Numerical Impulsive Response Analysis of a Frame Structure – A MATLAB Computational Project-Based Learning

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ABSTRACT

The aim is to implement a computational project-based learning methodology in engineering courses that can contribute to the success and improvement of students in engineering and engineering technology programs. This learning methodology is based on both computational and programming process that can enhance student's mathematical understanding, critical thinking, and problem solving skills through the core engineering courses.

A computational project based on both closed-form and numerical techniques with MATLAB programming will be implemented to study an impulsive response of a frame structure. The closed-form response of such system can be evaluated through convolution integral. For the case that a structure is subjected to an irregular impulsive forcing function such as seismic vibration, the numerical integration can be implemented with MATLAB programming to determine the impulsive response of such system.

In this investigation we will analyze the system response through a finite difference numerical technique. The technique can be implemented to investigate the impulsive response for both undamped and damped system with much less complex mathematical involvement. A MATLAB script based on finite difference technique can be developed to generate numerical results of such excitation during and after acting impulsive force.

Keywords: Impulsive response, Mixed finite difference, Numerical-based MATLAB Script

1. INTRODUCTION

The excitation of a system due to a suddenly non-periodic applied force that can only stay with the system for a very short period of time is known as transient impulsive response. A closed-form solution through convolution integral can be implemented to determine the impulsive excitation under an acting transient impulsive force. For an irregular forcing function the numerical integration (Mario Paz, 1980; Singiresu S. Rao, 2003) of convolution integral is implemented to determine the impulsive excitation. A MATLAB numerical-based script is developed to study and determine response of one-story frame structure during and after acting impulsive force.

In this investigation a mixed forward-backward finite difference techniques (H. Rahemi, 2009) is implemented to develop numerical form of the governing equation of motion due to a suddenly applied impulsive force. A MATLAB script is developed based on a finite difference numerical formulation to determine response of such system for both damped and undamped cases with an irregular acting impulsive force.

However any numerical solution involved with error, through Taylor's series, using both forward and backward finite difference, the truncation error been estimated to be in order of step size. To increase the accuracy of the numerical results, one should consider the step size as small as possible.

In this investigation we will study two cases of impulsive vibration; (1) Undamped System, (2) Damped System. For the case of undamped system the results by finite difference technique will be compared by the numerical integration of convolution integral.

2. COMPUTATIONAL PROJECT-BASED LEARNING MODEL

The aim is to implement a methodology based on computational project-based learning model to enhance student's learning objectives (analytical, numerical, critical thinking, and problem solving skills) through the engineering core courses (H. Rahemi, 2009). Figure 1 is a graphical model of computational project-based learning.

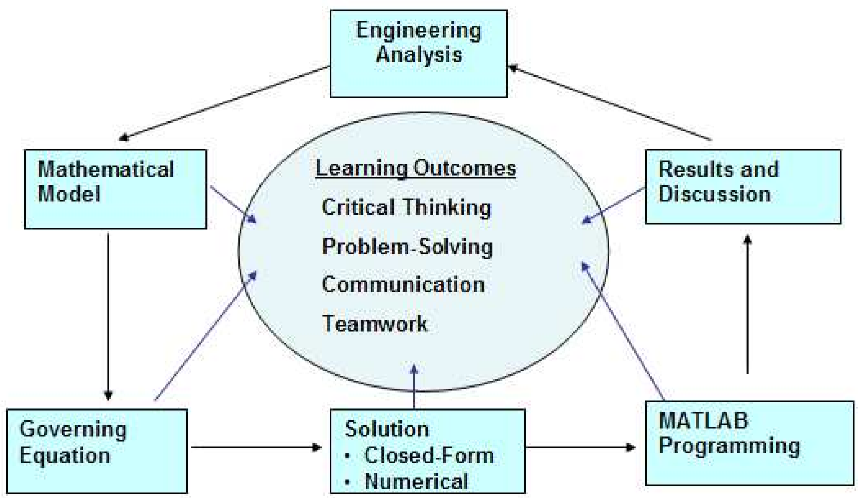

Figure 1: Computational Project-Based Learning Model

The behavior of many engineering systems (deformation, vibration, heat transfer, and others) can be identified by the solution of the governing equation of the body (H. Rahemi, 2007). The focus of this study is given to both closed-form and numerical solution using MATLAB programming. In this investigation, we will analyze the impulsive response of one-story frame structure by both closed-form numerical integration and a mixed (forward-backward) finite difference numerical technique.

