N};\ -70{,}1672\text{N});\ F_{64} = (38{,}7031\text{N};\ 10{,}1672\text{N})$$

IV. CONCLUSIONS

A method that uses a combination of analytical and graphical methods to perform the kinetostatic analysis of a fourth-class Assur group is presented.

The results obtained through the proposed and the analytical methods are in complete agreement; on the other hand, the results of the commercial software have a maximum difference of 0,8%, this can be explained by the fact that the commercial software uses numerical methods to obtain the solution. The proposed methodology requires the solution of a system of four equations with four unknowns; the traditional method requires solving a system of equations of 12th order.

The developed method can be used for the kinetostatic analysis of planar mechanisms including such groups. This method allows to develop a modular procedure for the kinetostatic analysis of mechanisms. This is especially appropriate for pedagogical purposes.

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