Seventh LACCEI Latin American and Caribbean Conference for Engineering and Technology (LACCEI'2009)
"Energy and Technology for the Americas: Education, Innovation, Technology and Practice"
June 2-5, 2009, San Cristóbal, Venezuela.

A New Wind Power Harvesting Interpolation Technique

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ABSTRACT

In today's world economy the new big player is Energy. The world addiction to it is having an incredible impact all over the globe. How much is being used, how much is left, and the consequences of its use are every day questions. A healthy economy certainly relies not only on the abundance of resources, including energy ones, but also on their quality for reduced environmental impact, cleanness, affordability, and sustainability. Much can be said about these, and many are the examples, but they all converge to the same conclusion: the world must stop its tremendous dependence on energy and turn its attention to renewable energy sources, as is Wind Power. This paper looks into historical wind trends for a given period and, through the use of a new mathematical interpolation technique based on current data, attempts to generate a wind speed curve for the next cycle period. The algorithm is based on an expansion around a minimum wind power point in the catchment regions of the cycle considered, through a Fourier series. Future wind speed values depend on some variables and past wind speed figures which are obtained from a meteorological station. The purpose of the research conducted is to have an insight into when high winds will occur, before storms even start to form. The usefulness is mainly to provide hurricane alerts, better use of wind resources for wind harvesting, and for power generation through the use of a renewable energy source as is Wind Power. This will allow having a better understanding of wind patterns and select dates for high wind speeds and power generation. In this work we present some preliminary results of the interpolation technique to predict wind power in Alachua County Florida. The results are very encouraging as the overall deviation is very small. Some comparison with current techniques, and future work are discussed.

Keywords
Wind Power, Energy Forecasting, Renewable Energy.

1. INTRODUCTION

An interpolation algorithm is created to predict the wind speed in miles per hour (mph) at a given location based on historical seasonal-wind trends and current wind data. Using historical data for wind speed, the algorithm which is a truncated version of the Fourier series is constructed to simulate a wind curve for a given period using only four data points, two in each catchment region.

For the study, data was collected about 4 times every 24 hours from a 10 meter high ultra sonic wind sensor in Alachua county Florida as displayed in Figure 1. It was found that for Alachua County there was a seasonal trend every 12 months (period), with traditionally high winds in the months of March, April and May, and historical high hurricane winds in the months of September and October, and low winds during the months of November and December.