3. IMPULSIVE RESPONSE ANALYSIS OF ONE-STORY FRAME STRUCTURE

In this investigation we will study a problem related to the impulsive vibration analysis of a one-story structure with four beam columns (W8x24) supporting structure. For this problem both closed-form numerical integration and numerical finite difference methods can be employed to determine the response of the structure. A MATLAB script can be developed to generate results and identify the vibration behavior of structure under an acting impulsive force as shown in Fig. 2.

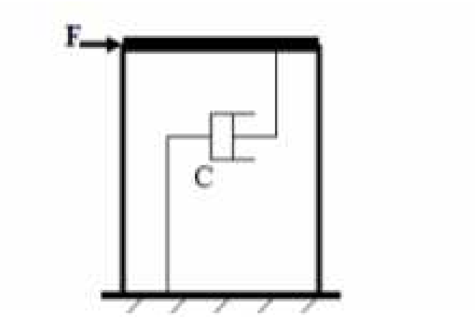

Fig. 2a: One-story frame structure

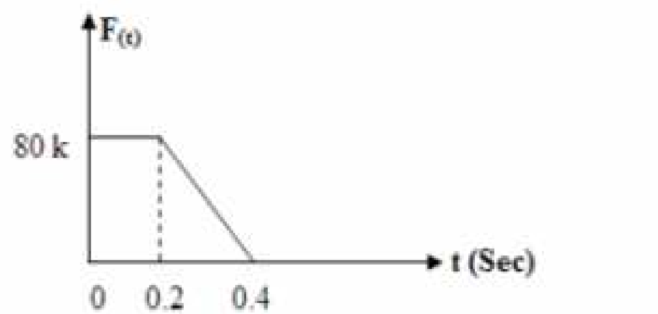

Fig. 2b: Structure Subjected to an impulsive force

Knowing that columns are rigid to the flour slab (Fig. 2C), the total spring rate (K_T) can be expressed as follow

$$K_T = 4K = \frac{48EI}{L^3} \qquad (1)$$

Let's consider W8x24 steel beam for the column With I = 82.8 in^4 and E = 30e6 psi, the spring Rate for four columns can be evaluated as

$$k_T = \frac{4(12)(30e6)(82.8)}{(19x12)^3} \cong 10{,}000\, lb/in$$

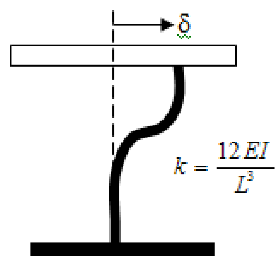

Fig. 2C: The displaced form of the frame structure

The weight of flour slab with all other dead loads can be considered as 38.7 kips, hence, the mass can be calculated as

$$m = \frac{W}{g} = \frac{38{,}700\, lb}{386.4\, in/s^2} = 100\, \frac{lb - s^2}{in}$$

3.1 NUMERICAL INTEGRATION

The impulsive response based on convolution integral (Mario Paz, 1980) for an undamped system can be expressed as

$$x_{(t)} = \frac{1}{m\varpi_n} \int_0^t F_{(\tau)} \sin \varpi_n (t - \tau) d\tau \qquad (2)$$

Above can also be expressed as

$$x_{(t)} = \frac{1}{m\varpi_n} \left\{ \sin(\varpi_n t) \int_0^t F_{(\tau)} \cos(\varpi_n \tau) d\tau - \cos(\varpi_n t) \int_0^t F_{(\tau)} \sin(\varpi_n \tau) d\tau \right\} \qquad (3)$$