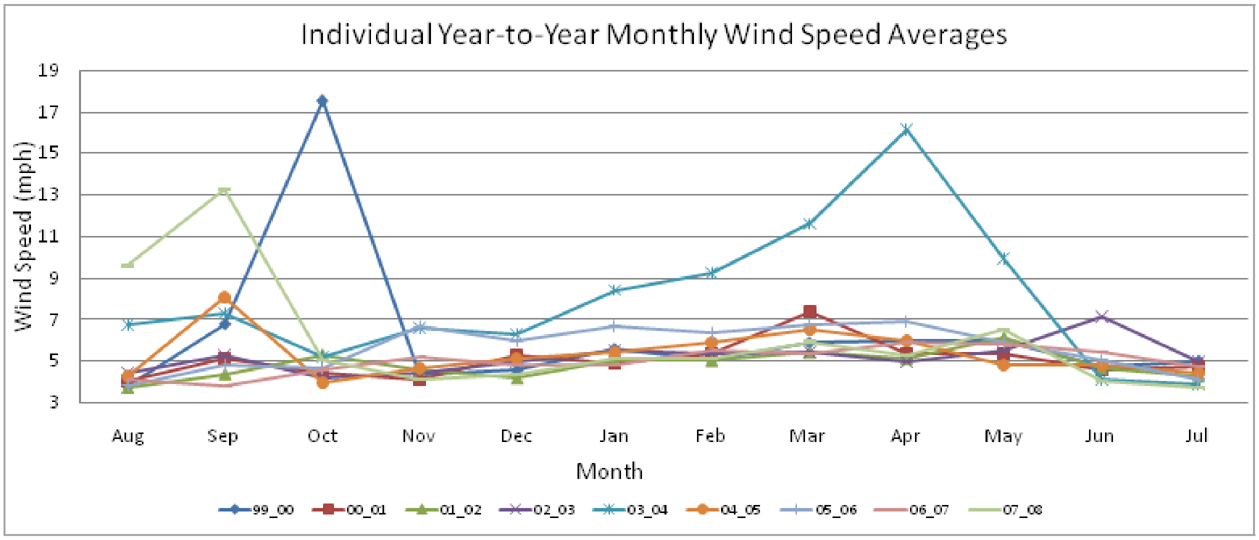

Figure 1: Annual wind speeds from 1999 to 2008 clearly depicting the higher wind speed months of March, April and May and the hurricane months of September and October

2. WIND SPEED MEASUREMENT

Instantaneous wind speed V can be described as a mean wind speed V_m plus a fluctuating wind component v [1] :

$$V = V_m + v \quad (1)$$

The mean wind speed V_m is typically determined as a 10 minute average value. The fluctuation of the flow is expressed in terms of the root mean square RMS value of the fluctuating velocity component $\left(\sqrt{\overline{v^2}}\right)$ and is defined as the turbulence intensity (T_u) given by:

$$T_u = \frac{\sqrt{\overline{v^2}}}{V_m} = \frac{1}{V_m}\left[\frac{1}{T}\int_0^T v^2 dt\right]^{\frac{1}{2}} \quad (2)$$

For very rough terrain (trees and buildings dense areas) the (T_u) intensity is in the range 0.15 and 0.2. For smooth terrain the intensity is typically 0.1. The wind power at different heights is obtained through the Power exponent function $V(z)$:

$$V(z) = V_r\left(\frac{z}{z_r}\right)^{\alpha} \quad (3)$$

Where z is the height above ground level, V_r is the wind speed at the reference height z_r above ground level, (V_z) is the wind speed at height z, and α is an exponent that depends on the roughness of the terrain.

3. WIND TURBINE BENEFITS

The power in the wind is proportional to the cube of the wind speed or velocity. It is therefore essential to have detailed knowledge of the wind and its characteristics if the performance of wind turbines is to be estimated accurately.

Data gathered from the Florida Automated Weather Network database[2]. In this study we use historical data from August 1999 to July 2008, and a nalyzed average monthly wind trends. The results (shown in Figure 2) indicated that there was a seasonal trend every 12 months (period), with traditionally high winds in the months of March, April and May, and historical high hurricane winds in the months of September and October, and low winds during the months of November and December. It is important to note that for those years when there were no hurricanes, the months of September and October displayed average wind speeds as low as those months of November and December. The database with information on wind speed is measured in mph at an altitude of 10 meter.

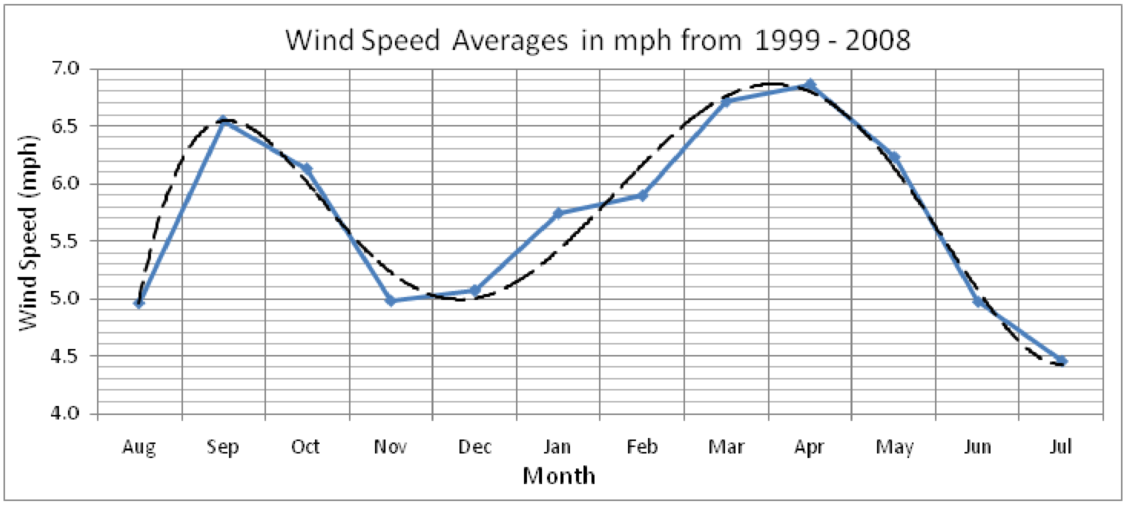

Figure 2: Average wind speed (solid) in mph and the trend line (dashed) where the equation of the line is $y = -0.00005x^6 + 0.0038x^5 - 0.0931x^4 + 1.0113x^3 - 5.1143x^2 + 11.119x - 1.954$ **and the corresponding** R^2 **value of 0.9605**

4. THE FOURIER SERIES

In general terms the Fourier series takes oscillating functions and dissects them into simpler periodic functions of mainly sines and cosines. The Fourier series was chosen because of the sinusoidal periodicity of the phenomena under study. Since the phenomena under study is symmetrical, it repeats itself without knowing directions, we consider here only the even function of the series. This is:

$$f(\alpha) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left[\frac{n\pi\alpha}{T}\right] \tag{4}$$

Where,

$$a_0 = \frac{2}{T}\int_0^T \left[f(\alpha)\cos\left[\frac{n\pi\alpha}{T}\right]\right] d\alpha \tag{5}$$

Using the Fourier Series from Equation (5), regression gives the values displayed in Figure 3, and the function expression shown in Figure 4 from equation (4).

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{2\pi x}{12}\right)dx=4.8929\times10^{-2}$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{3\pi x}{12}\right)dx=-0.22739$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{4\pi x}{12}\right)dx=-0.97694$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{5\pi x}{12}\right)dx=-0.76089$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{6\pi x}{12}\right)dx=-0.56354$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{7\pi x}{12}\right)dx=-0.49267$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{8\pi x}{12}\right)dx=-0.34445$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{9\pi x}{12}\right)dx=-0.32473$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{10\pi x}{12}\right)dx=-0.2288$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{11\pi x}{12}\right)dx=-0.22652$$

$$\frac{2}{12}\int_0^{12}\left(-.00005x^6+0.0038x^5-0.0931x^4+1.0113x^3-5.1143x^2+11.119x-1.954\right)\cos\left(\frac{12\pi x}{12}\right)dx=-0.16208$$

Figure 3: The values of for $\alpha_0$using equation (5)

$$y=\frac{12.632}{2}-1.8783\cos\left(\frac{\pi x}{12}\right)+4.8929\times10^{-2}\cos\left(\frac{2\pi x}{12}\right)-0.22739\cos\left(\frac{3\pi x}{12}\right)-0.34445\cos\left(\frac{8\pi x}{12}\right)-0.32473\cos\left(\frac{9\pi x}{12}\right)-0.97694\cos\left(\frac{4\pi x}{12}\right)-0.76089\cos\left(\frac{5\pi x}{12}\right)-0.56354\cos\left(\frac{6\pi x}{12}\right)-0.49267\cos\left(\frac{7\pi x}{12}\right)-0.2288\cos\left(\frac{10\pi x}{12}\right)-0.22652\cos\left(\frac{11\pi x}{12}\right)-0.16208\cos\left(\frac{12\pi x}{12}\right)$$

Figure 4: The results from Figure 3 used as the inputs for $F(\alpha)$ in equation (5)

The results from equation (4) of $F(\alpha)$ are plotted and shown in Figure 5a. Figure 5b shows the graph for the actual regression line.