The numerical integration of an impulsive response based on a piecewise linear forcing function has been studied by many researchers (Mario Paz, 1980; Singiresu S. Rao, 2003). Here, we will develop a MATLAB script based on such numerical integration to investigate the impulsive response behavior of one-story fame structure. Fig.3, is a representation of a piecewise liner forcing relationship between time t_i and t_{i+1}, that is

$$F_{(\tau)} = F_{(t_i)} + \frac{\Delta F}{\Delta t}(\tau - t_i) \qquad (4)$$

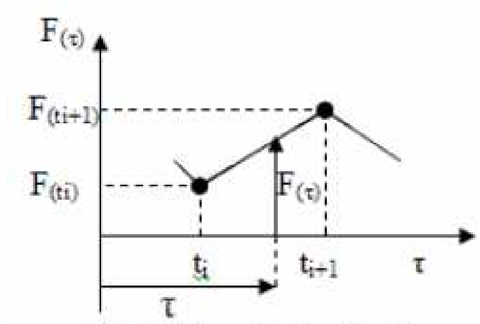

Fig.3. Linear Forcing Function

Hence, the numerical form of response (Mario Paz, 1980) can be expressed as

$$x_{(t_{+1i})} = \frac{1}{m\varpi_n}\left\{\sin(\varpi_n t_{i+1})\ M(t_{i+1}) - \cos(\varpi_n t_{i+1})\ N(t_{i+1})\right\} \qquad (5)$$

where $M(t_{i+1}) = M(t_i) + \int_{t_i}^{t_{i+1}} F_{(\tau)} \cos(\omega_n \tau) d\tau$

$$M(t_{i+1}) = M(t_i) + \left[F(t_i) - t_i \frac{\Delta F}{\Delta t}\right] \frac{[\sin(\varpi_n t_{i+1}) - \sin(\varpi_n t_i)]}{\omega_n}$$

$$+ \frac{\Delta F}{\omega_n^2 \Delta t}\left\{\cos(\omega_n t_{i+1}) - \cos(\omega_n t_i) + \omega_n\left[t_{i+1}\sin(\omega_n t_{i+1}) - t_i \sin(\omega_n t_i)\right]\right\}$$

and $N(t_{i+1}) = N(t_i) + \int_{t_i}^{t_{i+1}} F_{(\tau)} \sin(\omega_n \tau) d\tau$

$$N(t_{i+1}) = N(t_i) + \left[F(t_i) - t_i \frac{\Delta F}{\Delta t}\right] \frac{[\cos(\varpi_n t_i) - \cos(\varpi_n t_{i+1})]}{\omega_n}$$

$$+ \frac{\Delta F}{\omega_n^2 \Delta t}\left\{\sin(\omega_n t_{i+1}) - \sin(\omega_n t_i) - \omega_n\left[t_{i+1}\cos(\omega_n t_{i+1}) - t_i \cos(\omega_n t_i)\right]\right\}$$

A MATLAB script based on above numerical integration is written to generate the numerical response of a system under impulsive force. In this study, the response behavior of a one-story frame structure based on an impulsive forcing function as shown in figure 4 has been investigated.

$$F_{(\tau)} = F_0 \quad for\, TD1 \le \tau \le TD2$$

$$F_{(\tau)} = F_o\left(\frac{TD3 - \tau}{TD3 - TD1}\right) \quad for\, TD2 \le \tau \le TD3$$