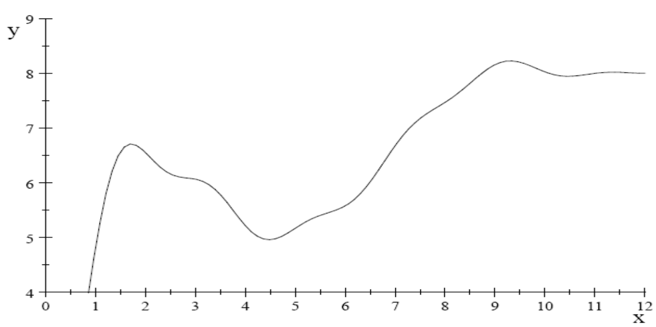

Figure 5a: Using Fourier series approximation

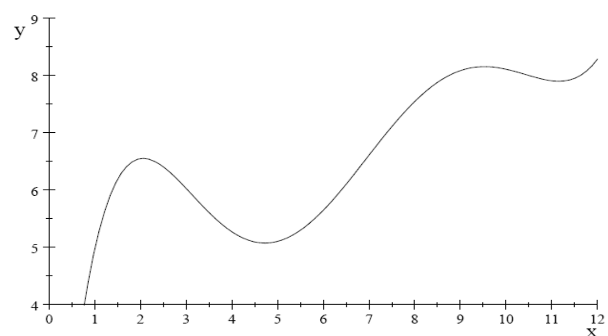

Figure 5b: Using actual regression from Figure 2.

The series does reproduce the function as fitted, requiring a great high input data. The algorithm, described in the next section, requires only four points to reproduce the data, providing additional insight on the phenomena.

5. PROPOSED ALGORITHM

We will use a truncated Fourier Series to represent an expression for the Wind Force (F) around a given point in time (α) . The symmetry of the system is considered as of location and periodicity. The period will be considered as half interval [0, α]. We will expand F around an initial point F_0, in terms of the even function of the Fourier Series. Since the space is symmetric $F(\alpha) = F(-\alpha)$. We consider a system of periodicity , that has a symmetry

$b = \frac{\pi}{p}$. For convenience we use $\frac{\pi}{b}$, where: $b = \frac{E}{2}$ The general concept for building the algorithm is shown in Figure 6.

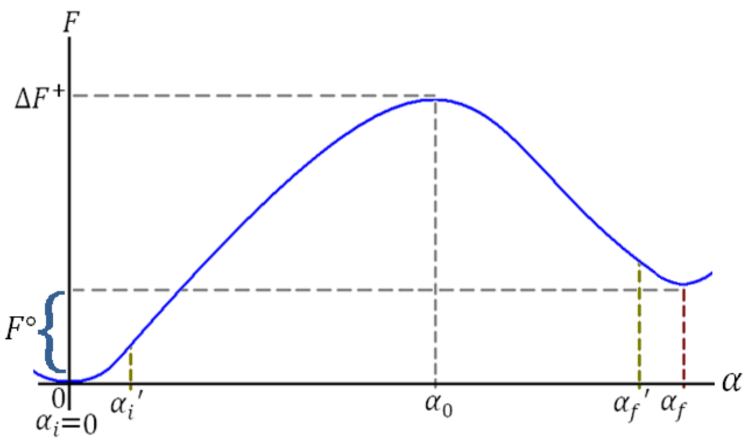

Figure 6: The general algorithm setup which will be used when analyzing wind speed data

With these considerations, we will write a general expression of the wind power F as a function of the symmetry parameter b and the time coordinated α, as follows:

$$F(b\alpha) = F_0 + \sum_{k=1}^{3} F_k \cos[kb\alpha] \tag{6}$$

Where F_0 is the independent term and the coefficients $F_k$are to be evaluated. Truncated to $k = 3$, because of the four available points (2 in each minima), we derive expressions for the first and second derivatives. This is:

$$F(b\alpha) = \frac{\partial F(b\alpha)}{\partial \alpha} = -b^2 \sum_{k=1}^{3} kF_k \sin[kb\alpha] \tag{7}$$

and,

$$K(b\alpha) = \frac{\partial^2 F(b\alpha)}{\partial \alpha^2} = -b^2 \sum_{k=1}^{3} k^2 F_k \cos[kb\alpha] \tag{8}$$

$K(b\alpha)$ can be associated with the local constraints of the wind power phenomenon. We shall return to this issue later on. To correlate the position α, first and second derivatives must be obtained. Let's now consider a Taylor Series expansion, truncated to the second order, of the Force of Wind around a given point α_j:

$$F(b\alpha) = F(b\alpha_j) + F_j(\alpha - \alpha_j) + \frac{1}{2} K_j(\alpha - \alpha_j)^2 \tag{9}$$

Here, α represents the point close to α_j and the subscript refer to the initial and final states $(j = i, f)$. These points, as well as at the minimum and maximum, $f(\alpha) = 0$ and, consequently, will give the harmonic approximation.

$$K_j = 2\left[\frac{F(b\alpha) - F(b\alpha_j)}{(\alpha - \alpha_j)^2}\right] \tag{10}$$

6. THE STEP

For simplicity, we conveniently define a step, (in time-degrees), as follows:

$\alpha_i{}' = \left(\frac{10^{\circ}}{b}\right) - \alpha_i$ (11a) and $\alpha_f{}' = \alpha_f - \left(\frac{10^{\circ}}{b}\right)$ (11b)

It is important to note that we have assumed a fixed step of $10°$ (or about 20 days). This is of crucial importance for the algorithm since it will indicate the performance as a function of the data. Let's now consider the wind power difference between the initial and final states $(\Delta F°)$:

$$\Delta F° = F(b\alpha_f) - F(b\alpha_i) \tag{12}$$

or,

$$\Delta F° = \sum_{k=1}^{3} F_k[\cos(kb\alpha_f) - \cos(kb\alpha_i)] \tag{13}$$

In the reference state (initial state), the following holds: $\alpha_i = 0°$ and $F(\alpha_i) = 0$. Therefore we now write for $\Delta F°$:

$$\Delta F° = -\sum_{k=1} F_k[1 - \cos(kb\alpha_f)] \tag{14}$$

A close examination of the values of $1 - \cos(kb\alpha_f)$ needs to be made in order to determine which ones will survive, as a function of b and k. We observe that the coefficient F_2 vanishes, and that only odd terms survive, for all values of b, giving rise to the following expression for $\Delta F°$:

$$\Delta F° = -2(F_1 - F_3) \tag{15}$$

Since $\Delta F°$ is a constant, we can write for F_1 $(or\ F_3)$:

$$F_1 = -F_3 - \frac{1}{2}\Delta F° \tag{16}$$

At this point, to find the expressions to evaluate the Fourier Series coefficients, we turn to the second derivative expressions. We must note, however, that there are constants $(K_i \text{ and } K_f)$ determined from the Taylor Series:

$$K_i = K(b\alpha_i) = -b^2 \sum_{k=1} k^2 F_k \cos(kb\alpha_i) \tag{17}$$

$$K_f = K(b\alpha_f) = -b^2 \sum_{k=1} k^2 F_k \cos(kb\alpha_f) \tag{18}$$

Lets now consider both the sum and the difference of K_i and K_f. The expression for the sum is as follows:

$$-\left[\frac{K_i + K_f}{b^2}\right] = \sum_{k=1} k^2 F_k[1 + \cos(kb\alpha_f)] \tag{19}$$

Which coefficients will survive? The answer is F_2, given the following expression for all values of b:

$$F_2 = \frac{-(K_i + K_f)}{8b^2} \tag{20}$$

From here we find F_3, using equation (9):

$$\frac{-(K_i - K_f + \Delta F°)}{b^2} = 3F_2[1 - \cos(2b\alpha_f)] + 8F_3[1 - \cos(3b\alpha_f)] \tag{21}$$

A close examination of F_2 and F_3 indicates that only the odd coefficients survive, therefore:

$$F_3 = \frac{-[K_i - K_f - b^2\Delta F°]}{16b^2} \tag{22}$$

Using the expression to find F_1 as written in equation (10), we get:

$$F_1 = \frac{[W_i - W_f - 9b^n \Delta F^o]}{16b^2} \tag{23}$$

Finally, and for the reference state $F(0^o)$ we write:

$$F(b\alpha_t) = F(0^o) = 0 = F_0 + F_1 + F_2 + F_3 \tag{24}$$

Or in a general form:

$$F_0 = -\sum_{k=1}^{3} F_k \tag{25}$$

Finally, from equation (2) we estimate the time/date (α_0) at which the strength of the wind will reach the peak.