$$F_{(\tau)} = 0 \quad for\, \tau > TD3$$

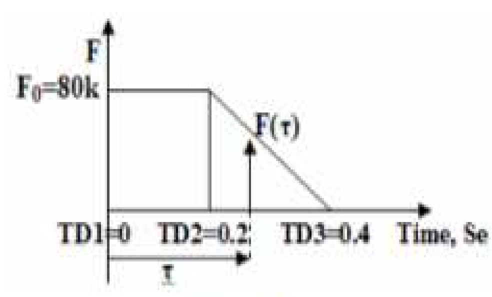

Fig.4: Impulsive Force

MATLAB Program - Numerical Integration

```
% Numerical Analysis of Convolution Integral
% Impulsive Vibration Analysis of an Undamped System
% For an acting rectangular force up to time TD2
% and a Triangular decreasing force up to time TD3
% h must be ≤ TD/10 to generate smooth response graph
ti = input('Initial time ti=');
tf = input('Final time tf=');
TD = input('Time Duration of Impulsive
            force Row Matrix TD=');
h = input('Time step size h=');
m = input('Mass, m =');
k = input ('Spring Cons. k = ');
Fo = input ('Force, Fo =');
wn = (k/m)^.5;
n = 1+((tf-ti)/h);
t=zeros(n,1); x=zeros(n,1);
t(1)=ti; M(1)=0; N(1)=0;
j=1;
while (1)
    t(j+1)=t(j)+h;
    if t(j)<= TD(1,2);
        F(j)=Fo;
        F(j+1)=F(j);
    elseif (t(j)<=TD(1,3) & t(j)>=TD(1,2))
        F(j) =(Fo/(TD(1,3)-TD(1,2)))*(TD(1,3)-t(j));
        F(j+1)=((TD(1,3)-t(j+1))/(TD(1,3)-t(j)))*F(j);
    else
        F(j) =0;
        F(j+1)=F(j);
    end
    M(j+1)=M(j)+(F(j)-t(j)*((F(j+1)-F(j))/h))*(1/wn)*(sin(wn*t(j+1))-
           sin(wn*t(j)))+((F(j+1)-F(j))/((wn^2)*h))*(cos(wn*t(j+1))-
           cos(wn*t(j))+wn*(t(j+1)*sin(wn*t(j+1))-t(j)*sin(wn*t(j))));
    N(j+1)=N(j)+(F(j)-t(j)*((F(j+1)-F(j))/h))*(1/wn)*(cos(wn*t(j))-
           cos(wn*t(j+1)))+((F(j+1)-F(j))/((wn^2)*h))*(sin(wn*t(j+1))-
           sin(wn*t(j))-wn*(t(j+1)*cos(wn*t(j+1))-t(j)*cos(wn*t(j))));
    x(j+1) =(M(j+1)*sin(wn*t(j+1))-N(j+1)*cos(wn*t(j+1)))/(m*wn);
    if t(j+1) >= tf, break, end
    j=j+1;
end
disp ('        Time      Response     ')
disp ([t x ])
plot(t,x)
```

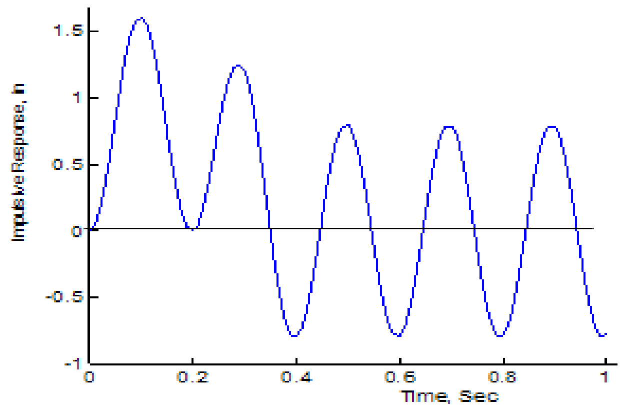

Fig. 4: Response due to impulsive force

The conditional loop can be adjusted to consider the effect of a specific applied impulsive force.

Upon execution the results can be generated as shown

Initial time ti=0
Final time tf=1
Time step size h=0.01
Time Duration of Impulsive force Row Matrix TD=[0 0.2 0.4]
Mass, m =100
Spring Cons. k = 100000
Force, Fo =80000

Table 1: Response by Numerical Integration

Time	Response
0	0
0.1000	1.5998
0.2000	0.0007
0.3000	1.1958
0.4000	-0.7920
0.5000	0.7905
0.6000	-0.7886
0.7000	0.7864
0.8000	-0.7839
0.9000	0.7810
1.0000	-0.7778

3.2 Finite Difference Numerical Analaysis

The governing equation of a system under an external force can be expressed as

$$\frac{d^2x}{dt^2}+\frac{c}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{1}{m}F_{(t)} \qquad (6)$$

For an acting impulsive force with various force functions, it is not feasible to determine the analytical solution. However, a solution based on finite difference techniques can be implemented with MATLAB to determine response of the system during and after acting impulsive force.