$$F_1 \sin(b\alpha_0) + 2F_2 \sin(2b\alpha_0) + 3F_3 \sin(3b\alpha_0) = 0 \tag{26}$$

From here, and after some algebra we find a relation for α_0:

$$\alpha_o \;=\; b \cos \{(F_2 \pm ½ \sqrt{[F_2 + 3F_3(4F_2+3\Delta F^o)]})/6F_3\} \tag{27}$$

7. PRELIMINARY RESULTS

To test the algorithm, the Alachua County wind data is used to conduct the analysis. The average wind speed curve is considered, and is divided into two distinct curves. Each of these curves has a different p-value and thus a different b-value. These values were calculated as fractions of π, which correspond to the period of the cosine function and to a 12 month cycle. These 180° were divided into two parts, the first part encompasses the months from August to December while the second one the months from December to July as shown in Figure 7.

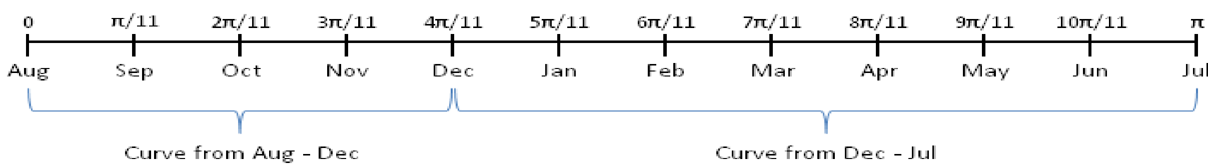

Figure 7: The divided periods

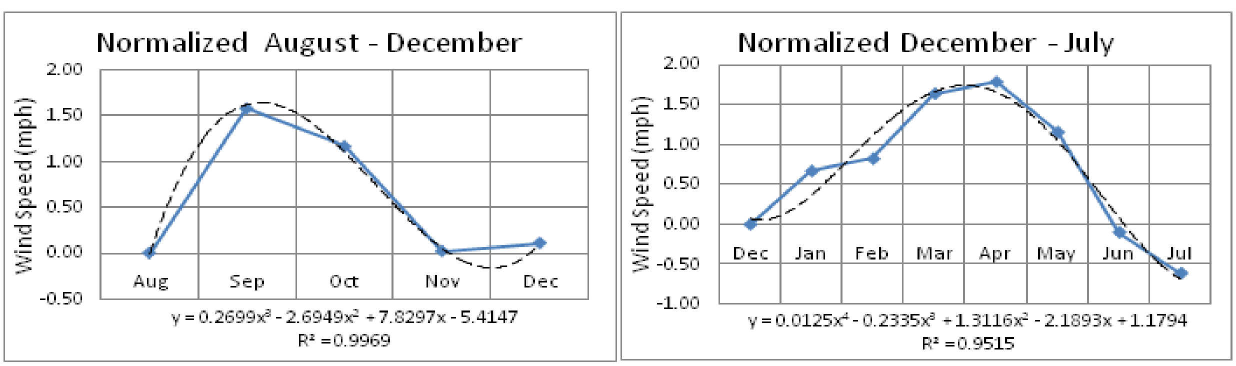

Figure 8: The initial curve being broken up into two individual curves with their respective trend line (dashed) and the R^2values, for the months of August – December(a), and December – July (b)

The curve from August to December has a periodicity $p = \frac{4\pi}{11}$ radians and a symmetry parameter $b = \frac{11}{4}$, which corresponds to 65.45°. Similarly, the curve from December to July has a periodicity $p = \frac{7\pi}{11}$ and a symmetry parameter $b = \frac{11}{7}$, which corresponds to 114.54°. Finally, the data for the curves is normalized so that the starting points are (0,0) for the (x,y) coordinates in both curves, as shown in Figure 8a and 8b. With the values of p and b we calculate the values of K_i, K_f and ΔF^o that in turn allows us to calculate F_1, F_2, F_3 and F_0 as shown below.

8. PRELIMINARY RESULTS

a) Period of August – December: Input and out data are shown in Table 1

Table 1: Input data for the initial and final points, and output data showing the second derivatives, Fourier series coefficients and the wind speed difference between extreme points. Period is August – December.

Initial Point	α_i	X(0,0)	$F(\alpha_i)$	α'_i	X(0,0)	$F(\alpha'_i)$
Deg	0.0	0.00	-0.0100	10.00	0.22	0.6203
Rad	0.0			0.1745		

Final Point	α_f	X(0,0)	$F(\alpha_f)$	α'_f	X(0,0)	$F(\alpha'_f)$
Deg	57.1	3.51	-0.1583	53.5	3.29	-0.1138

Output Data

K_i = 41.3865	K_f = 2.9233	ΔF^o = -0.1483	
F_o = 0.6582	F_1 = 0.4013	F_2 = -0.7324	F_3 = -0.3271

*Aug - Dec [Step size of 5 degrees F(x) vs. F(beta*alpha)]*

x (0,0)	x	Alpha	*F(x)*	*F(beta*alpha)*	Deviation
0.00	1.00	0	-0.01	0.00	1.00
0.31	1.31	5	0.8193	0.1524	0.8139
0.62	1.62	10	1.3387	0.5514	0.5881
0.92	1.92	15	1.5956	1.0461	0.3444
1.23	2.23	20	1.6370	1.4549	0.1112
1.54	2.54	25	1.5102	1.6371	-0.0840
1.85	2.85	30	1.2622	1.5432	-0.2226
2.15	3.15	35	0.9404	1.2244	-0.3020
2.46	3.46	40	0.5918	0.7987	-0.3496
2.77	3.77	45	0.2636	0.3947	-0.4970
3.08	4.08	50	0.0031	0.0994	-31.2380
3.38	4.38	55	-0.1427	-0.0657	0.5393
3.69	4.69	60	-0.1265	-0.1324	-0.0463
4.00	5.00	65	0.0988	-0.1483	2.5006

Table 2: Abbreviated version (step size of $\alpha = 5^o$) of the actual table used for Figure 9 where a step size of $\alpha = 0.1^o$ is used. The values of equation (6) as F(bα) are shown.

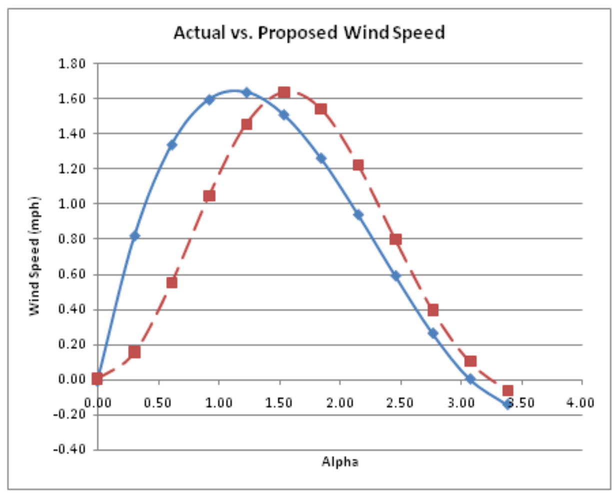

Figure 9: Actual (solid) and proposed (dashed) points from August to December

The Calculated Points and Deviation are shown in Figure 9. We show the original curve for the wind data function and those values obtained through the algorithm.

b) Period of December – July: Input and out data are shown in Table 3

Table 3: Input data for the initial and final points, and output data showing the second derivatives, Fourier series coefficients and the wind speed difference between extreme points. For the period December – July.