Above equation can also be expressed as

$$\frac{dV}{dt}+\frac{c}{m}V+\frac{K}{m}x=\frac{1}{m}F_{(t)} \qquad (7)$$

Using forward divided difference, first derivative can be approximated as

$$\frac{dV}{dt}=\frac{V(t_{i+1})-V(t_i)}{h}\pm O(h) \qquad (8)$$

The first forward difference of the governing equation will provide velocity at time t_{i+1}, that is

$$V_{(t_{i+1})}=\left[\frac{1}{m}F_{(t_i)}-\frac{c}{m}V_{(t_i)}-\frac{k}{m}x_{(t_i)}\right]h \qquad (9)$$

The first backward difference of above equation will provide response at time t_{i+1}, that is

$$x_{(t_{i+1})}=\left[\frac{1}{m}F_{(t_i)}-\frac{c}{m}V_{(t_i)}-\frac{k}{m}x_{(t_i)}\right]h^2+x_{(t_i)} \qquad (10)$$

A general MATLAB program based on mixed difference (forward-backward) approximation of above finite difference numerical formulations is developed to identify the vibration behavior of a damped and undamped structure under an impulsive acting force.

MATLAB Program – Mixed (Forward-Backward) Finite Difference Technique

```
% Impulsive Vibration, Mix Finite Difference
% h must be very small in comparison to TD (h«TD/10)
ti = input('Initial ti:'); tf = input('final tf:');
h = input('step size h:'); xi = input('Initial xi:');
m = input('mass, m =');
c = input ('Damping Coef., c = ');
k = input ('Spring Cons. k = ');
Fo = input ('Force ampl., Fo =');
vi = input('Initial vi:');
TD = input('Time Duration of Impulsive force Row Matrix TD=');
n = 1+((tf-ti)/h);
t=zeros(n,1); x=zeros(n,1); v=zeros(n,1);
t(1) = ti; x(1) = xi; v(1) = vi;
j=1;
```

```
while (1)
    if t(j)<=TD(1,2);
        F(j)=Fo;
    elseif (t(j)<=TD(1,3) & t(j)>TD(1,2));
        F(j)=((TD(1,3)-t(j))/(TD(1,3)-TD(1,2)))*Fo;
    else
        F(j)=0;
    end
    x(j+1) =((1/m)*F(j)- (k/m)*x(j)-(c/m)*v(j))*h^2 + v(j)*h + x(j);
    v(j+1) = ((1/m)*F(j)-(k/m)*x(j)-(c/m)*v(j))*h + v(j);
    t(j+1) = t(j)+h;
    if t(j+1) >= tf, break, end
    j=j+1;
end
disp ('        Time      Response  velocity  ')
disp ([t x v ])
plot(t,x)
```

Upon execution the results can be generated as shown

initial ti:0
final tf:1
step size h:0.002
Initial xi:0
mass, m =100
Damping Coef., c = 0
Spring Cons. k = 100000
Force ampl, Fo =80000
Initial vi:0
Time Duration of Impulsive force Row Matrix TD=[0 0.2 0.4]

Table 2: Undamped Response by finite difference technique

Time	Response	Velocity
0	0	0
0.1000	1.5993	-0.5369
0.2000	0.0018	1.0735
0.3000	1.1941	-9.6114
0.4000	-0.7896	2.1468
0.5000	0.7875	-2.6776
0.6000	-0.7850	3.2073
0.7000	0.7821	-3.7355
0.8000	-0.7789	4.2620
0.9000	0.7753	-4.7866
1.0000	-0.7714	5.3090

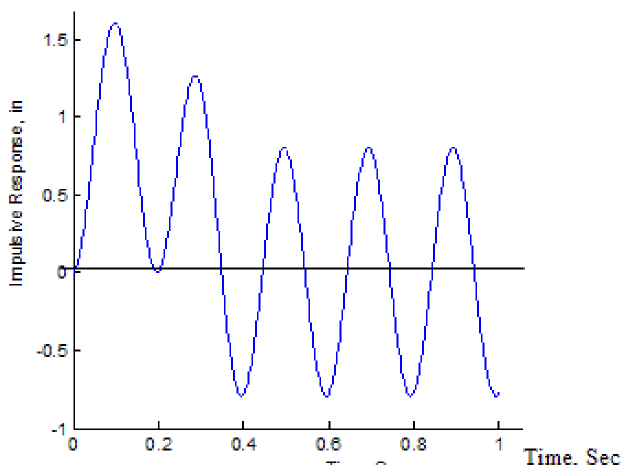

Fig. 5: Response due to impulsive force
Based on finite difference technique

For a time step size, h =TD/100, the results by finite difference technique (Table 2) in comparison to the results of closed-form numerical integration (Table 1) produce an error of ± 1% (error in order of step size).