Initial Point	α_i	X(0,0)	$F(\alpha_i)$	α'_i	X(0,0)	$F(\alpha'_i)$
Deg	0.0	0.00	0.0807	9.00	0.39	0.0899
Rad	0.0			0.1571		

Final Point	α_f	X(0,0)	$F(\alpha_f)$	α'_f	X(0,0)	$F(\alpha'_f)$
Deg	115	7.00	-0.7446	108.6	6.61	-0.5074

Output Data

K_i = 0.7444 K_f = 19.2286 ΔF^o = -0.8253

F_o = 0.5984 F_1 = -0.4163 F_2 = -1.0110 F_3 = 0.4163

*Dec - Jul [Step size of 5 degrees F(x) vs. F(beta*alpha)]*

x (0,0)	x	Alpha	F(x)	F(beta*alpha)	Deviation
0.00	1.00	0	0.08	0.00	1.00
0.30	1.30	5	0.0733	0.0031	0.9578
0.61	1.61	10	0.1634	0.0153	0.9061
0.91	1.91	15	0.3239	0.0449	0.8614
1.22	2.22	20	0.5301	0.1030	0.8058
1.52	2.52	25	0.7602	0.2008	0.7359
1.83	2.83	30	0.9945	0.3460	0.6521
2.13	3.13	35	1.2163	0.5400	0.5560
2.43	3.43	40	1.4112	0.7753	0.4506
2.74	3.74	45	1.5675	1.0352	0.3396
3.04	4.04	50	1.6760	1.2948	0.2274
3.35	4.35	55	1.7302	1.5239	0.1192
3.65	4.65	60	1.7260	1.6910	0.0203
3.96	4.96	65	1.6620	1.7679	-0.0637
4.26	5.26	70	1.5393	1.7346	-0.1268
4.56	5.56	75	1.3617	1.5832	-0.1627
4.87	5.87	80	1.1354	1.3202	-0.1628
5.17	6.17	85	0.8692	0.9668	-0.1122
5.48	6.48	90	0.5747	0.5568	0.0311
5.78	6.78	95	0.2657	0.1335	0.4975
6.09	7.09	100	-0.0411	-0.2563	-5.2326
6.39	7.39	105	-0.3266	-0.5681	-0.7391
6.70	7.70	110	-0.5691	-0.7653	-0.3447
7.00	8.00	115	-0.7443	-0.8247	-0.1080

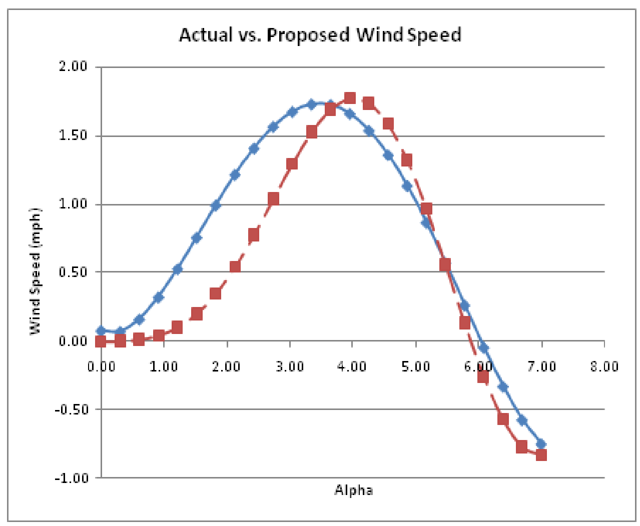

Table 4: Input data for (initial step of $\alpha = 5°$) for Figure 10. Sstep size of $\alpha = 0.1°$ is used. The values of equation (6) as F(bα).

Figure 10: Actual (solid) and proposed (dashed) points from December to July

9. CONCLUSIONS

We have presented an interpolation technique to represent, with minimal data, the wind speed in a given region for a 12 month period. The algorithm, although it has some minimal deviation with measured data, it represents the wind speed in the periods considered. The algorithm also provides additional insight on the potential strength of the winds. Furthermore, it gives an estimate of the location of the potential maxima given with small deviation from the real data. Second derivatives for the initial and final states, the coefficients and difference on wind speed from those states, can accurately be associated to the potential strength (speed) for the coming winds. The coefficients can be associated to the turbulence intensity (T_u) from equations (1), (2) and (3) and similarly to the location of the maximum strength of the winds (α_0). Although there is a deviation with respect to measurements, we believe results are encouraging. Our future work considers a better understanding of the algorithm and its variables.

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