By considering the air damping effect, C= 50 lb-sec/in, upon execution of MATLAB program with the entree of the following input data through command window, the impulsive vibration behavior can be identified for a damped one-story frame structure, that is

Initial ti:0
final tf:4
step size h:0.002
Initial xi:0
mass, m =100
Damping Coef., c = 50
Spring Cons. k = 100000
Force ampl, Fo =80000
Initial vi:0
Time Duration of Impulsive force Row Matrix TD=[0 0.2 0.4]

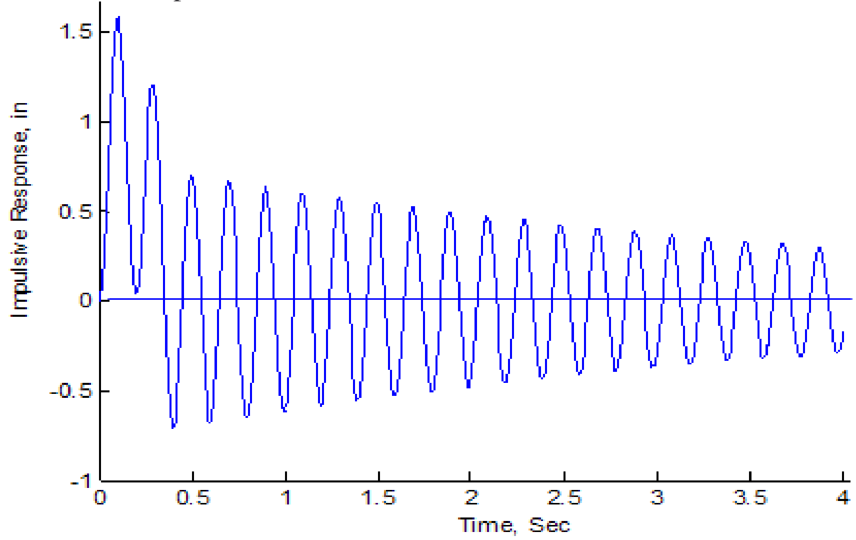

Fig.6: Damped response due to impulsive force

Table 3: Damped Response b finite difference technique

Time	Response	velocity
0	0	0
0.1000	1.5796	-0.5408
0.2000	0.0406	1.0547
0.3000	1.1407	-9.4460
0.4000	-0.7142	1.8123
0.5000	0.6948	-2.2509
0.6000	-0.6756	2.6656
0.7000	0.6566	-3.0572
0.8000	-0.6378	3.4264
0.9000	0.6193	-3.7739
1.0000	-0.6010	4.1004
1.5000	0.5133	-5.4406
2.0000	-0.4325	6.3515
2.5000	0.3590	-6.9059
3.0000	-0.2930	7.1692
3.5000	0.2345	-7.1992
4.0000	-0.1832	7.0470

In this case due to damping effect, the impulsive response eventually dies out after several vibrating cycles.

4. Conclusion

As it can be observed, the numerical mix finite difference technique with a time step size, h ≤ TD/100, provide results that are closely compatible with closed-form numerical integration approach. Development of finite difference numerical technique for impulsive response analysis is much less time consuming without any complex mathematical implication.

The MATLAB programming not only facilitates the computational process but also enable us to understand the behavior of an engineering system based on a given mathematical model. In this investigation the MATLAB programming enabled us to understand graphically the behavior of an impulsive response based on both the closed-form numerical integration and finite difference numerical approach. The results showed that the accuracy in finite difference numerical analysis can be achieved by selecting a proper step size, which is h ≤ (time duration of acting force)/100.

The computational project-based learning model with MATLAB programming delivered learning objectives that are required to motivate student's mathematical, critical thinking, and problem solving skills. It can be observed that MATLAB programming enable both researchers and students with a tool to understand different mathematical techniques in analyzing and studying behavior of an engineering system.

The presented analysis is based on an acting impulsive force on the structure. However, due to seismic ground excitation, a future study will consider an impulsive behavior of one-story frame structure based on earthquake ground excitation.

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